Covariances and Linear Predictability of the North Atlantic Ocean

Extract of a Longer Preliminary Draft

Carl Wunsch
Department of Earth, Atmospheric and Planetary Sciences,
Massachusetts Institute of Technology
Cambridge MA 02139 USA
email: cwunsch@mit.edu

January 31, 2011

Abstract

The problem of understanding linear predictability of elements of the ocean circulation is explored in the Atlantic Ocean for two disparate elements: (1) sea surface temperature (SST) under the storm track in a small region east of the Grand Banks and, (2) the meridional overturning circulation north of 30.5°S. To be worthwhile, any nonlinear method would need to exhibit greater skill, and so a rough baseline is the goal. The focus is a16-year ocean state estimate, under the assumption that oceanic variability is dominating externally imposed changes. Predictablity values obtained for SST are compared to the 28-year long SST record obtained from satellite data. Linear predictability exists for a few months in SST, and there are indications of some skill for a few years. [Omitted here.] Sixteen years is, however, *far too short* for an evaluation for interannual much less decadal variability and predictability, although orders of magnitude are likely stably estimated. The meridional structure of the meridional overturning circulation (MOC), defined as the time-varying vertical integral to the maximum meridional volume transport at each latitude, shows nearly complete decorrelation in the variability across about 35°N—the Gulf Stream system. Subtropical transport measurements would appear to have no descriptive or predictive skill for the subpolar region, although nothing can be said about the structure on time scales of many decades and longer—an issue which cannot be resolved with untestable long model runs.

(A shortened version of a longer paper in preparation.)

1 Introduction

The ability to predict future climate is, for many obvious reasons, high on the agenda of many scientists. Claims that climate should be predictable on some time-scale often rest upon the assumption that the long oceanic time-scales would provide much of the memory of the system—the atmosphere being assumed to lack such memory.

At the present time, more specifically, there is wide community interest in the possibility of decadal prediction of some elements of the ocean circulation, including sea level changes (e.g., Yin et al., 2009), surface temperatures, and volume transports (Msadek et al., 2010). That comparatively short time-scale does hold out the possibility of observational tests of actual predictions, something that is implausible with 50 to 100 year forecasts—which exceed both working scientific lifetimes and the durations of model credibility. The extent, however, of actual predictive skill for the ocean on the decadal time-scale remains obscure with e.g., divergences of IPCC model predictions, being a disquieting sign. (Some models are undoubtedly better than others, but which those are, and which fields are well-calculated, is unknown). Branstator and Teng (2010) review much of the existing discussion.

Predictability of the variability of any physical system involves several sub-elements: the extent to which boundary conditions are predictable; and the degree to which variations arise from internal fluctuations with fixed or known boundary conditions; and the degree to which that internal variability is fundamentally linear or non-linear. In particular, any discussion of oceanic predictability confronts the awkward fact that the ocean tends to react, rapidly and energetically, to shifts in the overlying atmosphere, particularly to changes in the wind-field, and most visibly responding in its upper reaches. (The most rapid response, however, is the barotropic one, which is almost instantaneous over the whole water column.) A literature has emerged showing the coupling of the North Atlantic circulation to the North Atlantic Oscillation (NAO, or Arctic Oscillation, AO) index; see e.g., Deser et al. (2010). Some of the most important elements of the ocean circulation, as they affect climate, such as the sea ice cover, or sea surface temperature (SST) are greatly modified by changing wind systems, and they in turn, modify the atmosphere. This inference then tends to direct attention to the more central question of whether the *atmosphere* is predictable on decadal time scales. For important oceanic phenomena such as sea level changes, one seeks, among other elements, an understanding of the predictability of future rates of land-ice melt, but which, on some time scales, are likely in part dependent upon the ocean itself.

The purpose of this paper is to explore some of the simpler aspects of the ocean prediction problem, focussing on changes that are assumed— absent strong evidence to the contrary—

that one is dealing with intrinsic ocean variability, rather than that induced by global warming or other external drivers. Despite some hysterical public pronouncements, it has been known for a long time (Veronis and Stommel, 1956) that, short of catastrophic disturbances, the basic stratification of the ocean outside of the equatorial band, such as the thermocline depth and temperatures, can be modified significantly only over many decades. Again, notwithstanding several overheated papers proclaiming major shifts in the ocean circulation, there is *no* evidence of observed changes in basic oceanic stratification or transport properties that lie beyond what are plausibly labelled "perturbations" and for which linearization about a background state is a plausible starting assumption. Alternatively, note that any major change in oceanic stratification—with its accompanying required geostrophic water movement—implies a corresponding major change in oceanic potential energy. "Rapid" changes (here meaning decadal) in large amounts of potential energy in turn require extremely efficient energy transfer mechanisms—mechanisms that in the observed ocean are remarkably weak (see the rate values in Ferrari and Wunsch, 2010). How those rates could become significantly larger has not been explained. In what follows, attention is focussed entirely on the perturbation problem and treated as linear.

Theoretical prediction skill is not very meaningful unless it is coupled with a discussion of the ability to detect it. Thus for example, a prediction that the meridional overturning circulation will weaken by 1 Sv in 10 years might be skillful, but if the present value is not known to that accuracy, at best one could say that the future value will not be distinguishable from the present one—given present observational capabilities. There is also the question, already alluded to, of what magnitude of change could be regarded as useful e.g., in producing a detectable contribution to a more directly experienced climate shift such as continental precipitation changes?

Underlying any discussion of prediction is the difficult question of what elements one is trying to predict and why? Myriad choices are phenomenological (sea surface temperature, sea level, meridional overturning (MOC),....), geographical (western North Atlantic, tropical eastern Pacific), seasonal (winter time SST versus summer time), and time horizon (SST with a one month lead time can be of intense interest to a weather forecaster, while the MOC state may be of interest only on 100-year scales). Here two fields of interest to different communities (North Atlantic SST and North Atlantic MOC), are chosen, simplified as far as possible, and the methodologies sketched that can be applied in seeking more definitive answers.

Ambitions are strictly limited. The goal is to quantify the magnitudes of the potentially predictable signals. Guided only by general physical understanding, an estimate of linear predictability skill is sought by empirical means. Prediction skill demands, as a prerequisite, quantitative description and that description is here also purely empirical. This approach thus differs radically from attempts e.g., Zhang and Wu (2010), to isolate physical mechanisms present in the signals (e.g., advection, diffusion, wave propagation, boundary currents, wind-driving) and to evaluate the potential predictability of the specific mechanisms. As will be seen, given the extremely limited duration of large-scale oceanic observations, even in the North Atlantic, for interannual predictability, one can hardly do more than state the problem. Resort to models can be made, but the same data duration limitations preclude any test for real skill. That is the central conundrum of understanding the ocean in climate change: what does one do when the data are inadequate for the goal?

