Transit-Time Distributions in a Global Ocean Model

SYNTE PEACOCK
Department of the Geophysical Sciences, The University of Chicago, Chicago, Illinois

MATHEW MALTRUD
Los Alamos National Laboratory, Los Alamos, New Mexico

(Manuscript received 27 September 2004, in final form 11 August 2005)

ABSTRACT

Results from a simulation of the ocean “transit-time distribution” (“TTD”) for global and regional ocean surface boundary conditions are presented based on a 5000-yr integration using the Parallel Ocean Program ocean general circulation model. The TTD describes the probability that water at a given interior point in the ocean was at some point on the ocean surface a given amount of time ago. It is shown that the spatial distribution of ocean TTDs can be understood in terms of conventional wisdom regarding time scales and pathways of the ventilated thermocline and the thermohaline circulation–driven deep-ocean circulation. The true mean age from the model (the first moment of the TTD) is demonstrated to be very large everywhere, because of very long-tailed distributions. Regional TTD distributions are presented for distinct surface boundary subregions, and it is shown how these can help in the interpretation of the global TTD. The spatial structure of each regional TTD is shown to become essentially the same at relatively long times. The form of the TTD at a given point in the ocean can be very simple, but some regions do exhibit more complicated multimodal distributions. The degree to which a simple functional approximation to the TTD is able to predict the spatial and temporal evolution of selected idealized tracers (for which interior sources and sinks are known or zero) with knowledge of only the tracer surface boundary condition is explored.

1. Introduction

A number of recent studies have highlighted the need for a more detailed knowledge of the “age spectrum” of the ocean if we are to develop a better understanding of time scales of pollutant transport, or the movement of a chemical species from one point in the ocean to another (e.g., Waugh et al. 2003). Rather than rely on a single-tracer-derived age to estimate the time scales of transport, it is necessary in an advective–diffusive environment to recognize that a water parcel can reflect a whole range of transit times, and that these can be characterized by a probability density function.

An early application of the concept of transit time distributions (TTDs) was in better constraining stratospheric transport time scales (Hall and Plumb 1994; Hall and Prather 1995). The concept has also been used in groundwater (Cook and Solomon 1997), lakes (Waugh et al. 2002) and ocean (Haine and Hall 2002) studies. Khatiwala et al. (2001) presented a 200-yr TTD simulation of using a North Atlantic Ocean model, and, more recently, Primeau (2005) has predicted the TTD using an offline transport code with a stationary flow field. In this paper, we present results from a 5000-yr integration of the transit-time distribution in a global ocean general circulation model. We show results for both the case of a global uniform surface ocean boundary condition, and for 11 surface ocean subregions.

The paper begins by briefly reviewing the concept of the TTD and discussing potentially important oceanic applications. In section 3, the model used to simulate the TTD is described. The characteristics of the simulated global surface ocean TTD are discussed in section 4. Results from the regional TTDs are presented in section 5. The degree to which the simulated TTD approximates a simple two-parameter analytical function is taken up in section 6. The paper ends with a discussion of the results in section 7.
2. Transit-time distributions

a. Background

A detailed introduction to transit-time distributions is given by Holzer and Hall (2000), and the reader is referred to this paper for a more thorough overview. These authors show, starting with a tracer continuity equation

\[ (\partial_t + L) c(\mathbf{r}, t) = Q, \]

where \( L \) is the linear (advective plus diffusive) transport operator, \( Q \) represents sources or sinks, and \( c(\mathbf{r}, t) \) is the concentration of a passive tracer, that for a steady-state ocean with no interior sources or sinks, the solution to Eq. (1) can be written

\[ c(\mathbf{r}, t) = \sum_i \int_0^\infty \, d\xi \, c_i(t - \xi)G'(\mathbf{r}, \xi|\Omega_i), \]

where \( c_i \) is the tracer mixing ratio, assumed to be spatially uniform over each surface region \( \Omega_i \) (see also Haine and Hall 2002). It also holds that

\[ (\partial_t + L)G'(\mathbf{r}, t|\Omega_i, t') = 0, \]

where \( G' \) is a propagator of boundary conditions that depends only on \( \xi = t - t' \) because of stationarity. Thus, one can define \( G'(\mathbf{r}, t|\Omega_i, t') = G'(\mathbf{r}, \xi|\Omega_i) \). The boundary condition is \( G'(\mathbf{r}, \xi|\Omega_i) = \delta(t - t') \) if \( \mathbf{r} \) is on \( \Omega_i \), and \( G'(\mathbf{r}, \xi|\Omega_j) = 0 \) otherwise.

