

# Assessment of the importance of ice-shelf buttressing to ice-sheet flow

T. K. Dupont and R. B. Alley

Department of Geosciences and EMS Earth and Environmental Systems Institute, The Pennsylvania State University, University Park, Pennsylvania, USA

Received 17 November 2004; revised 5 January 2005; accepted 19 January 2005; published 25 February 2005.

[1] Reduction or loss of a restraining ice shelf will cause speed-up of flow from contiguous ice streams, contributing to sea-level rise, with greater changes from ice streams that are wider, have stickier beds, or have higher driving stress. Loss of buttressing offsetting half of the tendency for ice-stream/ice-shelf spreading for an ice stream similar to Pine Island Glacier, West Antarctica is modeled to contribute at least 1 mm of sea-level rise over a few decades. These results come from a new, simple model that includes relevant stresses in a boundary-layer formulation, and allows rapid estimation of ice-shelf impacts for a wide range of configurations. **Citation:** Dupont, T. K., and R. B. Alley (2005), Assessment of the importance of ice-shelf buttressing to ice-sheet flow, *Geophys. Res. Lett.*, 32, L04503, doi:10.1029/2004GL022024.

## 1. Introduction

[2] The non-floating portions of the Antarctic and Greenland ice sheets represent the largest potential sources of sea-level rise on time-scales of human economies. Response times to some environmental forcings are reassuringly long [e.g., Alley and Whillans, 1984], but recent observations from numerous regions show short-time-scale changes (few-annual and shorter) with potential to affect sea level rapidly [Zwally et al., 2002; Anandakrishnan et al., 2003; Thomas et al., 2004; Scambos et al., 2004; Rignot et al., 2004; Joughin et al., 2004; Shepherd et al., 2004]. Of particular importance is the ice-sheet response to changes in their floating extensions, called ice shelves. Shearing of ice shelves past slower-moving ice or rock causes a back-stress [Thomas and MacAyeal, 1982], so ice-shelf thinning or loss leads immediately (stress transmission at the speed of sound) to acceleration of ice-sheet flow contributing to sea-level rise. Ice shelves can be affected rapidly by environmental changes, including increase in basal melting of O(10 m/year) for warming of sub-ice-shelf waters by 1°C [Shepherd et al., 2004], and very rapid collapse (order of days or less) when meltwater wedges open crevasses. Speed-up of ice flow in response to ice-shelf changes is strongly implicated in changes now occurring in places including Jakobshavn Isbrae in Greenland, the former site of the Larsen B ice shelf along the Antarctic Peninsula, and the glaciers draining the West Antarctic ice sheet into Pine Island Bay.

[3] We have developed a simple, fast tool for assessing the importance of ice-shelf buttressing to inland-ice behavior, and the early stages of response to loss of that buttressing. For a reference case similar to the Pine Island Glacier

(P.I.G.) ice stream, West Antarctica, loss of buttressing offsetting half of the tendency for ice-stream/ice-shelf spreading leads to sea-level rise of about 1 mm from the ice stream itself, with a response time of about a decade. Response will be greater for ice streams with more buttressing, less side drag, more basal drag, and higher driving stress.

## 2. Model Description

[4] We use a mass- and momentum-balance model of a coupled ice-stream/ice-shelf system, loosely following MacAyeal [1989] as derived by Dupont [2004], solved using what we believe is a glaciologically novel Petrov-Galerkin approach [Dupont, 2004] providing high accuracy rapidly. All variables are non-dimensional unless otherwise noted, with dimensional scales listed in Table 1. The primary variables solved for are the ice thickness  $h(x, t)$  and along-flow velocity  $u(x, t)$ . As shown in Figure 1, the ice stream flows from  $x = 0$  to  $x = 1$  from left to right, through a parallel-sided, unit-width channel. Lateral thickness variation is neglected, so that ice is always the same thickness on the sides and in the stream. Side and basal drags are applied in boundary layers, with basal shear replaced by water pressure where ice is afloat.