2 An Ocean State Estimate

To proceed, we use the ocean state estimate ECCO-GODAE, v3.73, discussed in detail by Wunsch and Heimbach (2007); Wunsch et al. (2009), and in many other papers listed on the website, http://www.ecco-group.org. This particular state estimate is a global one over 16 years, least-squares fit by the method of Lagrange multipliers to the very large oceanographic data sets that became available beginning about 1992 in the World Ocean Circulation Experiment and its aftermath. The terminology "state estimate" is used to distinguish the result from estimates based upon variations of meteorological forecast ("data assimilation") techniques—which lead to estimates with unphysical jumps. In the particular estimate used here (denoted version 3.73), the data sets included the monthly estimates by SST by Reynolds and Smith (1995). Vinogradova et al. (2010) discuss the global behavior of SST (particularly its rate of change) in the ECCO solutions. Over the vast bulk of the oceans, the model is in a slowly time-evolving thermal wind balance, largely constrained by in situ hydrography, Argo float profiles, and altimetric variability.

Sixteen years is an extremely short period for attemping understanding of multi-year or decadal prediction. The restriction to that time period is dictated by the extreme paucity of oceanic data prior to about 1992. Ocean state estimates over intervals before 1992 (e.g., Wang et al., 2010) are from nearly unconstrained ocean models. Furthermore, the meteorological forcing fields used, even the most recent ones, are similarly greatly suspect; see e.g., Bengttson et al. (2004) or Bromwich et al. (2007).

Because of the short-duration, a comparison will be made to the longer duration (28 years) Reynolds and Smith (1995; hereafter RS) SST estimate used, separately, without the intervening ECCO system. Such estimates are, however, not available for other fields of interest (the meridional overturning, the corresponding oceanic heat transports, etc.) and one must use the state estimates. The even longer historical reconstructions of SST obtained prior to the arrival of globally orbiting satellites are avoided here, as the space-time sampling errors are far worse than they remain today.

3 Sea Surface Temperature (SST)

(Omitted here. Available in the full manuscript.)

3.1 A Formalism

Assume an autoregressive process, e.g. an AR(2), so that,

$$\xi\left(t\right)=a_1\xi\left(t-1\right)+a_2\xi\left(t-2\right)+\varepsilon\left(t-1\right), \tag{1}$$

{ar3}

where a_1, a_2 are regression constants and $\varepsilon\left(t\right)$ is near-Gaussian white noise of zero mean and variance σ_{ε}^2. Unless otherwise stipulated, t, denotes the present time, and the time-steps, Δt are implicit in all expressions. The coefficients in Eq. (1) are normally found by least-squares, leading to the so-called Yule-Walker and related equations. Here we explicitly write the set of simultaneous equations,

$$\begin{aligned}
\xi\left(t\right)&=a_1\xi\left(t-1\right)+a_2\xi\left(t-2\right)+\varepsilon\left(t-1\right) \qquad (2)\\
\xi\left(t-1\right)&=a_1\xi\left(t-2\right)+a_2\xi\left(t-3\right)+\varepsilon\left(t-2\right)\\
\xi\left(t-2\right)&=a_1\xi\left(t-3\right)+a_2\xi\left(t-4\right)+\varepsilon\left(t-3\right)\\
&.\\
&.\\
\xi\left(t-N\right)&=a_1\xi\left(t-N-1\right)+a_2\xi\left(t-N-2\right)+\varepsilon\left(t-N-1\right),
\end{aligned}$$

{ar4}

for N equations in $N+2$ unknowns (a_1, a_2, and N of the $\varepsilon(r)$). Re-write Eq. (2) in standard matrix vector notation as,

$$\mathbf{Ex}=\mathbf{y}, \quad \mathbf{E}=\left\{\begin{array}{cccccc} \xi(t-1) & \xi(t-2) & 1 & 0 & . & 0 & 0 \\ \xi(t-2) & \xi(t-3) & 0 & 1 & . & 0 & 0 \\ . & . & 0 & 0 & . & 0 & 0 \\ . & . & . & . & . & . & . \\ \xi(t-N-1) & \xi(t-N-2) & 0 & 0 & . & 0 & 1 \end{array}\right\}, \tag{3}$$

{ls2}

$$\mathbf{x}=\left[\begin{array}{c} a_1 \\ a_2 \\ \varepsilon(t) \\ \varepsilon(t-1) \\ . \\ \varepsilon(t-N-1) \end{array}\right], \ \mathbf{y}=\left[\begin{array}{c} \xi(t) \\ \xi(t-1) \\ \xi(t-2) \\ . \\ . \\ \xi(t-N) \end{array}\right],$$

a formally underdetermined problem and which can be solved in numerous ways, including those normally used for this type of regression problem (e.g., Box et al., 2008; Priestley, 1982). This form differs from the conventional least-squares approach (Priestley, 1982, P. 346) only in treating the $\varepsilon(r)$ as explicitly part of the solution, rather than as residuals of the formally over-determined problem for a_1, a_2 alone. Here, for several reasons, we choose this approach (Wunsch, 2006): the formal regression problem, when many more physical variables are reasonably introduced (e.g., the SST time series at all latitudes, or the wind field), rapidly becomes very underdetermined in the conventional forumulation; least-squares makes simple the computation of uncertainties in the parameters $(a_1, a_2, \varepsilon(r))$; and one can easily "color" the noise $\varepsilon(t)$ either by modification of the identity matrix appearing in $\mathbf{E}$ (which would make it an ARMA), or by introducing column weighting (solution covariance) matrices.

Any stationary univariate AR can be converted into a moving average (MA), of form,

$$\xi(t)=\sum_{p=0}^{\infty} b_p \varepsilon(t-p)=\varepsilon(t)+b_1\varepsilon(t-1)+b_2\varepsilon(t-2)+... \tag{4}$$

{ma1}

For known a_i, the b_i can be obtained by simple algebraic long division,

$$1+b_1 z+b_2 z^2+...=\frac{1}{1+a_1 z+a_2 z^2+a_3 z^3+....}, \tag{5}$$

{zpoly}

and vice-versa. The b_i can also be determined directly without first calculating the a_i. The MA form produces the $\tau-$ahead prediction error as,

$$\left\langle\left(\tilde{\xi}(t+\tau)-\xi(\tau+\tau)\right)^2\right\rangle=\sigma_\varepsilon^2\sum_{p=0}^{\tau} b_p^2, \quad b_0=1,$$

Variable	GBB SST (ECCO) $^{\circ}\mathrm{C}^2$	GBB (Reynolds & Smith) $^{\circ}\mathrm{C}^2$	MOC at 20°S Sv^2	MOC at 25°N Sv^2	MOC at 50°N Sv^2
Total Record	10.2	9.9	6.5	10.2	10.5
Annual cycle	9.45 (93%)	8.8	2.1	2.9	3.6
Record w/o annual cycle	0.78	0.7 (7%)	4.4	7.2	7.0
Annual averages	0.50 (5% of the total)	0.36(3.6%)	1.9	2.2	1.8
One month PE	0.2	0.3 (MA(3) and MA(10))	2.4 MA(4)	6.5 MA(4)	5.6 MA(4)
Six month PE	0.6	0.7	-	-	-
One year PE	0.2 (AR(1) with trend)	0.05 (MA(4))	0.5 MA(4)	0.2 MA(4)	0.8 MA(4)
Three year PE	0.4	0.3	-	-	1.5

Table 1: Summary statistics. Variances are either in $^{o}C^2$ (for SST) or Sv^2 for the meridional overturning circulation (MOC). PE is the abbreviation for the prediction error. The record variance is not the sum of the component variances because the monthly values include the low frequency variability. GBB denotes the Grand Banks Basin, the area employed for SST.

