Logical and empirical flaws in applications of simple climate-economy models

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Abstract. Simple climate-economy models are widely used for climate policy analysis, despite the limitations associated with their lack of regional and process detail. One of the main arguments brought forward in favour of these models is their transparency, which should enable researchers to easily interpret the simulation results and adapt the model design to their specific research interests. We investigate the degree to which this claim is supported in the case of the DICE model but most of our findings are relevant for other welfare-optimizing climate-economy models as well. Our review comprises the handling of time discounting in social welfare functions, the combination of different social welfare functions, the calibration of uncertain climate parameters, the representation of uncertainty about future climate change, and the evolution of carbon abatement costs over time. The unsettling conclusion of our reanalysis is that each of these aspects has been treated inconsistently in previous studies. We demonstrate that these flaws are not only of theoretical interest but they can also strongly affect the policy recommendations drawn from the simulation results. We provide recommendations how future climate policy analyses can avoid the problems uncovered in this paper. In particular, we present guidelines on the consistent use of social welfare functions in welfare-optimizing climate-economy models and on the calibration and probabilistic representation of uncertain model parameters. Our findings indicate that much more caution is needed in the development, application, and modification of simple climate-economy models, and in the interpretation of their results. The combined efforts of original model developers, analysts applying and adopting existing models, and peer reviewers are required to ensure that model applications are scientifically sound, and that the policy conclusions drawn from a particular model experiment are actually supported by the simulation results.

Keywords: climate change, integrated assessment, DICE model, social welfare function, growth discounting, abatement costs, probabilistic analysis

Abbreviations: CRRA – constant relative risk aversion; DU – discounted utility; GCM – general circulation model; GHG – greenhouse gas; GMT – global mean temperature; GWP – gross world product; PDF – probability density function; PV – present value; PVC – present value of consumption; PVO – present value of output; SWF – social welfare function; THC – thermohaline circulation

1. Introduction

Simple climate-economy models are still being used for a variety of climate policy analyses. While researchers are well aware of the limitations of these highly aggregated models resulting from their lack of regional and process detail, three advantages are often emphasized to justify their continued use (see, *e.g.*, Kolstad, 1998). First, many researchers favour simple models in which reduced-form functions replace complex processes that we are not able to model explicitly, and whose parameters can be easily varied in sensitivity analyses. Second, the computational efficiency of simple models allows the performance of sophisticated probabilistic analyses. Third, the *simplicity* of these models is generally equated with *transparency* in the sense that individual researchers can fully understand the model design and interpret the simulation results. In this paper, we investigate the degree to which the last argument is actually supported by previous studies with simple welfare-optimizing climate-economy models.

Our analysis focuses on the DICE (Dynamic Integrated model of Climate and the Economy) model, one of the most widely used global climate-economy models. DICE is a global integrated assessment model of the economics of climate change that was developed by W.D. Nordhaus and collaborators. DICE links a neo-classical Ramsey–Cass–Koopmans optimal-growth model of the world economy in which a central planner maximizes intertemporal welfare subject to certain constraints to a description of anthropogenic climate change with the implied economic impacts. Economic output is described by a constant-returns-to-scale Cobb-Douglas production function with labor and capital as input factors. Applications of DICE in a cost-benefit or cost-effectiveness framework maximize the discounted utility from consumption by determining the optimal division of economic output over time into consumption, investment, and emissions abatement.

Three versions of DICE are distinguished in the literature: DICE (Nordhaus, 1992; Nordhaus, 1993), DICE-94 (Nordhaus, 1994), and DICE-99 (Nordhaus and Boyer, 2000). The main developer of DICE has also developed a probabilistic version of DICE: PRICE (Nordhaus and Popp, 1997), a version of DICE that considers induced innovation: R&DICE (Nordhaus, 1999), and a series of regionally disaggregated climate-economy models: RICE-96 (Nordhaus and Yang, 1996), RICE-99 (Nordhaus and Boyer, 2000), and RICE-2001 (Nordhaus, 2001b). The DICE and RICE models have been used for climate policy analysis not only by their original model developers but also by other researchers who frequently adapted them to investigate a variety of scientific and policy questions (Roughgarden and Schneider, 1999; Keller et al., 2000; Mastrandrea and Schneider, 2001; Azar and Lindgren, 2003; Buonanno et al., 2003; Mastrandrea and Schneider, 2004; Moles et al., 2004; Newell and Pizer, 2004; Popp, 2004; Yohe et al., 2004; Keller et al., 2004; Keller et al., 2005; Fankhauser and Tol, 2005; Schlesinger et al., 2005; Smirnov, 2005; Bosetti and Gilotte, 2005; Yohe et al., 2006). In this paper, we focus on the original DICE-99 model as described by Nordhaus and Boyer (2000) and on a modified version applied by Yohe et al. (2004) and Yohe et al. (2006).

The maximization of intertemporal welfare, as implemented by the DICE models, has been criticized, among others, for its perfect-market assumption, its assumption of full substitutability between market commodities and environmental goods and services, its neglect of the allocation of rights, its implicit assumption that intergenerational compensation is actually feasible, its inability to account reliably for deep uncertainty or catastrophic outcomes, and the weak empirical basis of widespread practices such as assuming representative agents and decision-makers that maximize global intertemporal welfare, applying logarithmic utility, and exponential time discounting (Lind et al., 1982; Taylor, 1982; Lind, 1995; Lind and Schuler, 1998; Spash, 2002; Azar and Lindgren, 2003; DeCanio, 2003; Yohe, 2003; Gowdy, 2005). Other studies have questioned various parameterizations of the DICE models, such as its assumptions on demographic and economic development, the representation of abatement costs, the formulation of the carbon cycle, and the representation of climate change impacts (Chapman et al., 1995; Grubb et al., 1995; Kaufmann, 1997; Schultz and Kasting, 1997; Courtois, 2004).

The focus of the discussion in this paper is on logical and methodological inconsistencies in applications of DICE and other optimizing climate-economy models. In the context of this discussion, we do accept the welfare maximization paradigm. Consequently, this paper is particularly relevant for those analysts who see some merit in using aggregated climate-economy models as it helps them steer clear of several important inconsistencies. The problems discussed in this paper occur in different versions of the DICE model (*e.g.*, the 'original' DICE-99 model *vs.* the version applied by Yohe et al., 2004), and some of them relate to inconsistencies between different model implementations (*e.g.*, GAMS *vs.* Excel implementation of DICE-99). For that reason, most of them cannot be addressed by some quick fixes to the model code. We do, however, provide recommendations how future climate policy analyses can avoid the problems uncovered in this paper.

This paper is structured as follows. Sect. 2 reviews the application of welfare metrics in DICE and other optimizing climate-economy models. The discussion addresses time discounting and its link to the index number problem, inconsistencies between welfare metrics due to different aggregations across time and states of the world, and recommendations for the consistent use of welfare metrics in optimizing climate-economy models. For a more extensive and theoretical discussion of many topics addressed in this section, see Füssel (2006). Sect. 3 addresses several other inconsistencies in applications of DICE, including the extrapolation of uncertain climate parameters beyond their physically plausible range and the undocumented specification of radically different abatement cost curves. While this discussion is more specific to DICE, it provides important lessons for other models as well. Sect. 4 concludes the paper.

2. Welfare metrics applied in global climate-economy models

The ultimate goal of climate policy analysis according to the welfare maximization paradigm is to assess alternative policies according to a predefined social welfare function (SWF). A SWF is an algebraic formulation that assigns numerical social utility to each possible social state. In the context of this paper, we use this term broadly to refer to any mathematical formulation that assigns a numerical value to a stream of economic output and/or consumption derived from a global climate-economy model. The models considered here assume that individual utility is determined by a single economic good, and that all individuals can be characterized by the same utility function.

Ideally, a SWF would be derived from the revealed preferences of the individuals concerned. However, Arrow's Impossibility Theorem (Arrow, 1951) shows that there is no unique method for aggregating individual preferences into social preferences. Even if such an aggregation was theoretically possible, it would not be practical in the case of climate policy analysis, which needs to consider future generations who cannot reveal their preferences today. In this situation, the SWFs applied in the climate change context should aim to reflect the implicit or explicit preference structure of current decision-makers.

The main SWFs that have been used by climate-economy models for assessing the costs and benefits associated with alternative climate policies are discounted utility of consumption (DU), present value of consumption (PVC), and present value of economic output (PVO). These SWFs has been defined for different assumptions regarding time discounting. In this section, we analyze several welfare metrics that have been used in connection with the DICE model. Even though the examples are drawn from the DICE family of models, this discussion applies to all optimizing climate-economy models. 'We note that the (uncritical) application of welfare economics in the climate change context has been criticized for various reasons not addressed in this paper (see Sect. 1 for selected references). Our goal in this section is to help avoid the introduction of *additional* inconsistencies that may be caused by choosing flawed SWFs or by combining SWFs inappropriately.