[5] The elevation of the channel's bed  $z_r$  is specified as

$$z_r(x) = -r_{sw}^{-1} + \beta(x - 1) \quad (1)$$

where  $r_{sw}$  is the ratio of the density of seawater to ice, and  $\beta$  is the gradient in bed elevation. Note that given this bed elevation, the flotation thickness  $h_f$  is

$$h_f(x) = 1 + r_{sw}\beta(1 - x) \quad (2)$$

This is the maximum floating-ice thickness, such that ice is grounded if  $h > h_f$ , and floating otherwise.

### 2.1. Momentum Balance

[6] We model the momentum balance following MacAyeal [1989], as derived by Dupont [2004, equation (2.65)] with specified channel bed elevation:

$$\partial_x \left( 2h\nu\partial_x u - \frac{A}{2}h^2 \right) - G_s h u^{\frac{1}{2}} - \begin{cases} A\beta h + G_b u_m^{\frac{1}{2}}, & h > h_f \\ -\frac{A}{2r_{sw}}\partial_x h^2, & h \leq h_f \end{cases} = 0 \quad (3)$$

This non-dimensional, width-averaged and depth-integrated stress-equilibrium equation is appropriate for thin, channelized flow within ice streams and shelves. A fundamental

**Table 1.** Values of Scales, Constants and Parameters

Scale, Constant or Parameter	Value
$\rho_i$	$9.17 \times 10^2 \text{ kg m}^{-3}$
$\rho_{sw}$	$1.028 \times 10^3 \text{ kg m}^{-3}$
$g$	$9.81 \text{ m s}^{-2}$
$n$	3
$m$	1
$H$	$1 \times 10^3 \text{ m}$
$L_x$	$1 \times 10^5 \text{ m}$
$L_y$	$2 \times 10^4 \text{ m}$
$U$	$7.6 \times 10^{-5} \text{ ms}^{-1}$
$B_i$	$2 \times 10^8 \text{ Pa s}^{1/3}$
$\tau_b$	$7.3 \times 10^4 \text{ Pa}$
$\tau_y$	$5.5 \times 10^5 \text{ Pa}$
$r_{sw} = \rho_{sw}/\rho_i$	1.12
$T = L_x/U$	$1.3 \times 10^9 \text{ s} = 41 \text{ years}$

assumption here is that lateral and vertical strain rates only become important in narrow zones on the lateral and basal boundaries, respectively. Lateral and basal drag are then introduced through a boundary layer treatment, yielding a coupled balance between the longitudinal stress gradient and dynamic drag at the lateral and basal surfaces. Here  $\partial_x$  is the  $x$ -directed derivative,  $z_b = z_b(x, t)$  is elevation of the ice base,  $n = 3$  is the ice flow-law exponent, and  $m$  is the basal-material flow-law exponent.  $\nu$  is the strain-rate dependent effective viscosity, defined as

$$\nu \equiv |\partial_x u|^{\frac{1-n}{n}} \quad (4)$$

The dimensionless parameters  $A$ ,  $G_s$  and  $G_b$  are determined from the scales adopted as follows

$$A \equiv \frac{\rho_i g H}{B_i \left(\frac{U}{L_x}\right)^{\frac{1}{n}}} \quad (5)$$

$$G_s \equiv \frac{L_x \tau_y}{L_y B_i \left(\frac{U}{L_x}\right)^{\frac{1}{n}}} \quad (6)$$

$$G_b \equiv \frac{L_x \tau_b}{H B_i \left(\frac{U}{L_x}\right)^{\frac{1}{n}}} \quad (7)$$

where  $H$  is the thickness scale,  $L_x$  the longitudinal length scale,  $L_y$  the lateral or width length scale,  $U$  the velocity scale,  $\tau_y$  the lateral shear-stress scale,  $\tau_b$  the basal shear-stress scale, and  $B_i = A_i^{-n}$  is the ice hardness parameter, with  $A_i$  being the usual softness parameter. The dimensionless parameters  $A$ ,  $G_s$ , and  $G_b$  measure the respective strengths of the driving stress, lateral shear stress and basal shear stress relative to the stress required to produce a longitudinal strain rate  $U/L_x$ .