When integrating over \( \mathbf{r}_0 \) on the surface, \( G' \) becomes a pure “transit time probability distribution function” (TTpdf; or TTD). If the boundary over which the delta function is applied comprises the entire surface ocean, the TTD will describe the probability that a given interior water parcel was anywhere on the ocean surface a given time ago. If a subregion of the ocean surface is selected, the TTD will describe the probability that an interior point was somewhere in that subregion a given time ago. It also follows from this interpretation as a probability distribution function that the mean age at a given location is just the first moment (in time) of this distribution, while the second moment is a measure of the distribution’s width.

For a tracer with a spatially uniform surface ocean concentration at any given time, convolution of the ocean surface boundary condition and a given interior ocean TTD yields the time evolution of the tracer at that point in the ocean interior [Eq. (2), with \( i = 1 \)]. For a species that has a nonuniform spatial distribution in the surface ocean at any time (e.g., CFCs or most soluble gases), it becomes necessary to describe multiple TTDs for various ocean subregions and then to approximate the surface tracer concentrations as piecewise constant over each of the subregions (Haine and Hall 2002).

Technically, simulation of the boundary propagator requires a delta-function boundary condition on the upper boundary (Holzer and Hall 2000). In practice, in ocean general circulation model (OGCM) simulations, the delta function is applied over a finite interval of time. In studies to date (including this one), the selected time interval has been 1 yr (to smooth out seasonality; e.g., see Khatiwala et al. 2001), though there is no reason why a shorter period of time should not be chosen. In this paper, we present results from a model run that simulates the global surface TTD, as well as the TTDs from 11 surface subregions.

b. Applications

Knowledge of the spatial distribution of the actual ocean TTD would potentially be of great value in efforts to predict ocean uptake and redistribution of chemical tracers and pollutants. One of the most promising applications of the TTD is to improve estimates of anthropogenic CO₂ uptake by the ocean. Most current estimates of anthropogenic CO₂ uptake are based on the assumption that the flow is dominated by advection, and that a chlorofluorocarbon (CFC)-derived age characterizes the mean transit time (Gruber et al. 1996; McNeil et al. 2003). However, Hall et al. (2002) have recently shown in a simple box-model framework that use of the TTD yields significantly more accurate estimates of anthropogenic carbon uptake than do CFC-derived ages. They have also taken this a step further and shown that, if the ocean TTD can be described by a relatively simple functional form, current estimates of anthropogenic CO₂ uptake in the Indian Ocean are overestimated by about 30% (Hall et al. 2004).

For meaningful interpretation of model tracer simulations in general, knowledge of how a change in a tracer boundary condition will be reflected at various locations is important. An example of this is in the interpretation of various tracer-derived “ages.” It has been noted by various authors (e.g., Waugh et al. 2003) that such ages (e.g., the CFC-11 age of a water mass) vary with space and time. All measures of the water parcel age sample some portion of the TTD; most measures of age (such as CFC age) are biased toward the young part of the spectrum (Waugh et al. 2003). There are instances in which the temporal evolution of the CFC age at a given location has been interpreted as reflecting a fundamental change in the rate of water mass ventilation at that point (i.e., a change in the transport operator), such as the study by Watanabe et al. (2001). However, it is important to be able to disentangle interior tracer variability resulting from changes...
in surface boundary conditions from that from changes in circulation. The TTD framework provides a useful way to accomplish this.