This section is structured as follows. Sect. 2.1 presents the welfare metrics investigated here; Sect. 2.2 and 2.3 investigate the implications of different time discounting schemes for ordinal and cardinal welfare metrics, respectively; Sect. 2.4 discusses inconsistencies between several internally consistent SWFs; Sect. 2.5 presents four recommendations for the application of SWFs in climate policy analysis based on the welfare maximization paradigm; and Sect. 2.6 applies these rules to a critical reanalysis of two earlier analyses with DICE.

2.1. Welfare metrics applied in welfare-optimizing climate-economy models

A variety of welfare metrics have been used for comparing alternative policy strategies determined by the DICE model and other welfare-optimizing climate-economy models. In this section, we present six SWFs that take a finite output or consumption stream (expressed in currency, such as dollars) as input and calculate a scalar welfare value (expressed either in currency or in arbitrary 'utils') as output. All of them are defined as the discounted intertemporal sum of the welfare in each time step, and they assume deterministic future discount rates. We neglect the difference between the annual specification of the welfare functions defined here and the decadal time step of the DICE model.

The following notation is used in the definition of the SWFs:

Y	≥ 0	(net) economic output
C	≥ 0	consumption
I	≥ 0	investment
L	> 0	population

$\theta \geq 0$ intertemporal elasticity of substitution
g *actual* growth rate of per capita consumption
$\tilde{g}$ *assumed* growth rate of per capita consumption
$\rho \geq 0$ pure rate of time preference (aka 'utility discount rate')
r social discount rate

These variables and parameters may be supplemented with a time index t (*e.g.*, L_t denotes population in year t), whereby $t = 0$ refers to the present year and $t = T$ to the final year of a time series. If such an index is missing, the respective variable is assumed to be constant over time. $X_{u \cdots v}$ denotes the stream of variable X from time u to time v (assuming $u \leq v$). For notational convenience, we assume the value of the 'empty product' to be one, *i.e.*, $\prod_{t=1}^{0} X_t = 1$.

The definition of the SWFs in Eq. 1–6 includes parameters that reflect social value judgments about the distribution of wealth within and across generations. There is a wide range of literature on the most appropriate values of these parameters (Lind et al., 1982; Arrow et al., 1996; Nordhaus, 1997; Heal, 1997; Portney and Weyant, 1999; Toth, 2000; Howarth, 2003; Newell and Pizer, 2004). The standard value for θ in economic models of climate change is unity (Arrow et al., 1996; DeCanio, 2003). The corresponding logarithmic (or Bernoullian) utility function is also applied in the DICE models. There is more disagreement on appropriate values for ρ, and on the question whether this parameter should be constant over time. The default value in DICE-94 is $\rho = 3\%/yr$ (Nordhaus, 1994, p. 104), the original DICE-99 model assumes that ρ declines over time, from $\rho = 3\%/yr$ in 1995 to $\rho = 1.25\%/yr$ in 2335 (Nordhaus and Boyer, 2000, pp. 15–16), and the adaptation of DICE-99 applied in Yohe et al. (2004) assumes $\rho = 0\%/yr$.

$$\mathrm{DU}_{\mathrm{DICE}}(C_{0\cdots T}, L_{0\cdots T}; \rho_{1\cdots T}) = \sum_{t=0}^{T} \frac{L_t \cdot \ln(C_t/L_t)}{\prod_{t'=1}^{t} (1+\rho_{t'})} \tag{1}$$

$$\begin{aligned} \mathrm{PVC}_{\mathrm{DICE}}(C_{0\cdots T}, L_{0\cdots T}; \rho_{1\cdots T}) &= \sum_{t=0}^{T} \frac{C_t}{\prod_{t'=1}^{t} (1+\rho_{t'}) \cdot \frac{C_{t'}/L_{t'}}{C_{t'-1}/L_{t'-1}}} \\ &= \frac{C_0}{L_0} \cdot \sum_{t=0}^{T} \frac{L_t}{\prod_{t'=1}^{t} (1+\rho_{t'})} \end{aligned} \tag{2}$$

$$\mathrm{PVC}_{\mathrm{end}}(C_{0\cdots T}, L_{0\cdots T}; \rho_{1\cdots T}, \theta) =$$

$$\sum_{t=0}^{T} \frac{C_t}{\prod_{t'=1}^{t} \left(1 + \rho_{t'} + \theta \cdot \left(\frac{C_{t'}/L_{t'}}{C_{t'-1}/L_{t'-1}} - 1\right)\right)} \quad (3)$$

$$\mathrm{PVC}_{\mathrm{ex}}(C_{0\cdots T}; \rho_{1\cdots T}, \theta, \tilde{g}) = \sum_{t=0}^{T} \frac{C_t}{\prod_{t'=1}^{t} (1 + \rho_{t'} + \theta \cdot \tilde{g})} \quad (4)$$

$$\mathrm{PVO}_{\mathrm{ex}}(Y_{0\cdots T}; \rho_{1\cdots T}, \theta, \tilde{g}) = \sum_{t=0}^{T} \frac{Y_t}{\prod_{t'=1}^{t} (1 + \rho_{t'} + \theta \cdot \tilde{g})} \quad (5)$$

$$\mathrm{PVO}_{\mathrm{Yohe}}(Y_{0\cdots T}, L_{0\cdots T}) = \sum_{t=0}^{T} \frac{Y_t}{\prod_{t'=1}^{t} \left(1 + \ln \frac{C_{t'}/L_{t'}}{C_{t'-1}/L_{t'-1}}\right)} \quad (6)$$

$\mathrm{DU}_{\mathrm{DICE}}$ describes the logarithmic utility of consumption based on 'classic' utility discounting at the rate of pure time preference. This utility function is used as objective function in the original DICE-99 model (Nordhaus and Boyer, 2000, p. 181).

The other SWFs express welfare in monetary units (*i.e.*, currency). They apply some variant of growth discounting, which focusses on the social marginal utility of consumption today compared with consumption in the future and represents the 'classical' approach to time discounting (Arrow et al., 1996; Nordhaus, 1997; Heal, 1997; Tol, 1999; Toth, 2000). The conventional formula for social time preference, also known as the 'Ramsey growth discounting rule', is $r = \rho + \theta g$, whereby r is often referred to as social rate of time preference or utility discount rate. However, this formula is only an approximate solution of the Ramsey model (Füssel, 2006; DeCanio, 2003, Section 3.3.1).

$\mathrm{PVC}_{\mathrm{DICE}}$ describes the present value of consumption as calculated in the original DICE-99 model, which applies a variant of growth discounting.

$\mathrm{PVC}_{\mathrm{end}}$ describes the present value of consumption according to the conventional formulation of the Ramsey rule. In $\mathrm{PVC}_{\mathrm{end}}$, the discount rate is determined based on the *endogenously* determined growth rate of per capita consumption in each year. This SWF has been widely applied in global economic models of climate change (see, *e.g.*, Tol, 1999).

$\mathrm{PVC}_{\mathrm{ex}}$ also describes the present value of consumption according to the Ramsey growth discounting rule. In contrast to $\mathrm{PVC}_{\mathrm{end}}$, the discount rate is determined based on an *exogenously* specified assumed growth rate of per capita consumption. This welfare measure has been used to determine the total welfare effects of climate policies in DICE-99 (Nordhaus and Boyer, 2000, p. 127).

PVO_{ex} describes the present value of economic output according to the Ramsey growth discounting rule, whereby the discount rate is determined based on an exogenously specified assumed growth rate. PVO_{ex} is identical to PVC_{ex}, except that economic output is substituted for consumption. A special case of this welfare function (assuming $\theta = \rho_t = 0$) is applied in Fankhauser and Tol (2005), which apparently uses undiscounted gross world product (GWP) calculated by different versions of DICE-94.

PVO_{Yohe} describes the present value of economic output applying yet another variant of growth discounting. This SWF has been applied in a modified version of the DICE model (Yohe et al., 2004, and Yohe, pers. comm.) Neither θ nor ρ are contained in the specification of PVO_{Yohe} since Yohe et al. (2004) assumes $\theta = 1$ and $\rho = 0$. We note that Yohe et al. (2004, SOM pp. 1–2) states that *"In this Policy Forum, the pure rate of time preference is set equal to zero. With an elasticity of marginal utility equal to unity, the social discount rate is simply the endogenously determined rate of annual growth of per capita consumption."* This text suggests that monetary values were discounted using the discounting scheme from PVC_{DICE} or PVC_{end} (the two are identical for $\theta = 1$ and $\rho = 0$). If this had indeed been the case, PVC would be identical across all consumption scenarios (see Sect. 2.2). However, the model code that was kindly provided by G. Yohe revealed that the SWF that has actually been used in determining discounted GWP and selecting the costs of alternative policies is PVO_{Yohe}.