Some prediction error values are omitted as being of no particular interest. {TableKey}

and if the b_i are sufficiently small, there will be rapid convergence to the asymptote of the variance of $\xi\left(r\right)$: $\left\langle \xi^2 \right\rangle = \sigma_\varepsilon^2 \sum_{p=0}^{\infty} b_p^2$. Like the AR($M$), any practical MA will have a finite order, N. Generally speaking if M is small, N will be large, and vice-versa, and with the tradeoff becoming part of the discussion of representational efficiency. Note that stationarity, which we are assuming, is easily shown to require that the polynomials in Eq. (5) should both be convergent when $|z| = 1$ (they are "minimum phase" in the signal processing terminology).

4 Months-Ahead Prediction

{table}

4.1 SST Predictability—A Caveat

The reader is reminded that the study is based upon a "hindcast" skill, meaning that the same data are used to determine the time series structure as are used to test its prediction skill. Hindcast skill is inflated relative to true forecast skill by a significant amount. Davis (1976) has a particularly clear discussion of the issue. As he notes, an accurate estimate of the skill inflation is only simple with large-sample statistics and, in particular, for interannual behavior, the 16 years of estimated SST and MOC used here is a very small sample. It is useful, in many cases, to withhold part of the data set as a way of emulating an independent record for testing skill, perhaps by dividing it into two pieces—an identification section and a test section. But the "red" nature of the spectra observed shows that there will exist significant correlations between the used and withheld portions of the time series, and again a rigorous calculation becomes very difficult. For present purposes, we will leave the discussion at this point—as a warning that estimates here particularly of the interannual forecast skill are likely optimistic.

5 The Meridional Overturning Circulation (MOC)

Determining the volume or mass transport in the North Atlantic can be done only by use of a model, albeit a number of papers (e.g., Lorbacher et al., 2010) claim the existence of useful covariances between its values and some observables such as sea surface height. Unlike SST, an immediate issue is the definition of what is meant by the MOC, as the literature contains usages calculating it at very different latitudes, integration depths and times. Here we take advantage of a global system to define it—in the Atlantic Ocean—as a function of all latitudes from the Cape of Good Hope (about 30°S) northward to the northern limits of the present model (79.5°N). It is, more specifically, calculated from the zonal integral, continent to continent of the meridional velocity, the density being treated as constant, consistent here with the Boussinesq version of the model,

$$V\left(y,z,t\right)=\int_{0}^{x_{L}(y)}v\left(x,y,z,t\right)dx \tag{6}$$

{meridtrans1}

(in practice, spherical coordinates are used). At any latitude, at any time, the MOC is then defined here as the maximum of the integral from the surface to a time and space varying depth $z_{\max}\left(y\right)$,

$$V_{moc}\left(y,t\right)=\max_{z_{\max}(y,t)}\int_{z_{\max}(y,t)}^{0}V\left(y,z,t\right)dz \tag{7}$$

{moc1}

Fig. 2 displays the time average value, $\left\langle V_{moc}\left(y,t\right)\right\rangle$ as well as the depth, $z_{\max}$, where, on average, the maximum is reached. A geographical maximum of about 15Sv is reached at northern mid-latitudes and drops rapidly with latitude beyond about 50 degrees. At the present time, it is not possible to provide a useful uncertainty estimate on these values, but the general structure appears very robust to both variations in the data base and in model parameters. Fields, V, were discussed in some detail by Wunsch and Heimbach (2006, 2009).

How much does $V\left(y,z,t\right)$ vary with time? Jayne and Marotzke (2001) infer, consistent with what is found here, that the annual volume variability is largely that of the surface Ekman layer. Fig. 3 shows its January anomaly values every two years, and indicating variations of up to about 4Sv, but only very locally. The variations in the anomaly of $V_{moc}\left(y,t\right)$ are shown in Fig. 4 at three latitudes, where the integration depth is kept fixed at $z_{moc}\left(y\right)$, that is not time-varying. These integrals have a range, except in the far north, of about 10Sv (±5Sv). The same figure shows the 30° of latitude differences in $V_{moc}\left(y,t\right)$, and which are typically less than

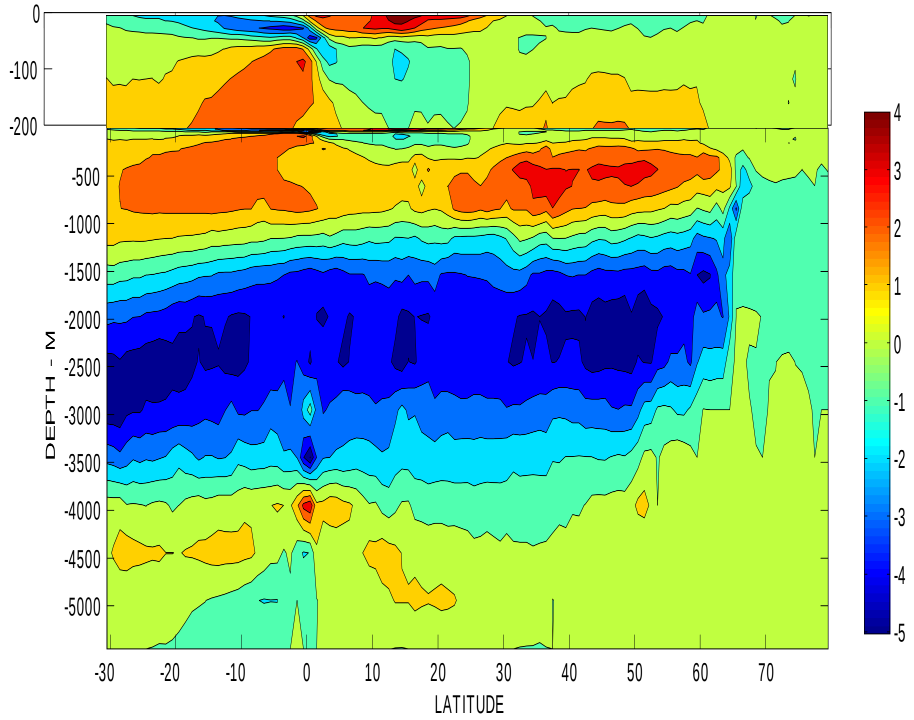

Figure 1: Zonal and time mean meridional transport (not the stream function), with an expanded scale for the upper 200m showing particularly the complex structure at low latitudes (see Wunsch and Heimbach, 2009).