3. Ocean model details

The model we used to simulate the TTDs is a coarse-resolution configuration of the Parallel Ocean Program (POP), a z-coordinate OGCM with a variable-free surface (Dukowicz and Smith 1994). The grid is made to be fully global (including the Arctic) by smoothly displacing the northern computational pole into Greenland (Smith et al. 1995), while the Southern Hemisphere remains a regular latitude–longitude grid. At the equator, the longitudinal spacing is 3.6°. In the Southern Hemisphere, the latitudinal spacing is just under 1° at the equator and expands to just under 2° at midlatitudes. Grid distortion in the Northern Hemisphere makes it harder to define the grid spacing in traditional terms, but it is instructive to note that overall the average grid spacing is about 200 km, which is approximately what one would obtain from a global 3° latitude × 3° longitude grid. There are 25 nonuniformly spaced vertical levels, varying from 12 m at the surface to 450 m at depth, and bathymetry is interpolated from the 5-Minute Gridded Global Relief (ETOPO5) database.

Because of the coarseness of the horizontal grid, several state-of-the-art parameterizations of subgrid-scale mixing have been implemented. The Gent–McWilliams (GM) formulation is used for the eddy mixing of tracers (Gent and McWilliams 1990), while a grid-aligned anisotropic viscosity is used for momentum (Smith and McWilliams 2003). Vertical mixing coefficients for momentum and tracers are calculated by an implementation of the K-profile parameterization (KPP) (Large et al. 1994) that includes the use of large diffusion coefficients (0.1 m² s⁻¹) to resolve gravitational instabilities. A polynomial United Nations Educational, Scientific and Cultural Organization (UNESCO) equation of state is used for calculating density.

When performing passive tracer studies with an OGCM, it is desirable to use an advection algorithm that does not allow spurious negative values that can result from the discretization of the transport equation. However, any scheme with this property will introduce nonlinearity into the advection term by making the operator depend on the value of the tracer, not just the velocity. This effect turns out to be quite small in the vast majority of the ocean, and does not affect the conclusions of this work. We have chosen to use the multidimensional positive definite centered difference (MPDCD) advection scheme (Lafore et al. 1998) in order to eliminate advective undershoots with the passive tracers (such as the TTDs), while retaining standard linear-centered advection (with no flux limiting) for temperature and salinity.

Surface forcing has been formulated in a manner very similar to Large et al. (1997), using monthly mean fields from a variety of sources. Wind stresses, atmospheric temperature, humidity, and wind speed are a 1985–88 climatology from the National Centers for Environmental Prediction–National Center for Atmospheric Research (NCEP–NCAR) reanalysis (Kalnay et al. 1996). Cloud fraction (used for calculating longwave radiative flux) and shortwave radiation are from the International Satellite Cloud Climatology Project (ISCCP) (Rossow and Schiffer 1991), and precipitation is a blended Xie–Arkin Microwave Sounding Unit (MSU) product (Xie and Arkin 1997; Spencer 1993). Heat fluxes are calculated using the Large and Pond bulk formulas (Large and Pond 1982). Surface restoration of salinity and temperature to annual values with a time scale of 15 days is performed under climatological ice. There is also weak surface restoration of salinity in the open ocean with a time scale of 100 days. In this study the surface boundary condition for the TTD was 1 yr⁻¹ for the first year and zero thereafter. In the 11-region experiment this boundary condition was applied to each individual region, with a boundary condition of zero for all time everywhere outside of the region of interest. The model was initialized with mean annual temperature and salinity fields from Levitus and Boyer (1994), and then was spun up for 500 yr before introducing the TTD at the upper boundary. A time step of 1 h was used. The run was computationally fairly intensive: the first 2000 yr of the simulation (which carried a total of 21 tracer fields) took roughly 135 days of continuous run time on 110 processors of a Silicon Graphics, Inc. (SGI), Origin 3000 (roughly 360 000 processor hours).

It is important to note that the TTD distributions to be described are dependent on the fidelity of the model circulation. We have included in the appendix several measures used to assess the model in order to establish that the circulation is acceptable for this study.