2.2. Growth discounting in ordinal SWFs

In this subsection, we analyze how the discounting schemes applied in the *monetary* SWFs applied in connection with the DICE model (Eq. 2–6) rank alternative consumption paths. To this end, we employ a monotonicity criterion that requires a SWF to assign higher welfare to a constant-growth consumption scenario with a strictly higher growth rate (and thus strictly higher consumption) than to one with a lower growth rate, everything else being equal. This analysis assumes consumption paths with a constant growth rate of the form

$$\mathrm{C}(t; C_0, g) \;=\; C_0 \cdot (1+g)^t, \tag{7}$$

whereby C_0 denotes initial consumption at $t = 0$ and g the rate of consumption growth. In addition, we assume constant population, savings rate, pure rate of time preference, and elasticity of the marginal utility of consumption. Formally, a discounted welfare metric DW fulfills the monotonicity criterion if

$$\mathrm{DW}(\mathrm{C}(t; C_0, g), t) \; < \; \mathrm{DW}(\mathrm{C}(t; C_0, g + \Delta g), t)$$

for all positive C_0, g, Δg, and t.

The motivation for this monotonicity criterion is our firm conviction that the vast majority of climate policy-makers seeking advice from optimal growth models would clearly prefer a policy scenario with consistently higher consumption growth over one with lower consumption growth, everything else being equal. This assumption is also made implicitly in most climate policy analyses with optimal growth models. Fankhauser and Tol (2005), for instance, apply DICE to compare indirect climate impacts under different assumptions, using future loss in *undiscounted* GDP as the main decision criterion. Their conclusions are, therefore, dependent on the assumption that a higher-growth pathway is always preferred. Therefore, we consider SWFs that violate the monotonicity criterion to be inconsistent with the preference structure of the target users and thus unsuitable for comparing alternative climate policies.

We find the following behaviour for the SWFs presented above (for a proof, see Füssel, 2006):

$\mathrm{DU}_{\mathrm{DICE}}$, $\mathrm{PVC}_{\mathrm{ex}}$, $\mathrm{PVO}_{\mathrm{ex}}$ and $\mathrm{PVO}_{\mathrm{Yohe}}$ always prefer the high-growth scenario over the low-growth scenario. Hence, they do fulfill the monotonicity criterion.

$\mathrm{PVC}_{\mathrm{DICE}}$ is insensitive to the consumption levels after the initial period. Hence, this SWF does *not* fulfill the monotonicity criterion. We note that the "stationarity axiom" proposed by Koopmans (1960) is violated as well. This axiom demands that if two sequences have the same start, then eliminating that common start and bringing the rest forward does not change their ranking.

$\mathrm{PVC}_{\mathrm{end}}$ fulfills the monotonicity criterion for $\theta < 1 + \rho$ but not for $\theta \geq 1 + \rho$. (For $\theta = 1$ and $\rho = 0$, $\mathrm{PVC}_{\mathrm{end}}$ is identical to $\mathrm{PVC}_{\mathrm{DICE}}$.)

$\mathrm{PVC}_{\mathrm{end}}$ fulfills the monotonicity criterion for some values of θ only. We note that the 'switch value' for the dimensionless parameter θ depends linearly on the dimensional rate parameter ρ. For instance, the numerical value of ρ is more than ten times larger when it is expressed per decade rather then per year. As a result, $\mathrm{PVC}_{\mathrm{end}}$

may or may not fulfil the monotonicity criterion depending on the (arbitrary) choice of time step for its specification. The reason for this inconsistency is that growth discounting in PVC_{end} is based on an approximate solution rather than the exact solution of the Ramsey model (see Füssel, 2006).

In summary, PVC_{DICE} and PVC_{end} violate the monotonicity criterion for many plausible parameter choices, including for $\theta = 1$ and $\rho = 0$. Consequently, we consider these two SWFs unsuitable for evaluating and comparing alternative climate policies.

2.3. Growth discounting in cardinal SWFs

The monotonicity criterion discussed in Sect. 2.2 only considers the ranking of alternative policies, *i.e.*, it regards the welfare metrics as *ordinal*. In this subsection, we analyze how the discounting schemes applied in the various monetary SWFs presented in Sect. 2.1 value the difference in present value (PV) between alternative policies. Monetary welfare metrics are, by definition, *cardinal* (*i.e.*, rational-scaled). The crucial question addressed in this context is as follows: *"Is the practice of using different discount factors in the PV calculations for alternative policies consistent, or not?"*

One opinion is expressed by the main developer of the DICE models: *"The present values are computed using the base case discount factors."* (Nordhaus and Boyer, 2000, p. 127) and *"In making welfare comparisons between two different policies, the same relative prices should be used to discount the future consumption streams that result from both policies. Thus, in constructing the comparison measures Total abatement cost of policy [. . .], we use the base case relative prices to discount both base case consumption and consumption under current policy."* (Nordhaus, 2001a, p. 19). Consequently, the original DICE-99 model applies PVC_{ex} to calculate the monetary welfare associated with different policy alternatives. Other analyses apply SWFs that determine the discount factors for each policy option endogenously: Yohe et al. (2004) applies PVO_{Yohe}, and FUND apparently applies PVC_{end} (Tol, 1999). Interestingly, the first authors of both studies are fully aware of the problems associated with growth discounting when the discount rates are determined endogenously. They find that this discounting approach may lead to infinite expected damages from climate change (and infinite expected marginal benefits of mitigation) if there is the possibility for a catastrophic outcome (Tol, 2003), or if the analyst

Table I. Present value difference for various discounting schemes between two consumption streams growing at different rates over a 10-year period.

	PVC_{DICE}, PVC_{end}	PVO_{Yohe}	PVC_{ex} $[\tilde{g} = 3\%]$	PVC_{ex} $[\tilde{g} = 0\%]$
$g = 3\%/yr$	1000.0	1001.9	1000.0	1146.4
$g = 0\%/yr$	1000.0	1000.0	878.6	1000.0
Difference	0%	-0.19%	-12.1%	-12.8%

assumes an infinite time horizon (Yohe, 2003). As a consequence, Yohe (2003, p. 243) concludes that *"we have added one more element to our list of reasons why it is inappropriate to use the expected value of discounted net benefits to judge mitigation policy"*.

Let us briefly demonstrate the practical consequences of applying different discounting approaches. Table I shows the relative present value differences for various discounting schemes between two finite consumption streams starting at $C_0 = 100$ and growing constantly over ten years at either $g = 3\%$ or $g = 0\%$ (as in Eq. 7). We assume $\theta = 1$ and $\rho = 0\%/yr$ since PVO_{Yohe} is only defined for these parameter choices. The *undiscounted* value of consumption for these two scenarios is 1146.4 and 1000.0, respectively (as in PVC_{ex} for $\tilde{g} = 0\%/yr$). PVC_{DICE} (and PVC_{end} for $\theta = 1$ and $\rho = 0\%/yr$) is insensitive to the differences between the two consumption streams. PVO_{Yohe} shows a small difference in PV between the two consumption streams, which is about 70 times smaller than the difference in undiscounted consumption. PVC_{ex} adequately reflects the difference in (undiscounted) consumption between the two scenarios, and it does that largely independent of the exogenous choice of the growth rate, $\tilde{g}$.

The theoretical arguments relevant to the discounting question considered here are discussed in Füssel (2006). This discussion concludes that PVC is a valid proxy for DU only when the same discount factors are used in the PV calculations of all policy options. Otherwise, the ranking of policy options (*i.e.*, consumption streams) according to PVC may be inconsistent with the ranking according to DU. Furthermore, it is shown that the discount factors applied to future welfare losses correspond to the relative prices of different goods in the index number problem, which *"can arise when an attempt is made to compare two [or more] sets of variables at two [or more] points in time using a single number since there are many different ways of aggregating variables into a single* *measure*" (Pearce, 1986). The index number problem has concerned economists and statisticians since the 19^{th} century at least (Jevons, 1865), and it has long been known that no unique solution exists (see, *e.g.*, Edgeworth, 1888, p. 347). However, there is unanimous agreement that a single set of prices has to be used for a meaningful comparison of quantities between different periods. This agreement provides another strong argument for using the same discount factors in PV calculations across all policy options considered. The final question then is how to determine reasonable discount factors in the absence of a unique 'correct' method for choosing them? In our view, the most reasonable approach is to pick a 'baseline' policy scenario, and to calculate the discount factors based on the growth rates of this baseline scenario, as in the original DICE-99 model (Nordhaus, 2001a, p. 19).

Since all available evidence indicates that PV calculations should use the same discount factors for all policy options under consideration, we consider $\mathrm{PVC}_{\mathrm{DICE}}$, $\mathrm{PVC}_{\mathrm{end}}$, and $\mathrm{PVO}_{\mathrm{Yohe}}$ as unsuitable for comparing the welfare implications of alternative climate policies. We note that $\mathrm{PVO}_{\mathrm{Yohe}}$ is subject to two other problems as well. First, it is only defined for $\rho = 0$, which is widely considered unrealistically low (Arrow et al., 1996). Second, there is no theoretical basis for the logarithmic relationship between consumption growth rate and discount rate assumed in Eq. 6.