{moc_timemean_

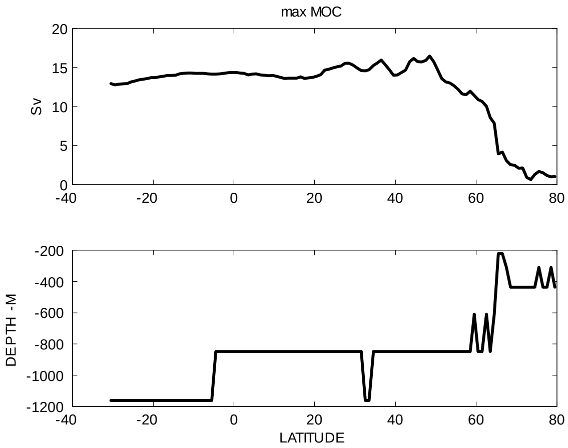

Figure 2: The maximum meridional transport, integrated from the sea surface, averaged over 16 years (upper panel). Lower panel shows the depth where the time-mean value is obtained.

{mocmax&deptht

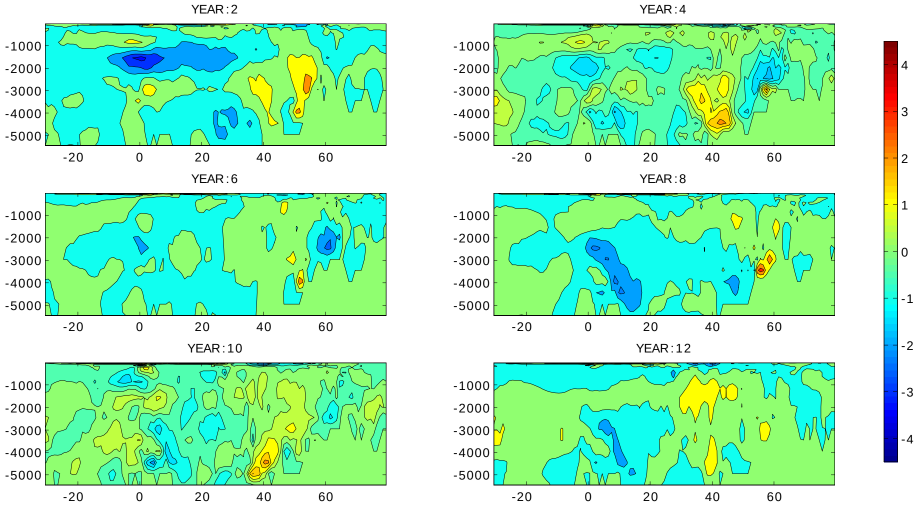

Figure 3: Monthly values of $V(y,z,t)$ for a succession of Januarys showing the typical interannual variability occurring at depth.

{moc_every2yea

about 5 Sv. Both sets of curves are very noisy on monthly time scales. The power densities for three latitudes are shown in Fig. 6. At most latitudes, there is a significant annual cycle, principally in the low-latitude Ekman layer, and its harmonics. Otherwise, the spectral densities are nearly white beyond the annual period—boding ill for linear predictability. The smallest low frequency energy is found at 50.5°N, consistent with linear dynamical behavior that implies a much longer time scale for wave-like motions. High latitude power densities are dominated by the annual cycle and not by the interannual variability (out to 16 years). In general, these spectra are "flat" by geophysical standards, not very far from white noise.

Variances of the MOC, computed for the monthly means over all 111 latitudes are $27\mathrm{Sv}^2 = (5.1\mathrm{Sv})^2$ and the annual means have variance $1.5\mathrm{Sv}^2 = (1.2\mathrm{Sv})^2$. At 50°N alone, the corresponding variances are $10.5\mathrm{Sv}^2 = (3.2\mathrm{Sv})^2$, and $1.8\mathrm{Sv}^2 = (1.3\mathrm{Sv})^2$.

A small visible trend appears in the values at some latitudes, a trend which disappears as one moves away from the starting time. Note that there are no data preceding the start time of 1992; hence the early years are much more weakly constrained than the later ones—which are controlled in considerable part by the data preceding the particular time of estimation.

Fig. 7 shows the correlation coefficient matrix, R_{ij}, between the annual mean variations in the MOC at all latitudes, i, j. Making the mildly optimistic assumption that each of the annual mean values is an independent variable at any latitude, at 95% confidence, one must have $|R_{ij}| > 0.5$, approximately, to distinguish the value from zero. A change takes place across

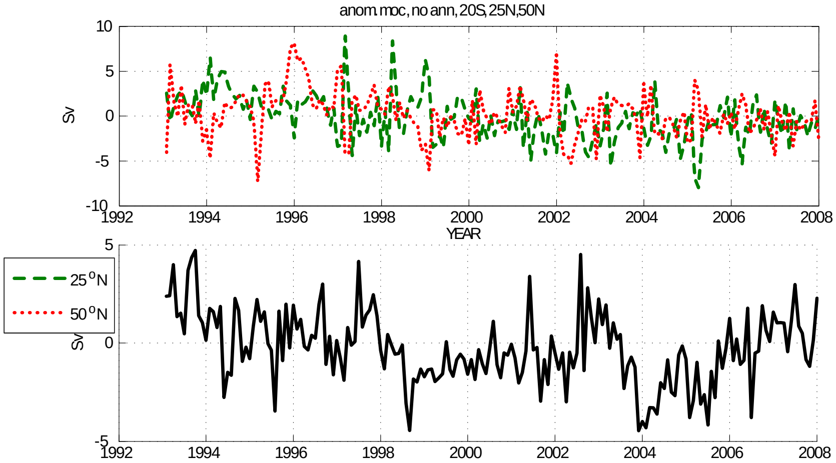

Figure 4: The maximum monthly MOC *anomaly* without the annual mean cycle at 20°S, 25°N and 50°N.

{moc_3lats_ts_

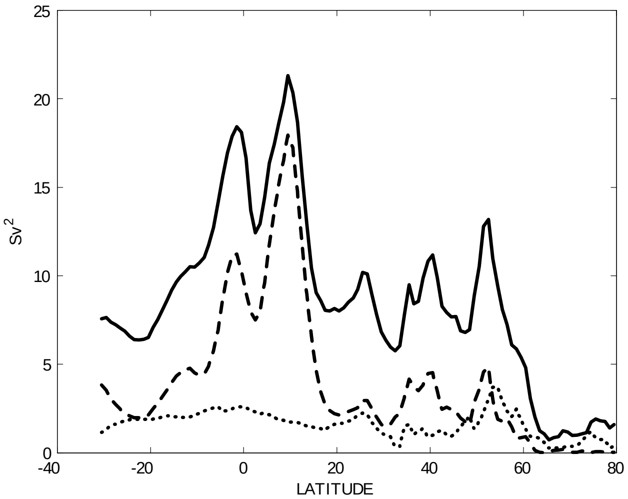

Figure 5: MOC variances (solid curve) and the annual contribution (with harmonics) as a function of latitude in Sv^2 (dashed line), and the residual after removal of the annual cycle (dotted).