4. Global transit-time distributions

a. General characteristics

The global TTD was simulated with the boundary condition of 1 yr⁻¹ for the first year and zero thereafter for the entire top level of the model, with no interior sources or sinks. Thus, the TTD is transported as a passive dye-like tracer, except for the fundamental dif-
ference of the surface boundary condition noted above. However, because the TTD resembles a passive dye tracer below the surface, it is constructive to investigate its evolution in light of what we know about the ocean's circulation and water masses. Figure 1 shows the evolution of the column integral of the TTD with time. At 10 yr, the highest column inventories are associated with shallow subduction along the boundaries of the subtropical gyres, while in the Pacific Ocean a maximum is also observed along the equator. Tracer is beginning to be seen in the North Atlantic Deep Western Boundary Current (DWBC) at this early time; by year 50 the DWBC has transported the tracer almost to the southern tip of South America. Deep-water formation in the North Atlantic gives rise to high column inventories in the Atlantic for the first 200 yr of the integration, after which time the Southern Ocean also becomes an important reservoir as deep water accumulates in the Antarctic Circumpolar Current (ACC). After roughly 1000 yr of model integration, the Pacific and Indian Oceans have become the dominant reservoirs, and by the end of the integration (5000 yr), most tracer resides in the deep North Pacific.

b. Globally averaged TTD

While it is interesting to investigate the spatial distribution of the TTD, it is important to keep in mind the interpretation of the temporal distribution at each model grid point, as a probability distribution function of ages. Because the surface value of the TTD is fixed at 1 yr\(^{-1}\) for the first year and set to zero for all time thereafter (i.e., the entire surface ocean becomes a sink after year 1), it is clear that the global ocean average value must monotonically decay with time (because there are no internal sources). The upper panel in Fig. 2 shows that the decay goes as \(t^{-1/2}\) for roughly the first 1500 yr, and is exponential thereafter. According to Haine and Hall (2002), the decay must ultimately be exponential at rate \(\lambda_{\text{min}}\) (where \(\lambda_{\text{min}}\) is the smallest eigenvalue of the transport operator). By definition, the time integral of the TTD must eventually converge to 1 for the case of a delta-function upper-ocean boundary

Fig. 1. Column integral of the global TTD at various times after the pulse input. Each plot has been scaled by the maximum value of the field at that time. Note that for this and all following map-view figures, we show a projection that reflects the logical-space layout of the grid.
condition and a perfect advection scheme. It can be seen that in this model simulation, the value of the cumulative sum of the globally averaged TTD slightly overshoots one at long time scales (lower panel of Fig. 2). This may be because of the approximation of a delta function with a yearlong step function, though numerical errors also likely contribute. The flatness of the integrated TTD by 5000 yr of integration indicates solution convergence.

c. Local descriptions of the global TTD distribution

An individual TTD at a specific grid point will generally, of course, look quite different from the global average. Traditional understanding of thermocline ventilation rates and pathways is borne out by the nature of the TTDs in the upper few hundred meters of ocean, such as along the north–south transect at 30°W and 245-m depth in the Atlantic Ocean shown in Fig. 3. The rapidly ventilated subtropical gyres have a relatively early peak and short-tailed distribution, and are bounded by regions to the north and south characterized by much longer-tailed, broader distributions that result from mixing with older upwelling waters. This pattern typifies the TTD distribution throughout the ocean’s thermocline, independent of the ocean basin.

What is not obvious from Fig. 3 is that the TTDs can sometimes be multimodal, with each local maximum indicating the arrival of a different water mass. Figure 4 shows the sign of the slope of the TTDs (note the very abbreviated time scale as compared with Fig. 3) at the same depth and longitude as Fig. 3. Whenever the shading transitions from white (negative slope) to gray (positive slope) with increasing time, the TTD starts building up to a new local maximum, which is reached at the time when the shade changes back from gray to white. Multimodal behavior can clearly be seen around 50°N, 15°S, and along the equator. In the Southern Ocean, multimodal distributions occur at virtually all depths and longitudes. Also, note that most TTDs are not monotonically decreasing after the first year, as must be the case with the full-volume-averaged TTD (Fig. 2). A more detailed description of some multimodal TTDs can be found in section 5.

The nature of the TTD distribution changes dramatically below the influence of the permanent thermocline, resulting in deep Atlantic TTDs that look very
different from those of the deep Indian and Pacific oceans. Even within a given basin, the symmetry seen in the wind-driven gyres is absent. The reason is straightforward; the subthermocline ocean is ventilated primarily by cold waters sinking in the Nordic seas and around the Antarctic perimeter. So, whereas the intermediate and deep Pacific is ventilated predominantly from the south by Antarctic Intermediate Water (AAIW) and Antarctic Bottom Water (AABW), the Atlantic is ventilated predominantly from the north by the North Atlantic Deep Water (NADW), all of which can be seen in Fig. 1.