[7] Two momentum-balance boundary conditions are applied, one at each end of the domain. At the upstream end ( $x = 0$ ) the velocity is a prescribed constant,

$$u(0, t) = u_0 \quad (8)$$

[8] The second boundary condition is

$$2\nu \partial_x u - \frac{A}{2} h^2 = \frac{A}{2} [(f-1)r_{sw}z_b^2 - fh^2], \quad x = 1 \quad (9)$$

where  $f$  is the buttressing parameter. When  $f = 0$ , the boundary condition is that appropriate for an appended free-floating ice shelf [Weertman, 1974]. For such an ice shelf the depth-integrated seawater pressure acting on the ice in the vertical plane must be balanced by the depth-integrated longitudinal stress at the point of attachment ( $x = 1$ ). This stress is the sum of the depth-integrated glaciostatic pressure and depth-integrated viscous stress. In this case the difference between the depth-integrated glaciostatic and hydrostatic pressures will always be positive, thus requiring viscous stretching to balance the forces. When  $f = 1$ , no static pressure difference is permitted, and the strain-rate is therefore zero. This is appropriate for an appended ice shelf that balances through lateral drag or localized basal drag the force producing the strain-rate in the  $f = 0$  case. This  $f = 1$  case we refer to as fully buttressed. The  $f = 0$  case we refer to as unbuttressed. Any value of  $f$  between these two cases is partially buttressed (the overbuttressed case, where  $f > 1$ , is not considered here).

## 2.2. Mass Balance

[9] The system mass balance is governed by the equation

$$\partial_t h = -\partial_x (uh) \quad (10)$$

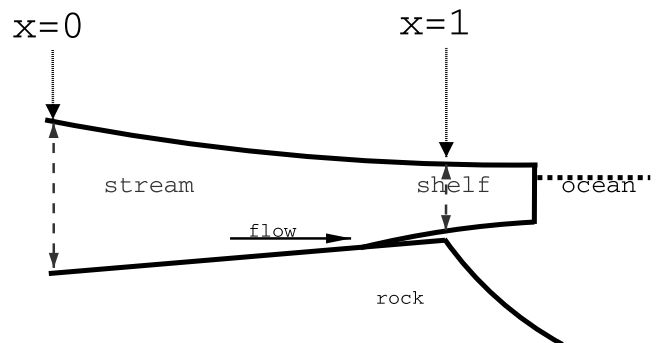
where accumulation is neglected and  $t$  is the non-dimensional time coordinate, whose dimensional scale is  $T$ . Equation (1) is derived from continuity and states that the time-rate-of-change in thickness is dictated by the convergence of depth-integrated flux.

[10] The upstream-end thickness is prescribed, serving as the one mass-balance boundary condition,

$$h(0, t) = h_0 \quad (11)$$

## 3. Experiments and Results

[11] We seek to assess the sensitivity of an ice-stream/ice-shelf system to buttressing changes. We discretize the equations spatially using linear finite elements, and tempo-



**Figure 1.** Schematic geometry. See color version of this figure in the HTML.

**Table 2.** Experiment Parameter Sets

Experiment	$G_s$	$\beta$	$G_b$	$A$	$h_0$	$u_0$
reference	15	0.3	40	50	1.9	0.53
$+G_s$	<b>18</b>	0.3	40	50	1.9	0.53
$-\beta$	15	<b>0.24</b>	40	50	1.9	0.53
$+G_b$	15	0.3	<b>48</b>	50	1.9	0.53
$+A$	15	0.3	40	<b>60</b>	1.9	0.53
$-A$	15	0.3	40	<b>40</b>	1.9	0.53
$+h_0$	15	0.3	40	50	<b>2.3</b>	0.53
$+u_0 = 1/h_0$	15	0.3	40	50	1.9	<b>0.63</b>

rally using semi-implicit finite differencing. Dupont [2004, Appendix B] gives the details of the what we believe is a novel (in the context of computational glaciology) Petrov-Galerkin approach employed for the discretization of the evolution equation (1).