{var_all_lats&

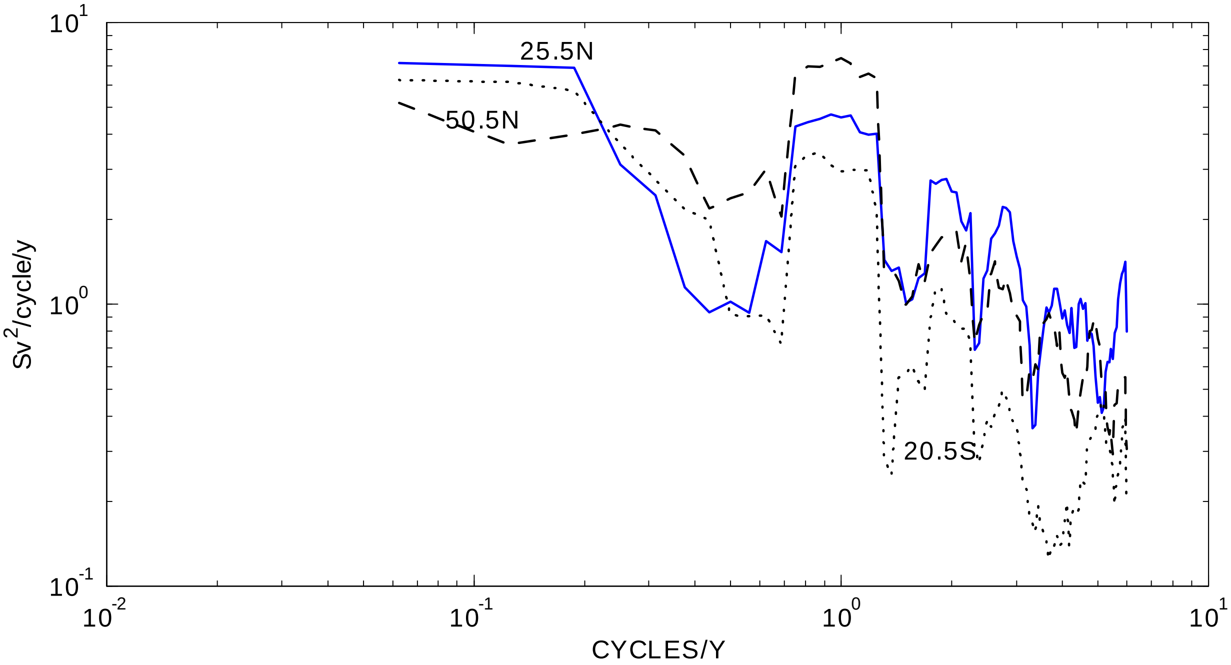

Figure 6: Power density of monthly MOC values at three latitudes. The annual cycle and its harmonics are visible, as is the low frequency asymptote toward white noise behavior.

{moc_pd_3lats.

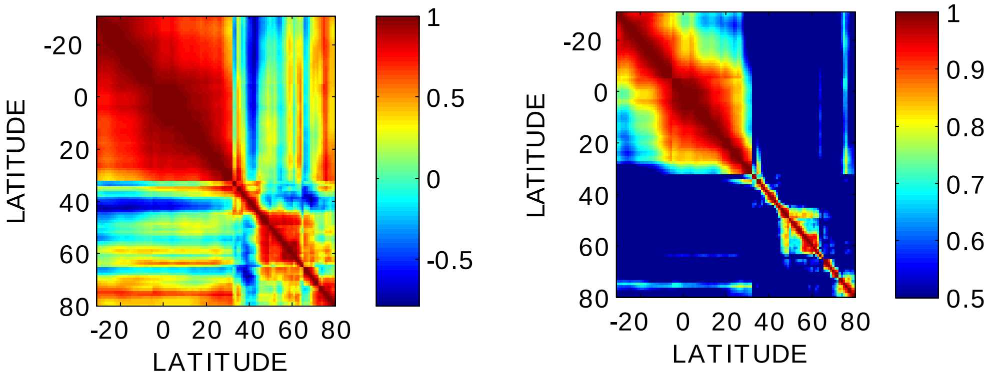

Figure 7: Correlation matrix with latitude of the annual mean MOC (left panel). Right panel is an expanded color scale version of the left panel, showing only the apparently statistically significant values. No negative correlations are significant.

{moc_latcorr_a

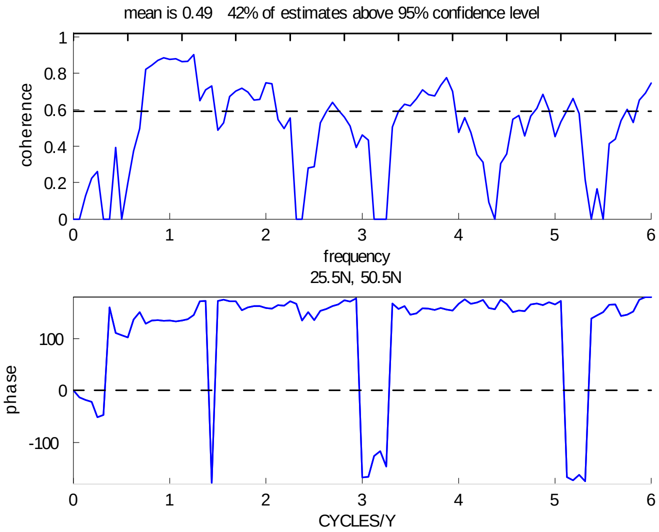

Figure 8: Coherence amplitude and phase between 20.5°N and 50.5°N. Significant coherence vanishes at periods longer than one year. High frequency coherence is in large part that of the annual cycle and its harmonics and for which the level-of-no-significance shown is inappropriate.

{moc_coher25n5

about 35°N where all linear correlation is lost between values on either side of that latitude (the approximate Gulf Stream position). The North Atlantic subtropical gyre shows some marginally significant correlation with the South Atlantic, but nothing in the North Atlantic subpolar gyre (consistent e.g., with the pure model results of Bingham et al., 2007). Within the subtropical gyre, correlation decays to insignificant levels over about 20°of latitude.

A problem with correlation analyses is that they lump together all time scales with often very diverse physics. One might hypothesize that the correlations here are dominated by noisy high frequencies. To address this issue in part, Figs. 8 and 9 show the coherence as a function of frequency between the 50°N MOC and its values at 25°N and 20.5°S. They show, to the contrary, that the only marginal coherence is at periods shorter than one year (at the annual period the conventional statistics do not apply). Evidently (on this decadal time scale), annual mean MOC determinations south of about 35°N carry no (linear) information about its behavior poleward of that latitude at any frequency now subject to test.