The manifestation of these ventilation pathways on the form of the TTDs at 30°W (Atlantic) and 160°W (Pacific) at 2300-m depth can be seen in Fig. 5. The upper plot clearly reveals the presence of NADW as the major peak in the Northern Hemisphere that crosses the equator in the DWBC after 30–50 yr, then reaches the Southern Ocean after 60–80 yr. AABW formed in the Weddell Sea is responsible for the smaller Southern Hemisphere peak at early times, resulting in a strongly bimodal structure south of about 40°S.

The Pacific TTD plot (lower panel of Fig. 5) looks very different from the Atlantic at the same depth, particularly the northern Pacific, because of the absence of deep convection here. The Southern Ocean sector of the Pacific is characterized by multiple and early peaks, then a slow exponentially decaying tail. The Pacific Ocean itself has a much more long-tailed distribution than does the Atlantic. The TTDs at about 75°N in the lower plot are from the Arctic Ocean, which at this depth are unimodal with maxima at a few hundred years and an exponentially decaying tail.

d. Moments of the TTD

The “mean age” of the water at a given location (which is related to the more commonly referenced “ideal age” as discussed in the appendix) is defined as the first moment of the distribution,

$$\Gamma(r) = \int_0^\infty tG(r, t) \, dt. \quad (4)$$

The mean age on various depth surfaces after 5000 yr of model integration is shown in Fig. 6. As is to be expected, there is a correspondence between the column inventories at long integration times (Fig. 1) and the mean age in the deep ocean; the deep Pacific shows the highest column inventory at long time scales, and the oldest mean age observed anywhere in the World Ocean.

In the upper few tens of meters, the regions with oldest age are those that are characterized by strong upwelling or vertical mixing. One example of a region characterized by high mixing is the Southern Ocean,
which has a mean age of about 200 yr at 38-m depth. This increases to about 500 yr by a depth of 245 m. The large diffusive contribution in this region yields a very long-tailed TTD, which makes the mean age very old. The signature of upwelling along the Peruvian coast is clear even at 38-m depth. By 245-m depth, the upwelling signature (old water) can clearly be seen to have spread westward across the Pacific basin.

At depths greater than 150 m, the signature of shallow subduction is very distinct. Water is subducted in to the thermocline at midlatitudes (Luyten et al. 1983) and is subsequently advected westward in a curved potential vorticity–conserving trajectory. This process creates a sharp boundary in mean age, which can be seen most clearly in the Pacific at 245-m depth (Fig. 6). Waters subducted and rapidly transported in the oceans subtropical gyres have a very small mean age. The other regions that stand out as having a very small mean age are regions of deep convection, where the deep ocean is rapidly ventilated by the surface ocean. In the Southern Ocean, small mean ages are seen in the Weddell Sea and along the entire Indian sector of the Antarctic coast. In the Labrador Sea the waters are also characterized by a very small mean age.

The signature of shallow subduction is more-or-less lost by 800-m depth (roughly the base of the permanent thermocline). Below 1000 m, the ocean has remarkably little variation in mean age with depth. The North and western Pacific consistently has the oldest water in the deep ocean, with a maximum age of 1700 yr at intermediate depths (around 2000 m). The deep Atlantic is characterized by a strong east–west gradient in age, with the rapidly ventilated western side of the ocean having a mean age of only a couple hundred years, while the eastern side of the ocean has a mean age of around 600 yr. The Southern Ocean at depths greater than 1000 m is remarkably uniform, with a mean age between 600 and 700 yr. The deep Indian Ocean shows a very slight north–south gradient, ranging from around 900 yr at the southern boundary to over 1200 yr in the north.

A measure of the width of the distribution is given by the second moment of the TTD. Following Hall and Plumb (1994), the width is defined as

\[ \Delta(r)^2 = \frac{1}{2} \int_0^\infty [t - \Gamma(r)]^2 G(r, t) \, dt, \]

and is shown in Fig. 7 at the same depths as \( \Gamma(r) \) in Fig. 6. There is a correlation between the first and second moment; in most general terms, where the mean age is
large, the width also tends to be large. Note, however, that the second moment is, for the most part, greater at 818 than at 2014 m in most of the ocean, whereas the reverse is true of the first moment.