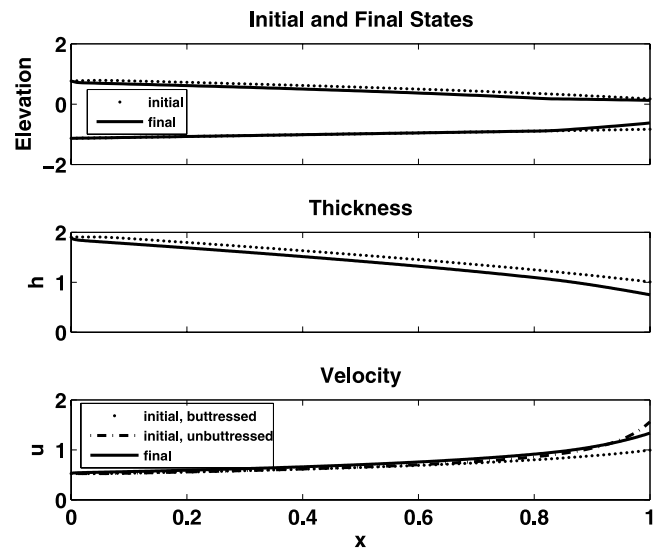
[12] Here, we start all the experiments from an initial steady state appropriate for  $f = 0.5$  (50% buttressing). We then set  $f = 0$  (unbuttressed) and let the system evolve to a new steady-state. Table 2 lists the parameter values adopted for each experiment. One experiment was designated the reference experiment. The values chosen for this reference experiment are roughly appropriate for P.I.G. Sensitivity of the results to our parameter-value choices is listed in Table 3.

[13] For the reference experiment, Figure 2 shows that the removal of buttressing increases velocity greatly at the downstream end. Thinning ensues until a new steady-state is achieved. Relative thinning  $>10\%$  extends upstream  $>30\%$ . Figure 3 indicates that the bulk of the evolution is accomplished within the first unit of non-dimensional time. This corresponds to a time scale of 41 years, given our P.I.G.-like length and velocities scales noted in Table 1; choosing a larger length scale or a smaller velocity scale would yield a longer time scale for the response. Also shown in Figure 3 is the evolution of the integrated volume of ice above flotation (VAF), which would directly contribute to sea-level rise. The grounding line retreated about 18% of the domain length in total, which would correspond to an 18 km retreat given a 100 km length scale. The net loss in VAF is 0.11 or roughly one third of the initial value of 0.342; this would correspond to roughly  $300 \text{ km}^3$  of ice (or 1 mm sea-level equivalent) added to the ocean given length, thickness and width scales of 100, 1, and 3 km respectively.

[14] Table 3 shows the sensitivity of the results to parameter-value changes within glaciologically reasonable scales. Results in all experiments shown are qualitatively the same as the reference experiment. Quantitatively, an increase in side drag, as measured by  $G_s$ , modestly decreases grounding-line retreat and VAF loss, whereas an increase in driving-stress or basal-drag strength, as measured by  $A$  and  $G_b$ , respectively, increases VAF loss. This

**Table 3.** Summary of the Results of Various Experiments

Experiment	Init'l g.l.	Final g.l.	g.l. Retreat	Init'l VAF	Final VAF	VAF Loss (% of Init'l)
reference	1	0.82	0.18	0.34	0.23	0.11 (33%)
$+G_s$	1	0.86	0.14	0.37	0.27	0.10 (27%)
$-\beta$	1	0.83	0.17	0.35	0.24	0.11 (32%)
$+G_b$	1	0.83	0.17	0.38	0.26	0.12 (32%)
$+A$	0.93	0.71	0.22	0.21	0.14	0.11 (43%)
$-A$	1	0.92	0.08	0.43	0.35	0.09 (20%)
$+h_0$	1	0.82	0.18	0.34	0.23	0.12 (34%)
$+u_0$	1	0.86	0.14	0.39	0.28	0.11 (28%)