5.1 Predicting the MOC

Here we explore the use of the two-dimensional (latitude and time) structure of the estimated annual mean MOC. Monthly predictions of the MOC, with no direct meteorological connection, are less obviously useful than for SST. Wunsch and Heimbach (2009) discuss the annual cycle

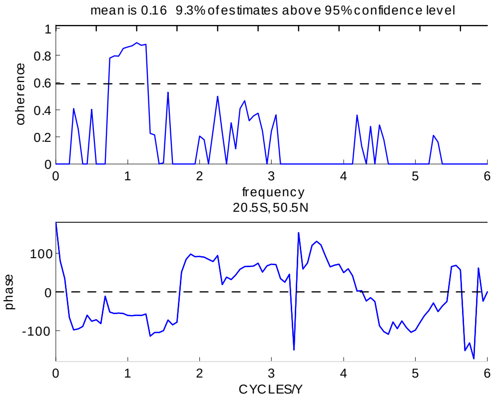

Figure 9: Coherence between the monthly MOC at 20.5°S and 50.5°N. Apart from the annual cycle, where the conventional statistics do not apply, there is no significant coherence.

{moc_coher21s5

of the MOC—and which is primarily a near-equatorial phenomenon, although extending to considerable depth. Hypothetically, one could imagine using each of the 111 time series at 1° latitude spacing, with time lags of one year and longer, as regression variables to predict e.g., the value at some specific latitude(s). With 16 sample points at any fixed latitude, one would be seeking the equivalent of the expansion of a 16-dimensional vector in 111 non-orthogonal vectors—a markedly underdetermined problem. Although we will return to this problem, for the moment, consider the more well-determined one of predicting from the present and past values at one particular latitude. The problem was already discussed above for SST, that of having only 16 samples,

The MOC at 50°N is arbitrarily chosen as the initial target prediction—on the basis that there is a large literature claiming that modifications in the high latitude transports are a key climate control parameter. This latitude is close to the one with the largest defined MOC and is just south of the region where the mean MOC declines very rapidly. Consider the problem, of predicting MOC(50°N,$t+1$), one year into the future, using the history at that latitude. As with SST, one cannot be definitive, only indicative. The spectral estimate in Fig. 6 is not very different from white noise, and thus one anticipates some rather modest degree of prediction skill. Fig. 10 shows the error growth using an AR(1) deduced from the measurements at 50°N alone.

Had there appeared significant correlations or coherences between 50°N and other latitudes, it would be reasonable to seek predictive power from variations in the MOC at all latitudes. The absence of such correlations shows that linear predictability will be slight. Experiments (not shown; the Appendix describes the approach), as expected did not produce any useful outcome.

Using knowledge of the MOC for 16 years at all latitudes except at 50°N, so as to predict

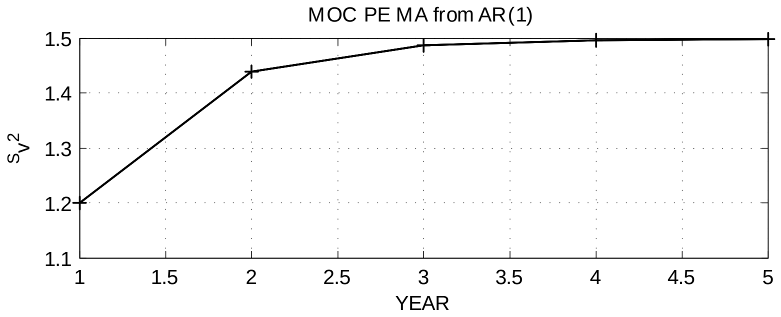

Figure 10: Prediction error as a function of year at 50°N from a univariate AR(1).

{pemoc50nfroma

the value at that latitude is a wildly optimistic hypothesis for any situation except with the use of a model. At the present time, direct measurements of meridional transports are being carried out at 26°N (e.g., Kanzow et al. (2009) and are planned for 30°S in the Atlantic (see AMOC website). It is, of course, possible that the existing 16-year interval is untypical of the behavior of the Atlantic Ocean and/or that linear predictive skill would emerge with much longer, multi-decadal plus, records, but these are pure speculations.

5.2 Correlation with SST

Study of the MOC has often been justified on the basis that its variability is linked to climate change, sometimes in truly dramatic fashion ("hosing"). Thus the question arises as to whether there is any relationship between the MOC variations estimated here, and the SST of the region previously discussed. One simple measure is the correlation coefficient between the MOC and GBB SST variations, depicted in Fig. 11, which repeats Fig. 7, such that the last row and column represent the annual mean SST time series. The calculation is shown for the case of the raw SST and where, also, its visible, linear, trend was removed by least-squares. One might infer that there is a marginally significant negative correlation between the low latitude MOC ($0 \pm 10°$ latitude) and the GBB SST. The result is, however, dependent upon the presence of the trend in SST, and which destroys the assumption of annually independent changes. Any inference of correlation is extremely fragile and not supportive of a simple relationship between MOC and SST at least on the time scales accessible here. Determining whether there is such a relationship on much longer time scales will have to await much longer durationobservations.

5.3 Comments on Ongoing Observational Programs

Any discussion of predictability of the MOC leads one inevitably to a discussion of observational programs directed, in some cases, at "early warning" of changes in the MOC, commonly by

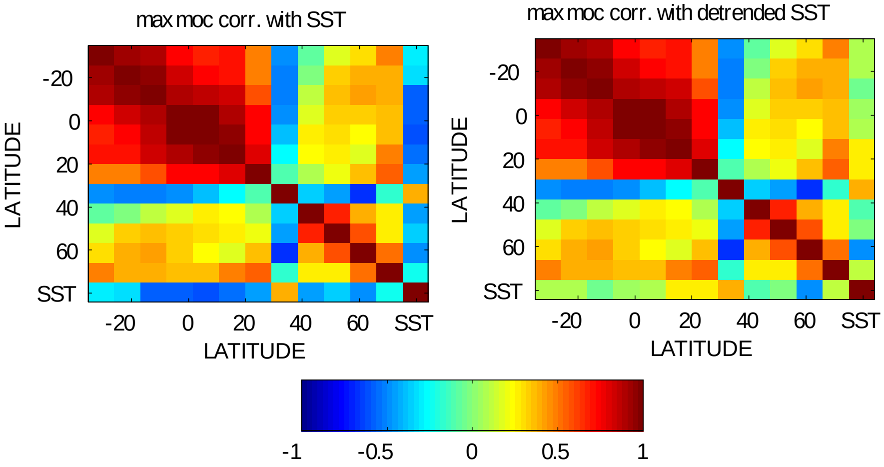

Figure 11: Correlation coefficient between the maximum MOC through time (annual means) with the GBB SST (left panel). The last row and column are the SST correlations. Omitting the last row and column repeats the values in Fig. 7. Right panel shows the same results but with a linear trend removed from SST, thus reducing the correlations. No values below magnitude 0.5 are statistically significant. (These correlations are with the MOC defined as integrated to the time-mean maximum depth. Results with the time-varying integration depth are indistinguishable.)