5. Multiple-region transit-time distributions

In addition to the “global TTD” tracer (boundary condition applied over the entire surface ocean), we...
also simultaneously carried 11 "regional" TTDs (boundary condition applied over a subset of the surface ocean) for the first 2000 yr of model integration. This was done in part to test the TTD approach to predicting the interior concentration of a tracer [Eq. (2)] whose surface boundary condition can be approximated as a (spatially) piecewise constant over each separate subregion (the subject of a separate paper). It was also hoped that parts of the ocean in which the global TTD shows pronounced multimodal behavior might be better understood in terms of contributions from different source areas. The regions used in this

---

**Fig. 6.** Mean age (yr) calculated from the first moment of the global TTD [Eq. (4)] on selected depth surfaces: (left) 38 and (right) 245 m.

**Fig. 7.** Distribution width (yr) calculated from the second moment of the global TTD [Eq. (5)] on the same depth surfaces as for mean age in Fig. 6.
study are based on those used in the “TransCom” experiments (Gurney et al. 2000) and are shown in Fig. 8. Note that, in theory, the sum of the regional TTDs is equal to the global TTD because the regions cover the entire ocean and the governing equations are linear. This sum no longer strictly holds when using a nonlinear advection scheme, in which case there will be small errors when adding up the regional curves. However, this does not affect the conclusions reached about the relative contribution of each region to the global sum. Figure 9 shows the evolution of the global TTD and various regional TTDs over 100 yr at the following three locations: 183-m depth, 10°S, 80°E in the Indian Ocean (top); 94-m depth, 10°N, 110°W in the Pacific Ocean (middle); and 2014-m depth, 42°S, 50°W in the Atlantic Ocean (bottom).

In the example from the South Indian Ocean, the global TTD appears unimodal. However, it is of interest to note that this apparently simple function results from the superposition of regional TTDs, which each have very different maximum values and first moments. The major contributor at this location is water that originated at the surface in the South Indian Ocean (region 5). This can be understood in terms of the subduction of water in the subtropics of the South Indian Ocean, and the subsequent westward transport with the South Equatorial Current, and then northward transport in the East Africa Coastal Current, where the South Equatorial Current splits at Madagascar at about 15°S. The Northern Pacific component (region 6) has its origin in water that was subducted in the northern Pacific, and subsequently advected equatorward, and then through the Indonesian Throughflow into the central Indian Ocean. It is of interest to note that the component from the west-central Pacific (region 7) is very small as compared with the contribution from region 6, and has a slightly earlier peak. This represents water that originated somewhere in the surface ocean in region 7, and was subsequently transported through the Indonesian Throughflow and subducted to a couple of hundred meters depth in the Indian Ocean.

The global TTD at 10°N, 110°W, at 94-m depth in the Pacific has a very pronounced bimodal structure, with the first and largest narrow peak at around 3 yr, and a slightly smaller broader secondary peak at around 13 yr. It is easily discerned from Fig. 9 that the first peak in the global TTD comes from water that originated in the surface ocean of the east-central Pacific (region 8) and reached 94-m depth through mixing (note the deep mixed layer in this region evident in Fig. A3). The Northern Pacific TTD (region 6) is interesting because it is itself bimodal. The first peak at about 3 yr and the second at about 14 yr strongly suggest an imprint from mixing within the mixed layer, while the second peak results from the roughly decadal transit time of water subducted in the North Pacific and advected southward.

**Fig. 8.** Map of the 11 regions of surface ocean used to propagate the “regional TTDs.” The letters A, B, and C denote the locations of time series shown in Fig. 9.
At 2014-m depth in the South Atlantic (lower panel in Fig. 9), the selected global TTD shows a multimodal pattern with an early weak double peak at around a decade, and a much stronger later peak at around 65 yr. When the Southern Ocean and North Atlantic regional TTDs are superimposed (regions 11 and 10, respectively), it is clear that the first peak is because of the arrival of AABW, and that the larger peak indicates the arrival of NADW at this location.

