AM205: Dynamic mode decomposition (due 5PM, October 12)

Please submit a PDF export of the Jupyter notebook and the notebook itself (do NOT zip them). You can write texts and \LaTeX{} in Jupyter notebook directly. Please also list the names of the students in the group.

1. **Write your own DMD function (in-workshop activity).** Complete Part I (discussion) and II (coding) in the Jupyter notebook to implement a DMD function.

2. **Apply DMD on vortex shedding.** Flow past a cylinder is a classical benchmark problem in fluid dynamics. The setup is simple: a cylinder is placed at the centerline of a channel, a unidirectional and uniform flow is passing through it. Under proper assumptions and simplifications, this problem can be solved analytically, which gives rise to the rich study in topics like potential flow, boundary layer theory, and lift and drag.

However, this simple setup can also lead to a complex fluid motion, called vortex shedding. *When a certain condition is met*, the flow will periodically create vortex pairs behind the cylinder, a pattern named after the famous fluid dynamicist Theodore von Kármán, “Kármán vortex street”\(^1\). Partially because the motion of vortex street is just simply mesmerizing to watch\(^2\), many computational fluid dynamics algorithms like to simulate flow past a cylinder, also because this problem has been well-studied and easier to make comparisons.

Yue is doing research on fluid–structure interaction, and no surprise she has extended her lattice Boltzmann fluid solver to simulate vortex shedding under 30 different settings. For this workshop, she selected 4 datasets from the 30 runs and uploaded them to the drive; but she did not name the folders according to the respective setting/parameter. In this problem, we will infer the parameter she tuned in each simulation with dynamic mode decomposition.

(a) Complete the functions to load and construct data matrices based on the vorticity field.

(b) With the given simulation dataset a, use your DMD function from QI to extract dominant spatial–temporal coherent structures from the vortex shedding simulation.

(c) Plot the first 20 eigenvalues from Part (b) on a complex unit circle and their corresponding eigenvectors (recall that each eigenvector is a column vector of the domain). Discuss your result. (e.g. Which modes are decaying/constant? Which mode is the most dominant?)

(d) Reconstruct the future vorticity field with the eigenvalues and eigenvectors from Part (c).

(e) As mentioned, vortex shedding only happens when a certain condition is met. This condition is controlled by a dimensionless number, the **Reynolds number**. For the purpose of this assignment, we do not need to care about the physical meaning behind this number, but a larger value typically corresponds to a more noticeable and faster vortex street. Yue used a different Reynolds number in each of the runs to generate a gallery of vortex street. Perform DMD on all 4 given datasets, can you rank the Reynolds number from smallest to largest based on the DMD modes? Please explain your reasoning.

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\(^1\)The *history* behind the vortex street is a good read. Many familiar names in fluid dynamics appeared in the story.

\(^2\)Interestingly, vortex shedding was also chosen as the *cover page image* for “*An Album of Fluid Motion*”, possibly the best book that documents different types of fluid flows with black–white photographs.
Figure 1: Vorticity field of the 30 vortex street simulations.