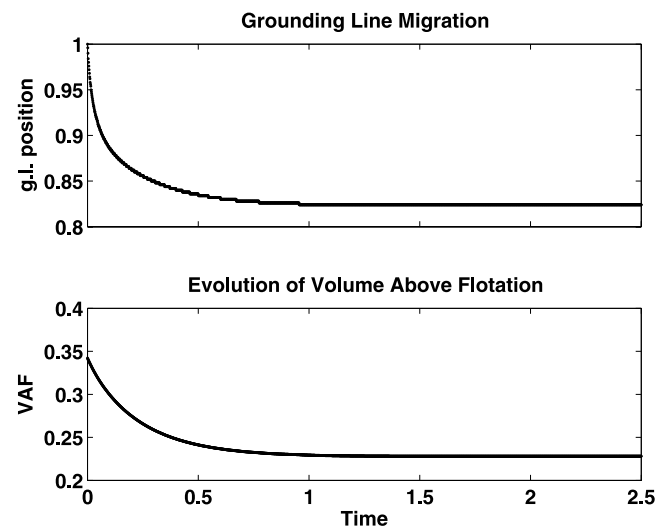


**Figure 2.** Plot showing initial and final states of the ice-shelf/ice-stream system for the standard or reference experiment. Upper panel shows surface and basal elevations at initial and final states, and lower two panels show initial and final thickness and velocity profiles.

suggests that, in terms of VAF loss and thus sea-level rise, buttressing changes matter more to systems with higher basal drag, higher driving stress, and lower side drag.

#### 4. Conclusions

[15] We find that a reduction in buttressing causes notable grounding-line retreat and loss in volume of ice above flotation. Response extends far upstream from where the buttressing change was imposed. In fact, the upstream extent of the response we estimate is a minimum given that we hold the upstream velocity and thickness (and thus the flux) constant in time, and we do not allow flow into the ice stream from the sides. Without these restrictions, which



**Figure 3.** Evolution of the grounding line (g.l.) and integrated volume above flotation (VAF) after removal of buttressing.

introduce an artificial stability to the system, we expect the thinning would continue upstream beyond  $x = 0$  in a manner similar to the diffusive response in kinematic wave theory [Kamb, 1964; Alley and Whillans, 1984], as well as laterally into the regions flanking the ice stream. The results are also conservative in light of our neglect of accumulation in equation (1), because ice ungrounding and exposing the basal surface to ocean water is likely to cause substantial melting [Jacobs *et al.*, 1996; Shepherd *et al.*, 2004; Payne *et al.*, 2004]. This feedback, which would represent the introduction of a strong mass-loss to the system, would tend to augment the grounding line retreat and loss of volume above flotation. Including such a feedback and freeing the upstream boundary may well allow for a much greater response [e.g., Hughes, 1980], perhaps faster than can be stabilized by negative feedbacks such as isostatic rebound. In addition, our choice of unbuttressing from an initial state of 50% buttressing is likely conservative for some ice streams, such as the essentially 100% buttressed Whillans ice stream [Whillans and Van der Veen, 1993].

[16] We find that sensitivity to buttressing changes is greater for systems with weaker side drag, larger basal drag, or larger driving stress strength, as measured by the dimensionless numbers  $G_s$ ,  $G_b$  and  $A$ , respectively. Thus, with all other things equal, the wider an ice stream, the ‘stickier’ its bed, or the larger its driving stress, the more sensitive it will be to perturbations in buttressing.

[17] The parameter values chosen here are appropriate for P.I.G. Recent observations there indicate rapid ice-shelf thinning from warming of subjacent water, leading to grounding-line retreat with grounded-ice speed-up and thinning over years, especially downglacier [Thomas *et al.*, 2004; Shepherd *et al.*, 2004]. We lack sufficient information on pre-perturbation buttressing and the timing of buttressing loss to conduct an accurate diagnosis of the ongoing changes, but the broad similarity of modeled and observed changes improves confidence in our model and in published diagnoses.

[18] Rutford Ice Stream has high driving stress and high basal drag similar to P.I.G., and thus might be expected to show similar sensitivity to buttressing changes. The Siple Coast ice streams generally have lower driving stress but some are wider, with opposing effects on sensitivity, so an additional suite of experiments will be required. Our tool can be used to assess the sensitivity of any other present or past [e.g., Hulbe *et al.*, 2004] ice streams to buttressing changes. It is evident from the experiments to date that buttressing cannot be ignored in assessing ice-sheet response to environmental change except in extreme limiting cases.