2.4. Differences between various internally consistent welfare metrics

Sect. 2.3 concluded that out of the six SWFs presented in Sect. 2.1, only $\mathrm{DU}_{\mathrm{DICE}}$, $\mathrm{PVC}_{\mathrm{ex}}$, and $\mathrm{PVO}_{\mathrm{ex}}$ are not obviously inconsistent. In this subsection, we investigate the differences between these internally consistent welfare metrics.

First, we look at the difference between SWFs based on (net) economic output (*i.e.*, $\mathrm{PVO}_{\mathrm{ex}}$) and SWFs based on consumption, which excludes investment in productive capital (*i.e.*, $\mathrm{DU}_{\mathrm{DICE}}$ and $\mathrm{PVC}_{\mathrm{ex}}$). While partial equilibrium models prescribe the investment rate, general equilibrium models such as DICE determine the optimal investment rate endogenously. In the present discussion, we neglect the problem of converting investment into consumption equivalents (see, *e.g.*, Lind and Schuler, 1998).

When $\mathrm{PVO}_{\mathrm{ex}}$ is substituted as objective function in DICE-99 for $\mathrm{DU}_{\mathrm{DICE}}$, the optimal (*i.e.*, PVO-maximizing) policy is characterized by an investment rate of 100% over the full time horizon. Since all economic output is used for investment, none remains for consumption, and utility becomes minus infinity. Thus the very policy that maximizes PVO_{ex} minimizes PVC_{ex} and DU_{DICE}. While this example is obviously unrealistic, Füssel (2006) shows that inconsistent rankings between DU_{DICE} and PVC_{ex} compared to PVO_{ex} may also occur for policy strategies determined by DICE-99 while maximizing its original objective function, DU_{DICE}.

Second, we investigate the difference between discounting monetary values (*e.g.*, consumption expressed in dollars or other currency) and discounting utility expressed in 'utils'. This distinction is often blurred, because the two are almost identical for marginal differences between policies.

Let us assume an agent that lives for two periods, and that has a baseline consumption of \$1 in each period. This agent is offered a choice between an additional consumption a now or $a \cdot (1 + \delta)$ in the next period. Füssel (2006) finds the following relationship between the equivalent discount rates for utility, r, and for consumption, δ:

$$r = \frac{\ln(1 + a \cdot (1 + \delta))}{\ln(1 + a)} - 1 \tag{8}$$

$$\delta = \frac{(1 + a)^{1+r} - 1}{a} - 1 \tag{9}$$

The two discount rates are very similar for marginal changes in baseline consumption (*i.e.*, $a \ll 1$). For non-marginal consumption differences, however, the utility discount rate is significantly smaller than the equivalent discount rate for consumption. In thise case, discounting utility is inconsistent with the use of a single discount rate for consumption (and vice versa).

The finding that discounting (logarithmic) utility is inconsistent with discounting consumption has implications for the ranking of alternative consumption paths by DU_{DICE} and PVC_{ex}. While these two SWFs produce identical rankings for the constant-growth consumption paths considered in Sect. 2.2, they may produce inconsistent rankings if the consumption trajectories concerned involve significant welfare deviations at different points in time (Füssel, 2006, for an example, see).

We have just shown that PVC and DU may rank alternative policies inconsistently because they aggregate welfare differently across time. In probabilistic analyses, these SWFs may also aggregate welfare differently across possible states of the world. Table II provides a simple

Table II. Consumption and logarithmic utility for two policies and two equally likely states of the world (see text). The preferred policy is indicated by bold face.

State of the world	SOW 1		SOW 2		Aggregated measures		
Welfare measure	C	U(C)	C	U(C)	E(C))	E(U(C)	C^*
Policy A	**1.5**	**0.405**	0.5	-0.693	**1.00**	-0.144	0.866
Policy B	1.0	0.000	**0.9**	**-0.105**	0.95	**-0.053**	**0.949**

example for this inconsistency between expected consumption and utility. The four left columns show consumption (C) and logarithmic utility ($\mathrm{U}(C) = \ln C$) for two equally likely states of the world (SOW 1 and 2) and for two different policies (A and B). Policy A is associated with higher consumption (and utility) under SOW 1 whereas policy B leads to higher consumption (and utility) under SOW 2. The three right columns show three welfare measures aggregated over the two SOWs. E denotes the expected value, and C^* denotes the certainty equivalent, which was calculated such that $\mathrm{U}(C^*) = \mathrm{E}(\mathrm{U}(C))$. A certainty equivalent is the certain monetary value that would make an individual with a given utility function indifferent between it and the uncertain outcome. We find that policy A has higher expected consumption but policy B has higher expected utility, and thus a higher certainty equivalent.

According to Arrow et al. (1996, p. 130), *"Most economists believe that considerations of risk can be treated by converting outcomes into certainty equivalents, [...] and discounting these certainty equivalents"*. While it is straightforward to compute certainty equivalents for individual time steps (as in Table II), determination of their present value raises the question of the proper discounting scheme. It can be shown that no discounting scheme exists for which the expected value of discounted logarithmic utility is consistent with the present value of a series of certainty equivalents.

2.5. Recommendations for the application of social welfare functions

Based on the findings in the previous subsections, we make the following recommendations for the consistent application of welfare metrics in optimizing climate policy analyses:

1. Welfare metrics should be chosen based on the (supposed) preferences of target decision-makers; this choice should be made explicit.

2. Welfare metrics expressed in monetary terms need to implement a consistent discounting scheme.

3. The same welfare metric and discounting scheme should ideally be used in all optimizations and to report the relative ‘desirability’ of alternative policies. If different welfare metrics are combined in an analysis, the analyst needs to demonstrate that the inconsistencies between these metrics do not affect the conclusions of the analysis.

What do these rules imply in practice? $\mathrm{PVC}_{\mathrm{ex}}$ is an appropriate SWF if (and only if!) an analyst assumes risk-neutral decision-makers. While economists find very little empirical support for risk-neutral behaviour in individuals (Arrow et al., 1996), the Arrow-Lind theorem (Arrow and Lind, 1970) holds that if risk can be pooled or spread in such a way that aggregate risk is negligible, governments can be considered risk-neutral. $\mathrm{DU}_{\mathrm{DICE}}$ is an appropriate SWF if an analyst believes that the degree of risk aversion and other preferences of target decision-makers are adequately reflected by the Bernoullian utility function. This SWF is indeed very commonly used in economic models of climate change (DeCanio, 2003, Table 2.4), and it can be easily modified to accommodate different degrees of risk aversion.

A disadvantage associated with non-monetary SWFs, such as $\mathrm{DU}_{\mathrm{DICE}}$, is that they are expressed in arbitrary utility units. As a result, *“Any economist doing this work will obviously feel a strong urge to discount the difference in the consumption streams to a present value”* (Lind and Schuler, 1998, p. 80). Following this “strong urge”, some analysts have attempted to convert utility differences into monetary costs, defined as the difference in present value between alternative policies (see Sect. 2.6). However, the combination of different welfare metrics in an analysis is likely to introduce inconsistencies because different welfare metrics aggregate differently across regions and population groups, across components of economic output, across time, and across possible states of the world. Analysts who nevertheless combine different welfare metrics (*e.g.*, by maximizing expected utility and reporting the present value of the certainty equivalents of consumption over time) need to demonstrate convincingly that the inconsistencies between these metrics do not affect the policy conclusions of the analysis. In the absence of such a demonstration, their analysis must be regarded as potentially inconsistent.

2.6. Application of social welfare functions in recent climate policy analyses

In this subsection, we review the use of welfare metrics in two recent applications of DICE that have not followed the recommendations from Sect. 2.5: Yohe et al. (2004) and Fankhauser and Tol (2005)

Yohe et al. (2004) perform a hedging analysis that aims to identify the optimum short-term policy under uncertainty about climate change and the long-term stabilization target. This uncertainty is characterized by several 'policy cases', which are characterized by a specific value for the climate sensitivity and an upper bound on the greenhouse gas (GHG) concentration level. Each policy case is assigned a probability based on an empirical probability density function (PDF) for climate sensitivity, assuming that all considered GHG stabilization levels are equally likely. It is further assumed that the 'true' policy case will be revealed in 2035. A modified version of DICE-99 is used to determine the 'optimal' decision strategy for each policy case by maximizing DU_{DICE} for different predefined levels of the carbon tax (until 2035) and without a constraint on the level of carbon taxes. For each of those utility-maximizing strategies, discounted GWP is calculated according to PVO_{Yohe}. The "discounted adjustment costs" for each policy case and initial carbon tax level are then defined as the difference in discounted GWP between the utility-optimal strategies with and without prescribing the initial carbon tax level. Finally, the "optimal" initial carbon tax level is determined by minimizing the *expected* discounted adjustment costs for each tax level, considering the probability of the various policy cases.

