AM205: Assignment 0

Assignment 0 is for your own edification, it should provide some problems for you to refresh/test/hone your Matlab programming. This assignment will not be assessed — you do not need to submit your answers.

Question 1

Find the angle, θ, between the vectors,

$$v_1 = (1.5, -2, 4, 10) \quad \text{and} \quad v_2 = (3.1, -1, 2, 2.5),$$

using the formula

$$\cos\theta = \frac{v_1 \cdot v_2}{\|v_1\|_2 \|v_2\|_2}.$$

Question 2

Evaluate and plot the Chebyshev polynomial of degree 5 at 100 evenly spaced points in the interval $x \in [-1, 1]$. Use the fact that we have the following recurrence relation for Chebyshev polynomials (where T_k denotes the Chebyshev polynomial of degree k):

$$T_k(x) = 2xT_{k-1}(x) - T_{k-2}(x), \quad \text{for } k \geq 2,$$

and $T_0(x) = 1$, $T_1(x) = x$.

Plot $T_3(x)T_5(y)$ on a 100×100 grid on the domain $(x, y) \in [-1, 1]^2$. Try different plotting functions in Matlab for this grid-based data e.g. `mesh`, `surf`, `contour`.

Question 3

Use the iteration

$$x_{k+1} = \frac{1}{2}\left(x_k + \frac{a}{x_k}\right).$$

to approximate $\sqrt{a}$. (This is known as Heron's formula[1] and in fact it is equivalent to Newton's method for $f(x) = x^2 - a$.) Choose an "initial guess" $x_0 = a$ and iterate until $|x_{k+1} - x_k| < \text{TOL}$. Determine the number of iterations required to compute $\sqrt{5}$ in the cases $\text{TOL} = 10^{-3}$ and $\text{TOL} = 10^{-9}$.

Question 4

In this question we examine the behavior of a finite difference approximation as $h \to 0$. [Aside: If $y = \alpha h^\beta$, then $\log(y) = \log(\alpha) + \beta \log(h)$. As a result, log-log axes are often the clearest way to show convergence results since if the error behaves like $O(h^\beta)$, then on log-log axes we will see a straight line with gradient β.]

Let $f(x) = \tan(x)$, and consider the second order finite difference approximation

$$f_{\text{diff},2}(x; h) \equiv \frac{f(x+h) - f(x-h)}{2h}.$$

Plot the relative error in the $f_{\text{diff},2}(x; h)$ approximation at $x = 1$ as a function of h for $h = 10^{-k}, k \in \{1, 1.5, 2, \ldots, 15.5, 16\}$. Make sure you use a "log-log" plot (use the command `loglog` in Matlab).

[1] Heron of Alexandria, 10–70 AD.

Overlay dashed lines $\alpha_1 h^2$ and α_2/h on the same log-log axes to compare to the error plot from above. (You should choose α_1, α_2 so that $\alpha_1 h^2$ and α_2/h are "close to" the error plot from above so that it is easy to see that the lines are aligned for certain ranges of h.)

Question 5

$y = \sin(x)$ is an analytic function, which means that the Taylor series

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

converges for any $x \in \mathbb{R}$. Write a Matlab function (call it `sinTaylorSeries`) to evaluate this series; the function should take arguments x (a vector of values to evaluate the function at) and N (the number of terms in the series) and should return a vector y of function values and a vector `err` of (absolute) error values with respect to Matlab's built-in $\sin(x)$ function.

Use your function to plot y and |`err`| on the intervals $[-\pi, \pi]$ and $[-10\pi, 10\pi]$ for $N = 10$ and $N = 100$. (Use a "semilog-y" plot for |`err`|, i.e. `semilogy` in Matlab.)