{mocmax_latcor

measurement of its values at one or more latitudes spanning the oceans. The results here are not encouraging of such efforts on time scales of less than many decades: (1) Latitudinal correlations of interannual variability are weak, and do not extend across about 35°N, roughly the zonal part of the Gulf Stream system. Because much of the justification for the "early warning" programs has relied upon inferences from "hosing" experiments where huge volumes of freshwater are imposed upon the subpolar gyre, there is little evidence that subtropical measurements could provide any immediately useful information, where "immediately" is used in the sense of less than about 15 years. How many years would be required is unknown but could be extremely long. This inference is generally consistent e.g., with the modeling results of Bingham et al. (2009), and also the much simpler coherence calculations (for the North Pacific) by Wunsch (2009) using altimetry by itself. (2) To the extent that trends are visible in the MOC, they are weak, and difficult to distinguish from artifacts resulting from inadequate data preceding 1992 and the changing observational base, or from numerical drifts of the model. Changes in the subpolar circulation apparently must be monitored in the subpolar region itself. More generally, climate change is a global phenomenon, integrating at any given location changes originating from diverse regions of the globe, and the responses represent a summation of perturbations arising locally and those generated far away and long before. Note too, that existing estimates of freshwater increments to the subpolar gyre from melting ice are a very small fraction of the precipitation rates there (see e.g., the table in Wunsch, 2010, QSR).

6 Discussion

It is useful to compare the results for the MOC here with the entirely different approach and inferences of Msadek et al. (2010) and who conclude that the MOC is predictable with skill out to 20 years. They used an unconstrained, coupled climate model run for 1600 years. Apart from the very much longer analysis time, their mean MOC is 25 Sv rather than the approximate maximum of 15 Sv found here. Their MOC spectrum (their Fig. 1) is steeply red from about about two year periods out to about 20 years, and culminating in a narrow-band spectral peak at about 20 years. Their larger inferred predictability is, not coincidentally, about 20 years, would be a consequence of that redder spectrum and, particularly, of the narrow peak—if it is real. This prediction skill is likely primarily a linear one, because low frequency narrow-band processes have an intrinsic long memory; as the peak-width becomes narrower, one converges to a deterministic component with an infinite prediction horizon. In contrast, the spectra computed here tend to indicate a white noise behavior beyond about 15 year periods with no indication of a narrow band spectral process, although no definitive statement can be made from the available observations. This disagreement between the two sets of results focuses one on the usual conundrum of climate change prediction: (1) It is difficult to compare a 16-year data-constrained estimated to a 1600-year unconstrained one. (In their study of 136 years of North Atlantic SST data, Tourre et al. (1999) did not report any obvious 20 year spectral excess, although all the caveats about data quality before the polar-orbiting satellite era will apply.) It is conceivable that the 16-year interval of the ECCO estimates is unrepresentative e.g., of the historical values of the MOC, and one might postulate that it is more typically closer to the 25Sv of the Msadek et al. (2010) model than to the ECCO values. Such an enhanced value, however, would imply a much increased geostrophic transport, which dominates the upper limb of the MOC (mostly in the Gulf Stream system), and within historical times such a large value is probably ruled out by existing coastal sea level and wind-strength records. (2) It is also conceivable that the more nearly white spectrum that we infer at periods of a few years is untypical of a hypothetical much longer record. Should one simply assume the reliability of climate model results obtained from runs that far outstrip any ability to test them?

A general comment, applicable also to the present results, is that most models are much less noisy than is the real world, either entirely lacking in the eddy field and internal waves, or usually underestimating it. In the present case (e.g., Wunsch, 2008; Kanzow et al., 2009) and in calculations such as Msadek et al. (2010), one should infer that all estimates of predictability skill are, yet again, upper bounds.

Poor prediction results in the fields discussed here does not mean that the corresponding variable is not predictable: the best prediction is often that of the sample mean, with a standard error given from the estimation variance. In other words, the best prediction may well be that the field will be the same as today—indistinguishable within its standard error, or that of the observations.

One can modify the methods here in a large number of ways. The singular value decomposition (see the Appendix) is identical in its $\mathbf{u}$ vectors to the conventionally defined EOFs, and emerges naturally as part of the regression problem. These individual orthogonal structures of the variability have been used by Davis (1978) and others. Generally speaking, any particular EOF (singular vector) will have a fraction, depending upon the degree of spatial correlation, of the total variance, and if it displays significant predictability (e.g., Branstator and Teng, 2010), it will only be for that fraction of the expected variance—perhaps large enough to be useful to someone.

The dual (adjoint) model calculations of Heimbach et al. (2010) represent a linearization of the governing equations. Regarded as Green function solutions, they can (and should) be used either directly in predictions, or as a guide in choosing the relevant regressor fields, locations, and time-scales. They do show the strong sensitivity of North Atlantic shifts to disturbances in distant ocean basins at earlier times.

As noted in the introduction, the present results apply only to the temporally stationary components. A major shift in controlling boundary conditions—such as a massive ice melt event, or an increase in greenhouse gases—would render the process non-stationary—changing its mean, and probably its statistics as well. The issue for those interested in decadal and longer predictability is whether those external controls are themselves predictable and whether they dominate the variance contributed by what here is assumed to be intrinsic changes in the ocean. Such external predictability, if it exists, is primarily independent of purely oceanic processes and its long memory components, which are often cited as the most likely source of true climate predictability. A long memory has the consequence, however, of producing changes today or in the future as the result of forcings and fluctuations occurring long ago (Heimbach et al., 2010), greatly complicating the interpretation of ongoing changes.

The linear analysis here does preclude inferences about external forcings that would drive the system out of the linear perturbation range. Note, however, that even the ongoing accelerated ice mass losses around Greenland (and possibly Antarctica) remain very small fractions of the normal variability e.g. of rainfall or wind-induced Ekman pumping.

The results here have all been biassed towards an optimistic outcome: using the estimated fields both to determine the optimal linear predictors and to test them; mainly retaining apparent trends; and by employing very large scale integrals such as basin-wide sea level or heat content or sea surface temperature. Consistent with the earlier study of the linear predictability of the North Atlantic Oscillation (NAO; Wunsch, 1999), little skill beyond a year is found. Major elements of the ocean circulation are of course, predictable far beyond that time interval: a robust prediction that the thermocline depth, the net heat content, etc. will be little changed in a decade or longer, probably undetectably so given the nature of the observing system and the natural noise, is a safe conjecture.

For SST in the GBB region,, use of a fixed annual cycle leaves a sub-annual variability error of $0.5°\mathrm{C}^2$. Linear prediction skill produces a one-year ahead prediction error of about $0.19°\mathrm{C}^2$ ($0.4°$C RMS) ^{2}. Any nonlinear estimation method must significantly reduce this value to be worth the extra computation.

The possibility of nonlinear skill remains, although distinguishing it from linear behavior with such short duration records is very difficult if not actually impossible. Note too, that many methods exist for rendering nonlinear time series into linear forms (e.g., taking the logarithm; see Hamilton (1994), etc.).