We argue that the use of welfare metrics in Yohe et al. (2004) involves several inconsistencies, with important implications for the results presented. Our first argument recalls the findings from Sect. 2.3, which shows that PVO_{Yohe} is internally inconsistent and underestimates the welfare differences between alternative policies by about two orders of magnitude compared to PVO_{ex}. Fig. 1, which reproduces two diagrams from Yohe et al. (2004), demonstrates the large practical relevance of this flaw. The discounted GWP difference between a 450 ppm CO_2 concentration target and a 900 ppm target in this figure is only 0.015% (left panel), which is about two orders of magnitude smaller than the cost estimates from most other studies (Metz et al., 2001). Furthermore, variation in expected discounted GWP across all considered tax levels is a mere 0.0004% (right panel). Our second argument recalls the findings from Sect. 2.4 that DU_{DICE} and PVO_{ex} may produce inconsistent policy

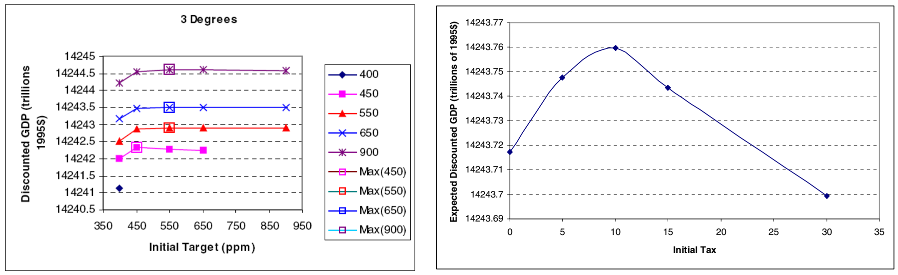

Figure 1. Present value of optimal climate policies for a range of stabilization targets and tax levels (reprinted from Yohe et al., 2004). *Left:* Discounted GWP for a range of greenhouse gas stabilization targets. *Right:* Expected value of discounted GWP for a range of initial carbon tax levels.

rankings. Yohe et al. (2004) initially maximizes DU_{DICE} but later use PVO_{Yohe} as the basis for selecting 'optimal' policies. In violation of the third recommendation from Sect. 2.5, there is no discussion whether these two SWFs produce similar rankings for the policies considered in that analysis, or what the potential implications of the inconsistent rankings are. Our third argument recalls the findings from Sect. 2.4 that expected consumption and expected logarithmic utility may produce inconsistent policy rankings due to different degrees of risk aversion. The same arguments hold in relation to Yohe et al. (2004), where expected output is maximized within a limited set of policy strategies that were initially determined by utility maximization in a deterministic context.

What are the implications of these flaws for the policy conclusions reported in Yohe et al. (2004)? This study concludes that *"An initial $10 tax policy is remarkably robust across the remaining possibilities"*, noting further that it is *"surprising that climate insurance over the near term can be so inexpensive and that an economically efficient near-term hedging policy can be so robust across a wide range of futures in comparison with doing nothing"*. We have shown that the reported costs of policies depicted in Fig. 1 are incorrect, most likely by about two orders of magnitude. We further note that even if the values depicted in Fig. 1 were correct, the tiny GWP variation across different policies would hardly support such a strong conclusion. Determining the correct optimal carbon tax level (subject to the assumptions of this particular analysis) would require a rerun of the whole modelling exercise in accordance with the recommendations from Sect. 2.5. Since such a reanalysis is beyond the scope of this paper, we cannot say for sure whether it would support the conclusions cited above.

Fankhauser and Tol (2005) apply DICE-94 to compare indirect climate impacts under different assumptions. This analysis defines several modifications to the production function of DICE-94, determines the optimal decision strategy for each model variant by maximizing the standard discounted utility function of DICE-94, DU_{DICE}, and presents the time paths and growth rates of undiscounted GWP (corresponding to PVO_{ex} for $\theta = \rho = 0$) for these decision strategies. Given the findings from Sect. 2.4 that DU_{DICE} and PVO_{ex} may produce inconsistent policy rankings, we have to consider the results of Fankhauser and Tol (2005) as potentially flawed.

3. Further flaws in the DICE-99 model and its application

In Sect. 2, we discussed the use of aggregated welfare metrics in climate policy analysis. While most examples referred to the DICE model and its adaptations, the results are more generally applicable to any welfare-maximizing economic model of climate change. In this section, we examine further flaws in DICE-99 and its application. Sect. 3.1 examines the flawed representation of uncertain climate parameters in two probabilistic model analyses, which leads to a significant underestimation of the risk of large transient climate change. Sect. 3.2 shows that the optimal climate policies determined by the Excel and GAMS implementations of DICE-99 are radically different, due to contradicting assumptions about the development of carbon abatement costs over time. Finally, Sect. 3.3 identifies some numerical errors in the GAMS version of DICE-99. Our motivation in this section is not to dwell upon these particular problems but to discuss the lessons to be learnt for model-based climate policy analyses.

3.1. Calibration of uncertain climate parameters

This subsection focusses on the calibration of uncertain climate parameters in DICE. In particular, we show that the inappropriate specification of the uncertainty about future climate change in two probabilistic analyses with DICE-99 (Yohe et al., 2004; Yohe et al., 2006) results in a very significant underestimation of the likelihood of large transient climate change, which has important implications for the conclusions drawn from one of these studies. We also provide recommendations for avoiding the various problems identified here.

The climate models applied in DICE-94 and DICE-99 are identical (Nordhaus and Boyer, 2000, p. 62–67), except for various changes in the names of model variables and parameters. This climate model is an adaption of the two-box model by Schneider and Thompson (1981):

$$\dot{T}_{up} = \frac{1}{R_1} \cdot \left(F - \frac{F_{2\times}}{T_{2\times}} \cdot T_{up} - \frac{R_2}{\tau_{12}} \cdot (T_{up} - T_{lo}) \right) \tag{10}$$

$$\dot{T}_{lo} = \frac{1}{\tau_{12}} \cdot (T_{up} - T_{lo}) \tag{11}$$

The three time-dependent variables and five parameters of this model are:

$T_{up}(t)$	[K]	temperature of the atmosphere and upper ocean
$T_{lo}(t)$	[K]	temperature of the deep ocean
$F(t)$	$\left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$	net change in radiative forcing
$T_{2\times}$	[K]	equilibrium increase in GMT from a CO_2 doubling
$F_{2\times}$	$\left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$	increase in radiative forcing from a CO_2 doubling
R_1	$\left[\frac{\mathrm{W\,yr}}{\mathrm{K\,m}^2}\right]$	thermal capacity of the atmosphere and upper ocean
R_2	$\left[\frac{\mathrm{W\,yr}}{\mathrm{K\,m}^2}\right]$	thermal capacity of the deep ocean
τ_{12}	[yr]	heat transfer rate from the upper to the deep ocean

The two most uncertain parameters are $T_{2\times}$, which determines the equilibrium change in GMT, and τ_{12}, which determines the speed of adjustment (Allen et al., 2000; Wigley and Raper, 2001; Knutti et al., 2005). The values of the other three parameters are relatively well known. In particular, R_1 can be easily determined from mixed layer depth. Recent research estimates average mixed layer depth at 70–100 m, with considerable global and seasonal variation (de Boyer Montégut et al., 2004). The reduced–form model applied in one of the studies reanalyzed here was calibrated to a GCM with a 60 m deep mixed-layer ocean model (Yohe et al., 2006, p. 62). The heat content of the atmosphere is approximately equivalent to a 2 m ocean layer and can thus be neglected in this context. Assuming a ratio of ocean surface to total Earth surface of $f_o = 71\%$ (Coble et al., 1987), a mixed layer depth of $h_m = 70\,\mathrm{m}$, a density of ocean surface water of $\rho_o = 1026\ \mathrm{kg} \cdot \mathrm{m}^{-3}$, and a heat capacity of ocean water of $c_{po} = 3996\ \mathrm{J} \cdot \mathrm{kg}^{-1} \cdot \mathrm{K}^{-1}$, we determine the heat capacity of the ocean mixed layer per area of the Earth's surface as

$$R_1 = f_o \cdot h_m \cdot \rho_o \cdot c_{po} = 204\,\frac{\mathrm{J}}{\mathrm{K\,m}^2} = 6.46\,\frac{\mathrm{W\,yr}}{\mathrm{K\,m}^2}. \tag{12}$$

The DICE climate model combines the five physical parameters from the model described in Eq. 10–11 into four parameters, most of which can no longer be interpreted physically: $\theta_1 = \frac{1}{R_1}$, $\theta_2 = \frac{F_{2\times}}{T_{2\times}}$, $\theta_3 = \frac{R_2}{\tau_{12}}$, and $\theta_4 = \frac{1}{\tau_{12}}$. In this model formulation, θ_2, θ_3, and θ_4 are associated with large uncertainty (since they depend on the highly uncertain parameters $T_{2\times}$ or τ_{12}), whereas θ_1 is much less uncertain (since it depends only on the relatively well known parameter R_1).