As with the global TTD, considering the regional TTDs as dyelike tracers or looking at column inventories at various depths can be very informative regarding large-scale circulation pathways and time scales, but presenting detailed figures here is prohibitive. At first, all of the regional TTDs evolve very differently in accordance with the flow regimes that characterize their particular region. For example, the deep-water signal at 50 yr in the Atlantic in Fig. 1 is due almost entirely to the contribution from region 10. At later times, however, the spatial structure of each regional TTD becomes the same as all of the others everywhere except for the Arctic. Figure 10 shows the global and all 11 regional TTDs at 819 m after 2000 yr of integration, normalized by the maximum value of each. Only in the Arctic and in the Southern Ocean are significant differences seen in the spatial patterns. In the Southern Ocean, the patterns can be grouped into two categories—one for the Pacific subregions (regions 6 through 9) and one for all of the other subregions. The Arctic regional TTDs show more pronounced differences (dif-

---

**Fig. 9.** Time series of global and significant regional TTDs at selected locations (denoted by A, B, and C in Fig. 8) in the (top) Indian, (middle) Pacific, and (bottom) Atlantic Oceans.
ficult to discern from Fig. 10, but evident on closer inspection); this may be because this region takes a very long time to achieve similarity because of its isolation. Not surprisingly, the degree of similarity after 2000 yr depends strongly on depth, with all 12 TTDs looking essentially identical at 245 m, and somewhat more dissimilar (though still strikingly alike) at 3000 m.

The behavior described above can also be seen in the globally averaged TTDs for each region. In Fig. 11, each regional TTD has been normalized by its value after 2000 yr of integration. Clearly, all of the regional TTDs evolve quite differently at first, but end up all having the same behavior after about 1500 yr. This, combined with the similar structure (Fig. 10), indicates that the regional TTDs need to be carried only for about the first 2000 yr of model integration, and that after this time each globally averaged TTD could be accurately predicted using the global TTD multiplied by a scale factor. The reason for the similarity of the TTDs at long time scales has to do with eigenmodes of the transport operator. In the limit of steady-state transport, these modes all ultimately decay exponentially in time, leaving just the gravest mode (T. Hall 2004, personal communication; also noted by Primeau 2005).

6. How close is the TTD to an inverse Gaussian?

One powerful application of the ocean TTD field is the potential ability to predict the spatial and temporal evolution in the ocean of an arbitrary tracer for which the surface boundary condition is known. Complications lie in tracers that have a nonuniform spatial sea surface boundary condition at a given time, and the question of how feasible it is to assume a stationary transport operator for the real ocean. These complications deserve extensive consideration, and will be the subject of a future paper.

It has been shown that, throughout much of the ocean, the TTD appears to be a fairly simple function with an early peak and long tail. However, there are regions, most notably the Southern Ocean, where the TTD is most frequently multimodal, sometimes assuming a fairly complex structure. It is important to know where one might expect the TTD to be sufficiently simple that it might be described by just one or two free parameters, because such simple forms would offer the potential for predictability based on tracer measurements. Hall et al. (2002) have shown that if the TTD takes one such form with two free parameters—the “inverse Gaussian” (IG)—that it can be fairly well con-

Fig. 10. The spatial distribution of the global log(TTD) and the 11 regional TTDs at 819-m depth after 2000 yr of integration. The contour interval is 0.2.
strained by a small number of distinct tracer measurements. The inverse Gaussian can be written as

$$G(t) = \sqrt{\frac{\Gamma^2}{4\pi \Delta^2 t^3}} \exp\left[-\frac{\Gamma(t - \Gamma)^2}{4\Delta^2 t}\right].$$

where $\Gamma$ and $\Delta$ are the first and second centered moments of the TTD (Waugh et al. 2003). In a recent paper (Hall et al. 2004), an inverse Gaussian functional form was assumed to characterize the global ocean surface TTD everywhere in the interior ocean. Existing tracer measurements were then used to infer the most appropriate IG for a given region. This inferred IG TTD was then convolved with the surface boundary condition for a series of outcropping regions to infer the uptake of anthropogenic CO$_2$ in the Indian Ocean. Clearly, in order to interpret the results from such studies it is desirable to have an idea of the error inherent in assuming that the actual TTD can be described by an IG.