[19] **Acknowledgments.** We thank D. MacAyeal, B. Parizek, S. Anandakrishnan, and the rest of the Penn State ice & climate group, as well as T. Hughes and an anonymous reviewer, for helpful comments. We gratefully acknowledge the support of the US National Science Foundation (grants 9870886 and 0126187) and the Gary Comer Foundation for partial support.

## References

- Alley, R. B., and I. M. Whillans (1984), Response of the East Antarctic ice sheet to sea-level rise, *J. Geophys. Res.*, *89*, 6487–6493.
- Anandakrishnan, S., D. E. Voigt, R. B. Alley, and M. A. King (2003), Ice stream D flow speed is strongly modulated by the tide beneath the Ross Ice Shelf, *Geophys. Res. Lett.*, *30*(7), 1361, doi:10.1029/2002GL016329.
- Dupont, T. K. (2004), Abrupt changes in ice shelves and ice streams: Model studies, Ph.D. thesis, Pa. State Univ., University Park.
- Hughes, T. J. (1980), The weak underbelly of the West Antarctic Ice Sheet, *J. Glaciol.*, *27*(97), 518–525.
- Hulbe, C. L., D. R. MacAyeal, G. H. Denton, J. Kleman, and T. V. Lowell (2004), Catastrophic ice shelf breakup as the source of Heinrich event icebergs, *Paleoceanography*, *19*, PA1004, doi:10.1029/2003PA000890.
- Jacobs, S. S., H. H. Hellmer, and A. Jenkins (1996), Antarctic ice sheet melting in the southeast Pacific, *Geophys. Res. Lett.*, *23*(9), 957–960.
- Joughin, I., W. Abdalati, and M. Fahnestock (2004), Large fluctuations in speed on Greenland’s Jakobshavn Isbrae glacier, *Nature*, *432*(7017), 608–610.
- Kamb, B. (1964), Glacier geophysics, *Science*, *146*(3642), 353–365.
- MacAyeal, D. R. (1989), Large-scale ice flow over a viscous basal sediment: Theory and application to Ice Stream B, Antarctica, *J. Geophys. Res.*, *94*(B4), 4071–4087.
- Payne, A. J., A. Vieli, A. P. Shepherd, D. J. Wingham, and E. Rignot (2004), Recent dramatic thinning of largest West Antarctic ice stream triggered by oceans, *Geophys. Res. Lett.*, *31*, L23401, doi:10.1029/2004GL021284.
- Rignot, E., G. Casassa, P. Gogineni, W. Krabill, A. Rivera, and R. Thomas (2004), Accelerated ice discharge from the Antarctic Peninsula following the collapse of Larsen B ice shelf, *Geophys. Res. Lett.*, *31*, L18401, doi:10.1029/2004GL020697.
- Scambos, T. A., J. A. Bohlander, C. A. Shuman, and P. Skvarca (2004), Glacier acceleration and thinning after ice shelf collapse in the Larsen B embayment, Antarctica, *Geophys. Res. Lett.*, *31*, L18402, doi:10.1029/2004GL020670.
- Shepherd, A., D. Wingham, and E. Rignot (2004), Warm ocean is eroding West Antarctic Ice Sheet, *Geophys. Res. Lett.*, *31*, L23402, doi:10.1029/2004GL021106.
- Thomas, R., and D. R. MacAyeal (1982), Derived characteristics of the Ross Ice Shelf, *J. Glaciol.*, *28*, 397–412.
- Thomas, R., *et al.* (2004), Accelerated sea-level rise from West Antarctica, *Science*, *306*(5694), 255–259.
- Weertman, J. (1974), Stability of the junction of an ice sheet and an ice shelf, *J. Glaciol.*, *13*(67), 3–11.
- Whillans, I. M., and C. J. Van der Veen (1993), New and improved determinations of velocity of ice streams B and C, West Antarctica, *J. Glaciol.*, *39*(133), 483–490.
- Zwally, H., W. Abdalati, T. Herring, K. Larson, J. Saba, and K. Steffen (2002), Surface melt-induced acceleration of Greenland ice-sheet flow, *Science*, *297*(5579), 218–222.

R. B. Alley and T. K. Dupont, 532 Deike Building, Department of Geosciences, The Pennsylvania State University, University Park, PA 16802, USA. (dupont@geosc.psu.edu)