Even though θ_1 is much less uncertain than θ_3 and θ_4, Nordhaus (1994, Chapter 3) sets out to calibrate $T_{2\times}$ and θ_1, using historical data as well as the results from GCM experiments. The joint PDF for these two parameters constrained by historical forcing and temperature data shows a negative correlation between $T_{2\times}$ (varied from 1–5 K) and θ_1 (varied from 0.01–0.1 $\frac{\mathrm{K\,m^2}}{\mathrm{W\,yr}}$), but the conditional PDF for θ_1 given $T_{2\times}$ is often rather flat (Nordhaus, 1994, p. 43). We consider the variation of θ_1 by a factor 10 as inconsistent with established knowledge about the physically plausible range of R_1. After finding that simulations by general circulation models (GCMs) and historical data disagree in constraining the uncertain parameters in the DICE climate model (most likely due to the lack of consideration of the cooling effect of aerosols in the GCM experiments considered), Nordhaus (1994, pp. 46–47) asserts that *"For the DICE model, we employ the parameter pair (Sc2) drawn from the results of the SJ (*Schlesinger and Jiang, 1990*) model"*. However, the value $\theta_1 = 0.0226\,\frac{\mathrm{K\,m^2}}{\mathrm{W\,yr}}$ used in DICE (Nordhaus, 1994, Table 2.4) does not agree with the corresponding value $\theta_1 = 0.048\,\frac{\mathrm{K\,m^2}}{\mathrm{W\,yr}}$ from the SJ model (Nordhaus, 1994, Table 3.2.B), nor with the value $\theta_1 = \frac{1}{R_1} = 0.155\,\frac{\mathrm{K\,m^2}}{\mathrm{W\,yr}}$ determined according to Eq. 12.

There are several problems associated with the calibration of the DICE climate model described above. First, the reformulation of the climate model by Schneider and Thompson (1981) in such a way that most parameters can no longer be physically interpreted made it more difficult to focus on the main sources of uncertainty, and to identify all available data for constraining the uncertainty of individual parameters. Second, the calibration of a parameter with low uncertainty while holding fixed more uncertain parameters contributed further to the calibration of this model parameter outside its physically plausible range. Third, the final choice of parameter values is not well documented.

A fourth problem, which is partly caused by the first and second one, occurs in Yohe et al. (2004) and Yohe et al. (2006). These probabilistic analyses with DICE-99 represent the uncertainty about future climate change by a single uncertain parameter. Analogous to the approach in

Nordhaus (1994, Chapter 3), $T_{2\times}$ and θ_1 are calibrated using a large ensemble of climate projections. A single value for θ_1 is then assigned to each value of $T_{2\times}$ (Yohe et al., 2004, Table S1) even though there is clear evidence (including from Nordhaus, 1994, Table 3.5) that θ_1 and $T_{2\times}$ are *not* perfectly correlated. Fig. 2 depicts GMT trajectories for the DICE-99 baseline emissions scenario calculated with the modified DICE-99 model that assumes perfect correlation between θ_1 and $T_{2\times}$. This figure from Yohe et al. (2004, Fig. S1) is essentially equivalent with Yohe et al. (2006, Fig. 2). The calculations represent a wide range of climate sensitivities from 1.5 to 9 K, which covers more than the 5–95% range of most published climate sensitivity PDFs (Baer and Mastrandrea, 2005; Meinshausen, 2005). Nevertheless, there is no discernible uncertainty in temperature change before 2050, and only moderate uncertainty (about 0.5 K) in 2100.

In contrast to Fig. 2, detailed probabilistic analyses find a large uncertainty range for 21st-century climate change. The width of the 5–95% range of GMT increase is estimated at 1.0 K by the 2020s independent of the emissions scenario (Stott and Kettleborough, 2002), at 1.5 K by the 2040s for the medium IS92a scenario (Allen et al., 2000), at 2.1 K (Stott and Kettleborough, 2002) and 2.2 K (Knutti et al., 2002) by 2100 for the low SRES B1 scenario, and at 3.9 K by 2100 for the high SRES A1FI scenario (Stott and Kettleborough, 2002). Similar results have been found by Wigley and Raper (2001) and by Knutti et al. (2005).

Fig. 3 (Cubasch et al., 2001, Fig. 9.15) depicts global mean temperature projections determined by the simple climate model MAGICC tuned to several GCMs for the six illustrative SRES emissions scenarios (Nakicenovic and Swart, 2000). The GHG emissions in the unmitigated reference scenario of DICE-99 are most closely resembled by the medium-low SRES B2 scenario. Fig. 3 projects a GMT increase for the SRES B2 scenario of approximately 0.7–1.1 K from 1990 to 2030 and 1.9–3.4 K from 1990 to 2100. The corresponding projections in Fig. 2 are only 0.5 K and 1.3–2.0 K, respectively, even though the underlying range of climate sensitivity considered is much wider. We conclude that the assumption of perfect correlation between the two uncertain climate parameters θ_1 and $T_{2\times}$ in Yohe et al. (2004) and Yohe et al. (2006) causes significant overconfidence in probabilistic projections of 21st-century climate change. In particular, we find a strong bias toward low estimates of transient climate change. Let us now look at the implications of this overconfidence for the policy conclusions drawn in Yohe et al. (2004) and Yohe et al. (2006).

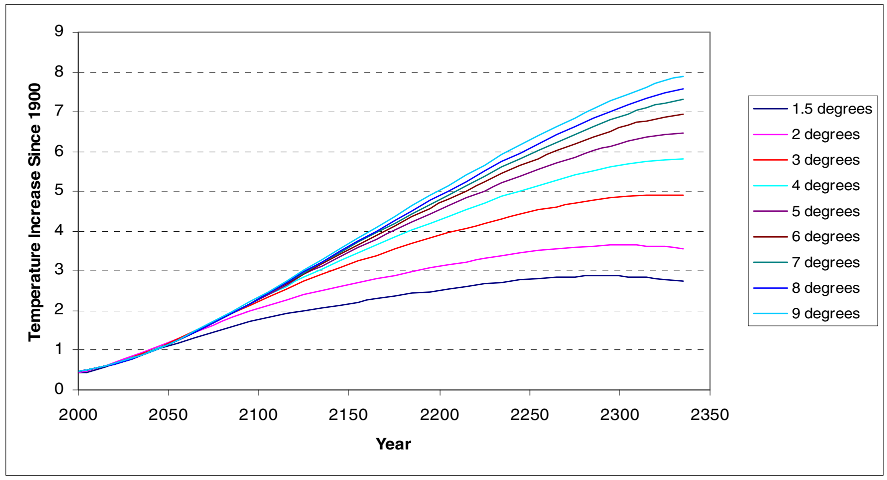

Figure 2. Global mean temperature trajectories for the DICE-99 baseline emissions scenario determined for alternative climate sensitivities from 1.5–9 K and associated calibrations of the heat capacity of the atmosphere and the upper ocean layer (reprinted from Yohe et al., 2004, Fig. S1).

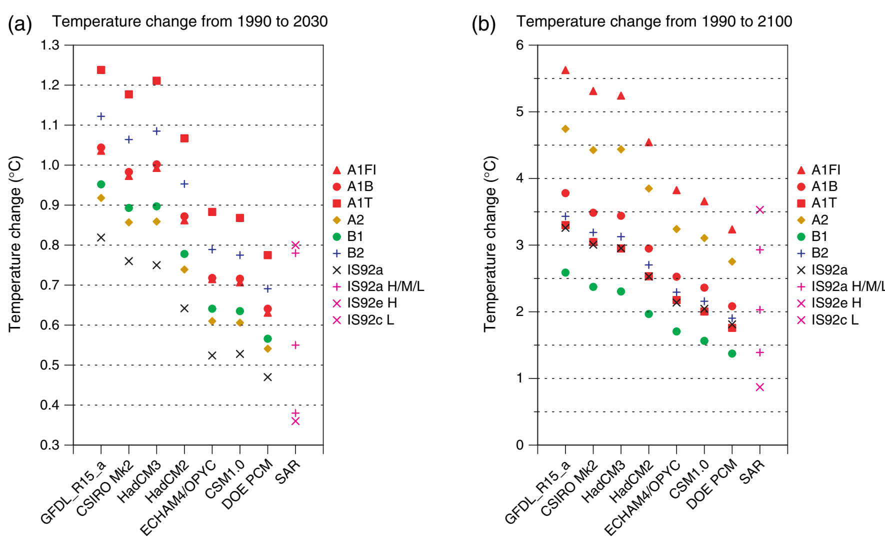

Figure 3. Global mean temperature projections determined by the simple climate model MAGICC tuned to several GCMs from 1990 to 2030 (left plate) and 2100 (right plate) for the six illustrative SRES emissions scenarios (reprinted from Cubasch et al., 2001, Fig. 9.15).