The general lack of covariance between subtropical and subpolar latitudes, with its consequences for observing systems intended to permit prediction, likely arises from at least two phenomena. Time scales for equilibrium dynamical response are far longer at high latitudes, and these are also regions where meteorological variability is greatest, involving not only the wind field, but also the precipitation patterns coupled to sea ice changes. The failure of observed Sverdrup balance (Wunsch, 2011) poleward of about 35°N is consistent with this inference. Final equilibria occurs ultimately on diffusive time scales, which can be extremely long.

References

Bengtsson, L., Hagemann, S., Hodges, K.I., 2004. Can climate trends be calculated from reanalysis data? Journal of Geophysical Research-Atmospheres, 109.

Bingham, R.J., Hughes, C.W., Roussenov, V., Williams, R.G., 2007. Meridional coherence of the north atlantic meridional overturning circulation. Geophysical Research Letters, 34.

Box, G.E.P., Jenkins, G.M., Reinsel, G.C., 2008. Time Series Analysis: Forecasting and Control. Hoboken, N.J.: John Wiley.

Branstator, G., Teng, H., 2010. Two limits of initial-value decadal predictability in a cgcm. Journal of Climate, 23, 6292-6311.

Bromwich, D.H., Fogt, R.L., Hodges, K.I., Walsh, J.E., 2007. A tropospheric assessment of the ERA-40, ncep, and JRA-25 global reanalyses in the polar regions. Journal of Geophysical Research-Atmospheres, 112.

Davis, R.E., 1976. Predictability of sea-surface temperature and sea-level pressure anomalies over North Pacific Ocean. Journal of Physical Oceanography, 6, 249-266.

Davis, R.E., 1978. Predictability of sea-level pressure anomalies over North Pacific Ocean. Journal of Physical Oceanography, 8, 233-246.

Davis, R.E., 1979. Search for short-range climate predictability. Dynamics of Atmospheres and Oceans, 3, 485-497.

Deser, C., Alexander, M.A., Xie, S.P., Phillips, A.S., 2010. Sea surface temperature variability: Patterns and mechanisms. Annu. Rev. Mar. Sci., 2, 115-143.

Ferrari, R., Wunsch, C., 2009. Ocean circulation kinetic energy: Reservoirs, sources, and sinks. Annual Review of Fluid Mechanics, 41, 253-282.

Hamilton, J.D., 1994. Time Series Analysis: Princton Un. Press.

Heimbach, P., C. Wunsch, R. M. Ponte, G. Forget, C. Hill, and J. Utke, 2010. Timescales and regions of the sensitivity of atlantic meridional volume and heat transport magnitudes: Toward observing system design. Deep-Sea Research, in press.

Kanzow, T., Johnson, H.L., Marshall, D.P., Cunningham, S.A., Hirschi, J.J.M., Mujahid, A., Bryden, H.L., Johns, W.E., 2009. Basinwide integrated volume transports in an eddy-filled ocean. Journal of Physical Oceanography, 39, 3091-3110.

Lawson, C.L., Hanson, R.J., 1995. Solving Least Squares Problems: SIAM, Philadelphia.

Lorbacher, K., Dengg, J., Boning, C.W., Biastoch, A., 2010. Regional patterns of sea level change related to interannual variability and multidecadal trends in the Atlantic meridional overturning circulation. Journal of Climate, 23, 4243-4254.

Msadek, R., Dixon, K.W., Delworth, T.L., Hurlin, W., 2010. Assessing the predictability of the atlantic meridional overturning circulation and associated fingerprints. Geophysical Research Letters, 37, -.

Nelles, O., 2001. Nonlinear System Identification.

Priestley, M.B., 1982. Spectral analysis and time series. Volume 1: Univariate Series. Volume 2: Multivariate Series, Prediction and Control: Academic, London.

Reynolds, R.W., Smith, T.M., 1995. A high-resolution global sea-surface temperature climatology. Journal of Climate, 8, 1571-1583.

Robinson, E.A., 1981. Time Series Analysis and Applications. Houston, Tex.: Goose Pond Press.

Storch, H.v., Zwiers, F.W., 2001. Statistical Analysis in Climate Research. Cambridge, UK ; New York: Cambridge University Press.

Sturges, W., Hong, B.G., 1995. Wind forcing of the Atlantic thermocline along 32-degrees-n at low-frequencies. Journal of Physical Oceanography, 25, 1706-1715.

Tourre, Y.M., Rajagopalan, B., Kushnir, Y., 1999. Dominant patterns of climate variability in the atlantic ocean during the last 136 years. Journal of Climate, 12, 2285-2299.

Van Huffel, S., Vandewalle, J., 1991. The Total Least Squares Problem. Computational Aspects and Analysis: SIAM, Philadelphia.

Veronis, G., Stommel, H., 1956. The action of variable wind stresses on a stratified ocean.

Wang, W.Q., Kohl, A., Stammer, D., 2010. Estimates of global ocean volume transports during 1960 through 2001. Geophysical Research Letters, 37.

Woollings, T., Hoskins, B., Blackburn, M., Hassell, D., Hodges, K., 2010. Storm track sensitivity to sea surface temperature resolution in a regional atmosphere model. Climate Dynamics, 35, 341-353.

Wunsch, C., 1999. The interpretation of short climate records, with comments on the north atlantic and southern oscillations. Bulletin of the American Meteorological Society, 80, 245-255.

Wunsch, C., 2006. Discrete Inverse and State Estimation Problems : With Geophysical Fluid Applications. Cambridge ; New York: Cambridge University Press.

Wunsch, C., 2008. Mass and volume transport variability in an eddy-filled ocean. Nature Geoscience, 1, 165-168.

Wunsch, C., 2009. The oceanic variability spectrum and transport trends. Atmosphere-Ocean, 47, 281-291.

Wunsch, C., 2011. The decadal mean ocean circulation and Sverdrup balance. Submitted for publication.

Wunsch, C., Heimbach, P., 2006. Estimated decadal changes in the north atlantic meridional overturning circulation and heat flux 1993-2004. Journal of Physical Oceanography, 36, 2012-

2024.

Wunsch, C., Heimbach, P., 2007. Practical global oceanic state estimation. Physica D-Nonlinear Phenomena, 230, 197-208.

Wunsch, C., Heimbach, P., 2009. The global zonally integrated ocean circulation, 1992-2006: Seasonal and decadal variability. Journal of Physical Oceanography, 39, 351-368.

Wunsch, C., Heimbach, P., Ponte, R.M., Fukumori, I., Members, E.-G.C., 2009. The global general circulation of the ocean estimated by the ECCO-Consortium. Oceanography, 22, 88-103.

Yin, J.J., Schlesinger, M.E., Stouffer, R.J., 2009. Model projections of rapid sea-level rise on the northeast coast of the United States. Nature Geoscience, 2, 262-266.

Zhang, H.H., Wu, L.X., 2010. Predicting north atlantic sea surface temperature variability on the basis of the first-mode baroclinic rossby wave model. Journal of Geophysical Research-Oceans, 115, -.