AM205: Fluid instability in coffee (due 5PM, November 29)

Please submit a PDF writeup. Please also list the names of the students in the group.

1. **Complete the derivation of the onset of Rayleigh–Bénard instability.** Follow the first 5 steps in the note “Derive the onset of instability for Rayleigh–Bénard convection”.

   (a) Perform nondimensionalization on Eq.(1), and prove the nondimensionalized governing equations for mass and energy in “Step 3” are:

   
   \[
   \begin{align*}
   & \text{mass: } \nabla \cdot \mathbf{u} = 0 \\
   & \text{energy: } \partial_t T + (\mathbf{u} \cdot \nabla) T = \nabla^2 T
   \end{align*}
   \]

   \( (1) \)

   (b) Linearize the nondimensionalized governing equations about the steady state solution. Complete the linearization calculations in “Step 5” on the linearized momentum \( \hat{\mathbf{y}} \), energy and mass equations. Prove that:

   \[
   \begin{align*}
   & \text{momentum } \hat{\mathbf{y}} : \frac{1}{Pr \bar{T}} \partial_t \hat{\mathbf{v}} = -\partial_y \hat{p} + \nabla^2 \hat{v} - Ra \hat{T} g \\
   & \text{energy : } \partial_t \hat{T} + \hat{v} \partial_y T^{SS} = \nabla^2 \hat{T} \\
   & \text{mass : } \partial_x \hat{u} + \partial_y \hat{v} = 0
   \end{align*}
   \]

   \( (2) \)

2. **Observe fluid instability in coffee.** Two fluid instability concepts are involved when we make coffee: boiling water in a kettle is an example of the Rayleigh–Bénard instability, and the Rayleigh–Taylor instability leads to the mesmerizing fluid motion when adding cream into the coffee. We will do two small hands-on activities to observe them in action.

   (a) Make an estimate about the Rayleigh number \( Ra \) of heating water. You could use any container size and look up the parameters like viscosity online. Make reasonable guesses on the temperature difference. If you have access to a stove, you are encouraged to heat water and observe the process. Please be careful if you are heating water or measuring the temperature difference! Explain your result. How is this heating-water setup different from the Rayleigh–Bénard convection setup in the workshop? If you can observe the water while it is heating up, what does the convection pattern look like from the top? How does it change with respect to the temperature difference, i.e. the Rayleigh number? You are also welcome to include photos of patterns in the discussion.

   (b) Adding cream or milk in coffee involves Rayleigh–Taylor instability, which is an instability of the interface between two fluids of different densities, e.g. heavier fluid over lighter. Watch this short video on Rayleigh–Taylor instability from FYFD and take some photos (or slow-motion video) of adding cream (or milk) into coffee (or tea). Have fun with filming the process and explain your photos with the concepts mentioned in the video. Discuss the shape of the interface. How long does it take for the cream to be well-mixed with coffee? Does the velocity of adding cream make any difference in the instability?