Table III. Sensitivity of the maximum likelihoods of THC collapse through 2105 and 2205, respectively, to the ranges of four uncertain climate parameters in the absence of a carbon tax (adapted from Yohe et al., 2006, Table 3 and 4).

THC collapse before	2105	2205
Kappa (K)	1–97%	30–100%
Alpha (α)	14–70%	25– 88%
Climate sensitivity (ΔT_{2x})	**38–50%**	**44– 78%**
Critical temperature (ΔT_c)	38–49%	45– 65%

The cost-effectiveness analysis presented in Yohe et al. (2004) applies constraints on GHG concentrations rather than temperature. Therefore, the only effect of climate change on the choice of cost-effective policies is through the smooth damage function applied in DICE-99. A cursory analysis indicates that the underestimation of the uncertainty about transient climate change does not have a large impact on the results reported in Yohe et al. (2004). (Note, however, the discussion of the inconsistent use of SWFs in this analysis in Sect. 2.6.)

The risk analysis presented in Yohe et al. (2006) estimates the likelihood of a collapse of the thermohaline ocean circulation (THC) before 2100 or 2200, respectively, under different levels of climate mitigation policy (expressed as carbon tax levels), subject to an empirical PDF for climate sensitivity and uniform distributions for three uncertain parameters of the THC model. Table III shows one of the key findings of this study. According to this table, the uncertainty about climate sensitivity is much less important for estimating the risk of a THC collapse than the uncertainty about two other uncertain climate parameters, K and α. It is further suggested that even for the highest value of climate sensitivity, the risk of a THC collapse before 2105 is 'only' 50%. As argued above, the flawed probabilistic representation of future climate change in Yohe et al. (2006) significantly underestimates the uncertainty of transient climate change, primarily by falsely excluding high values. Consequently, a more accurate probabilistic representation would show a higher importance of the uncertainty about climate sensitivity, and it would produce higher estimates of the likelihood of a THC collapse. While we do not have access to the collection of models applied in Yohe et al. (2006) to replicate their analysis with an improved probabilistic representation, we can still provide a rough estimate of the magnitude of the effect. The range of GMT change projected in Yohe et al. (2006)

for 2200 is reached in the detailed probabilistic analyses cited above already around 2100. Hence, we suspect that the high estimates for the likelihood of a THC collapse before 2205 reported in the bold-faced row in Table III (*i.e.*, 78% for a climate sensitivity of 9 K) are more indicative of the risk up to 2105 (which is estimated at a maximum of 50%).

The single most important measure to avoid the four problems mentioned here would be to retain the original formulation of the climate model by Schneider and Thompson (1981) and to focus the calibration on the two most uncertain physical parameters in that model ($T_{2\times}$ and τ_{12}), using all available data for constraining their PDFs. Since the two uncertain parameters calibrated in the DICE climate model show only a modest correlation, their uncertainties in a probabilistic analysis should preferably have been represented by their joint PDF. Interestingly, the uncertainty range for transient climate change determined in the detailed studies cited above is much better reproduced when only $T_{2\times}$ is varied and θ_1 is held fixed at its default value (as in Keller et al., 2004; Mastrandrea and Schneider, 2004; Keller et al., 2005) than when $T_{2\times}$ and θ_1 are varied assuming a deterministic relationship (as in Yohe et al., 2004; Yohe et al., 2006).

As a consequence of the various problems identified in this reanalysis, we make the following recommendations for the calibration and probabilistic representation of uncertain model parameters:

1. A model intended at mimicking a real-world system should be specified in such a way that (uncertain) model parameters correspond to observable properties in the real world.

2. The calibration of uncertain model parameters should use all available information for constraining them.

3. If the calibration of all uncertain model parameters is not possible, the calibration should focus on those parameters whose uncertainty is most important for the model results.

4. Probabilistic analyses involving two or more uncertain parameters should carefully consider the intercorrelation between these parameters, whenever such data is available. While it may sometimes be justified to treat uncertain parameters as independent or as perfectly correlated, many probabilistic analyses need to apply their joint PDF.

In the present example of the DICE climate model, Nordhaus (1994) violated the first three recommendations; the probabilistic analyses in Yohe et al. (2004) and Yohe et al. (2006) additionally violated the fourth recommendation.

3.2. Different abatement cost curves in two model implementations

DICE identifies the 'optimal' climate policy by solving an intertemporal optimization problem. One of the most important assumptions affecting the simulation results is the development of carbon abatement costs over time. The costs of emissions abatement in DICE-99 depend on the deviation of actual emissions from the unabated baseline emissions scenario at a specific point in time according to

$$\mathrm{Cost}(\mu_t, t) \;=\; \mathrm{b}_1(t) \cdot \mu_t^{\mathrm{b}_2} \cdot \mathrm{Y}^*(t), \quad b_2 = 2.15, \tag{13}$$

whereby $\mathrm{b}_1(t) \in [0, 1]$ is a time-dependent abatement cost factor that denotes the fraction of gross world product (GWP) that would be lost if carbon emissions were reduced to zero in year t, $\mu_t \in [0, 1]$ is the emission control rate ($\mu_t = 0$ refers to business-as-usual emissions and $\mu_t = 1$ refers to zero emissions), and $\mathrm{Y}^*(t)$ is the GWP in the unabated business-as-usual scenario in year t. Several authors have criticized the representation of abatement costs in DICE (Grubb et al., 1995) or have developed alternative formulations that consider the effects of induced technological change (Nordhaus, 1999; Buonanno et al., 2003; Popp, 2004). In contrast, the discussion here focusses on the temporal development of abatement costs in the original DICE-99 model.

DICE-99 assumes an *increase* in the abatement cost factor $b_1(t)$ over time: from 3% in 1995 to 11.1% in 2335 (Nordhaus and Boyer, 2000, Appendix E). Since the emissions intensity of the world economy is assumed to decrease 57-fold during that period, the increase is even stronger in absolute terms: from 110 US$/tC in 1995 to 23,000 US$ in 2335 (assuming constant 1995 dollars). The determination of the coefficients b_1 and b_2 is explained as follows (Nordhaus and Boyer, 2000, p. 104): *"The coefficients $b_1(t)$ and b_2 [...] were set so that the optimal carbon tax and emissions control rates in DICE-99 matched the projections of these variables in the optimal run of RICE-99."* Since RICE-99 determines the costs of carbon abatement differently than the DICE models (by including carbon-based energy as a separate factor in its production function), it is not possible to directly compare the specification of RICE-99 with the abatement cost curves of either DICE-98

Table IV. Time development of carbon abatement costs in two versions of DICE.

	DICE-99	DICE-98
File name	`dice99.xls`	`Dice020899.gms`
Platform	Microsoft Excel	GAMS/MINOS5
Reference	Nordhaus and Boyer (2000, App. E)	Nordhaus and Boyer (1999, App. C)
$dmiufunc$	-8	0.26
$b_1(1995)$	3.0%	4.5%
$b_1(2335)$	11.1%	1.5%

and DICE-99. However, since the elasticity of economic output with respect to energy decreases over time in RICE-99, we would expect the fraction of GWP lost for a given level of carbon abatement to decrease as well, in contradiction to DICE-99.

The unexplained assumption of increasing abatement costs over time is surprising enough. However, the issue is even more confusing as different implementations of 'DICE-99' make very different assumptions regarding the development of abatement costs over time (and some other model features). DICE-99 is available from the main developer's homepage `http://www.econ.yale.edu/~nordhaus/homepage/web/%20table%20of%20contents%20102599.htm` as Excel spreadsheet (`dice99.xls`) and as GAMS program (`Dice020899.gms`). In the following discussion, we denote the Excel and GAMS implementations as DICE-99 and DICE-98, respectively, for reasons that will be explained soon. The model developers assert that *"The Excel and GAMS versions are identical for the DICE-99 model"* (Nordhaus and Boyer, 2000, p. 107) but Table IV shows that this statement is not correct. The characterization of abatement costs in the Excel spreadsheet, which matches the description in the GAMS code in Nordhaus and Boyer (2000, Appendix E), assumes the abatement cost factor $b_1(t)$ to *increase* over time. The GAMS program, which is identical to the GAMS code in Nordhaus and Boyer (1999, Appendix C), assumes the abatement cost factor $b_1(t)$ to *decrease* over time. Since the latter reference is titled *Computer Code for DICE-98 Model*, we assume that the GAMS program `Dice020899.gms` refers to an earlier model version denoted as DICE-98, despite the contradicting file name. For comparison, b_1 was assumed to be constant in DICE-94 (Nordhaus, 1994, p. 193).

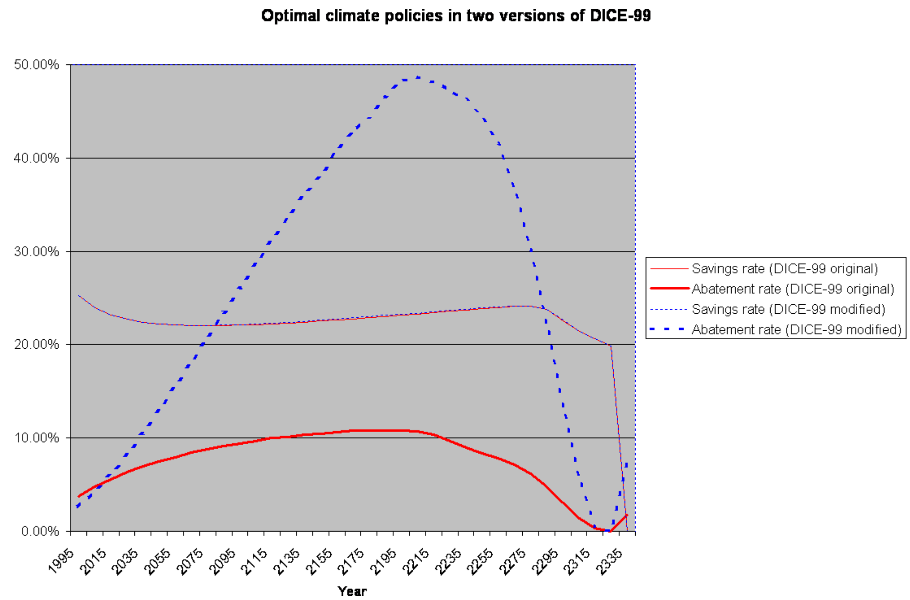

Figure 4. Optimal climate policies determined by the original DICE-99 model and by DICE-99 modified to apply the abatement cost function of DICE-98.

The fact that the two significantly different versions of the DICE-99 model are not distinguished in the literature is unsatisfying by itself. In fact, the column titled "DICE-99" in Table IV refers to two slightly different models, since the GAMS program described in Nordhaus and Boyer (2000, Appendix E) contains some numerical errors that are not present in the Excel spreadsheet `dice99.xls` (see Sect. 3.3). Consequently, there are actually three different models that are commonly referred to as DICE-99. Fig. 4 shows that the choice of one or the other abatement cost function has wide-ranging implications for the 'optimal' policy strategy determined by DICE-99. The thick solid line depicts the abatement rate of the original DICE-99 model, which recommends modest emission reductions only (peaking at 11%). The thick dashed line, in contrast, depicts the abatement rate of DICE-99 applying the decreasing abatement cost function of DICE-98, which recommends much larger abatement (peaking at 49%). The differences are even stronger for the lower discount rates applied in some studies (*e.g.*, Yohe et al., 2004). The two (almost identical and thus hard to distinguish) thin lines depict the optimal savings rates determined by the two model versions. In agreement with earlier studies (*e.g.*, Kaufmann, 1997), we find little variation in the optimal savings rate.

Two aspects of the representation of abatement costs in DICE-99/98 are particularly disturbing. First, at least one of the model versions grossly misrepresents the model developers' knowledge (or expectations) about the development of abatement costs over time. While the decreasing abatement cost function in DICE-98 seems to better agree with their verbal explanation, this representation has later been discarded in favour of the increasing abatement cost function of DICE-99. The model developers neither point to the existence of these different models nor do they provide an explanation how the widely diverging parameterizations were determined. Second, most scholars are not aware of the differences between the two model implementations available for download, which were described as "identical" by their developers. Hence, different analysts may unknowingly arrive at very different results, depending on whether they use the Excel or GAMS implementation of the model denoted as DICE-99.

The inconsistent assumptions regarding abatement cost curves are obviously specific to the DICE-99 model(s). However, we see this problem as an example of the more general challenge of ensuring the consistency of aggregated models where important parameters have to be calibrated to observational data or to the results of more complex models. In this context, we reiterate our first recommendation from Sect. 3.1 to specify a model in such a way as to minimize the number of parameters that cannot be checked against observations. In addition, we emphasize the importance of providing a "traceable account" (Moss and Schneider, 2000) explaining how the values of the remaining parameters were determined, including the results of alternative assumptions. Neither of these recommendations was followed in the case of DICE-99.

3.3. Numerical errors

The GAMS versions of DICE-98 (Nordhaus and Boyer, 1999, Appendix C) and DICE-99 (Nordhaus and Boyer, 2000, Appendix E) are distinguished not only by changes in the values of many empirical parameters but also by changes in the units in which these parameters are expressed. Most (but not all) flow parameters are expressed in *fractions per decade* in DICE-98, whereas most (but not all) of them are expressed in *percent per year* in DICE-99. For instance, the value of $GA0$ changed from 0.055 (fraction per decade) to 3.8 (percent per decade), and the value of $ET0$ changed from 11.28 (GtC per decade) to 1.128 (GtC per year); at the same time $ET0$ was renamed to $LU0$. Some inconsistencies apparently have been introduced to the GAMS version of DICE-99 during this conversion:

E, $INDEM$, $ETREE$, $LU0$: Equation EE, which defines the total carbon emissions per decade (E), applies a factor 10 to annual industrial emissions ($INDEM$) but not to annual land-use emissions ($ETREE$). Comparison with the Excel version of DICE-99 confirms that the definition of E incorrectly lacks the factor 10 for $ETREE$. Hence, land-use change emissions are underestimated by a factor 10 in the GAMS version of DICE-99. This error has a small effect in cost-benefit analyses with DICE-99, where land-use emissions are soon marginalized by the growing industrial emissions, but it can have significant effects in cost-effectiveness analyses. For instance, optimal carbon taxes for GMT stabilization at 2 °C above preindustrial levels are about 20% higher when land-use emissions are accounted for correctly.

$DELA$, GA: $DELA$ is defined as *(percent?) change per decade* but the definition of GA requires that $DELA$ is specified as *percent change per year*. Since the numerical value of $DELA$ is very small, this error has a negligible effect on model outcomes.

$DESIG$, $DESIG2$: $DESIG$ is defined as percent per *decade* but the definition of $GSIG$ requires that $DESIG$ is specified as percent per *year*. There is also confusion about $DESIG2$, for which no units are specified. The definition of $GSIG$ requires that $DESIG2$ is defined as fraction per year per decade, which would be a rather unusual choice of units.

MIU: Equation EE suggests that MIU is specified as percentage but its upper bound is set to 1.0, which only makes sense for pure numbers.

Once again, our main motivation for this discussion is to draw lessons for future analyses. The obvious lesson to be learnt from the errors identified in this subsection is that the units of all variables in a simulation model should be explicitly specified, either as documenting text or (preferably) using dedicated simulation software that can check the consistency of model equations in terms of the units involved.

4. Summary and conclusions

The main motivation for this paper has been to investigate the degree to which the simplicity of globally aggregated climate-economy models actually leads to transparency and consistency in their application. While our analysis focusses on the DICE model, most of it is relevant to other climate-economy models as well.

Sect. 2 reviews the application of social welfare functions in applications of DICE. We show that the inappropriate implementation of growth discounting has lead to the application of various internally *inconsistent* welfare metrics in climate policy analyses, and we discuss the link between growth discounting and the index number problem. We further show that the remaining internally *consistent* welfare metrics are generally not interchangeable, since they aggregate differently across regions, time, states of the world, and components of economic output. Based on these findings, we present several recommendations for the consistent use of welfare metrics in welfare-optimizing climate-economy models. A reanalysis of two climate policy analyses with DICE that violate these recommendations reveals, among others, cost estimates of climate policies that are off by a factor 100, questioning the validity of the policy conclusions drawn from the model results.

Sect. 3 identifies further empirical and numerical flaws in applications of DICE-99. These flaws include the calibration of uncertain climate parameters beyond their physically plausible range, the inappropriate specification of uncertainty about transient climate change, the undocumented specification of radically different abatement cost curves in two implementations of DICE-99, and several numerical errors. As in the previous chapter, we estimate the importance of these problems for model-based climate policy analyses, and we provide recommendations for preventing them in future analyses.

Our reanalysis of several published climate policy analyses demonstrates that the logical and empirical flaws identified in this paper are not only of theoretical interest. Most of them can strongly affect the policy recommendations drawn from the simulation results, *e.g.*, by significantly underestimating the range of 21-st century climate change. The existence of these flaws is particularly disturbing given that DICE has been publicly available for many years, and that this model has been used and adapted by many scholars.

Our findings indicate that much more caution is needed in the development, application, and modification of simple climate-economy models, and in the interpretation of their results. The combined efforts of original model developers, of analysts adopting existing models, and of peer reviewers are required to ensure that model applications are scientifically sound, and that the policy conclusions drawn from a particular model experiment are actually supported by the simulation results. Specific recommendations on how to prevent the various problems identified in this paper are provided in the text.

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