The inverse Gaussian distribution has an early peak and a long tail; this is the shape obtained from the 1D advection–diffusion equation in a semi-infinite domain (Hall and Haine 2002). To assess how good an approximation the IG assumption was to the simulated TTD, the first and second centered moments ($\Gamma$ and $\Delta$) were computed for each ocean grid point from the 5000-yr-long TTD at each point. The IG with these $\Gamma$s and $\Delta$s were then calculated for each grid point according to Eq. (A1).

Let us for the moment assume that all of the conditions necessary for the convolution integral described in Eq. (2) are satisfied. An interesting question to explore then becomes that of how sensitive the prediction of the interior ocean distribution of an arbitrary tracer at a given time is to the assumed functional form of the transit-time distribution. More specifically, we ask to what degree the predicted distribution of an arbitrary tracer is compromised if the IG is used in the convolution integral instead of the “actual” TTD. It must be borne in mind that the differences obtained in such an idealized experiment might be very different to those obtained in an attempt to predict the actual CFC distribution, for example, with the full TTD versus the IG approximation. In the case of actual CFCs one would need to convolve a suite of boundary conditions with the TTDs in the interior ocean and sum the results (as described in Hall et al. 2004). Here we are considering a much simpler case, where the idealized tracer has a spatially uniform surface boundary condition at any given time.

Assessment of the error incurred by using an IG over the full TTD is complicated because the answer will depend both on the shape of the upper-ocean boundary condition, and the time at which the comparison is made. Thus, the error between tracer distribution predicted by the TTD and the IG will vary spatially and temporally. We here consider two idealized cases: the distribution of a “CFC like” tracer in the ocean 50 yr after its first appearance in the atmosphere, and a “natural radiocarbon like” tracer after 1000 yr. The time-dependent amplitude of the spatially uniform boundary conditions for each of these tracers are shown in Fig. 12.
a. Idealized CFC-like tracer

The observed atmospheric CFC-11 history was used to construct an idealized sea surface CFC-11 boundary condition assuming 100% solubility and a mean temperature and salinity of 10°C and 34‰ in the surface ocean. This ocean boundary condition was then "propagated" into the ocean interior using first the model-simulated TTD, then using the IG obtained from the first and second moments of the 5000-yr model TTD.

The distribution of the CFC-like tracer in the ocean predicted by convolution of the IG with the idealized boundary condition ("predicted") at year 50 was compared with that obtained by convolution of the model global TTD with the CFC boundary condition ("actual"). Fifty years after introduction into the atmosphere, the tracer concentration ranges between 0 and 4.5 pmol kg\(^{-1}\) on the 245-m model depth surface, as is shown in the upper panel of Fig. 13. In the middle panel of Fig. 13, the absolute difference between the actual and predicted tracer distribution is shown, and the percent error between the actual and predicted tracer is shown in the lower panel. The agreement between actual and predicted tracer concentration is remarkably good in the subtropical gyres, but is less good in the equatorial and low-latitude regions. This is particularly clear when measured as percent error. High-error regions tend to have a fairly small actual tracer concentration (on the order of 1 pmol kg\(^{-1}\)), and an even lower predicted tracer concentration (on the order of 0.5 pmol kg\(^{-1}\)), leading to a percent error of the order of 100%, although absolute error is relatively small (on the order of 0.5 pmol kg\(^{-1}\)). In most regions, the IG-based prediction is significantly smaller than the TTD-based estimate.

b. Natural radiocarbon–like tracer

The boundary condition for the natural radiocarbon–like tracer differs from that for the CFC-like tracer in that it is constant in time, fixed at the preindustrial atmospheric \(^{14}\)C/\(^{12}\)C ratio of \(1.18 \times 10^{-12}\). This tracer also differs in that there is now a sink: radioactive decay. This is trivially incorporated into the predicted tracer field by adding a decay term to Eq. (2):

\[
C(\mathbf{r}, t) = \int_0^\infty C(\mathbf{r}_0, t - \xi)G(\mathbf{r}, \xi)e^{-\lambda \xi} d\xi,
\]

where \(\lambda\) is the radiocarbon decay constant \([\lambda = \ln(2)/\tau^{1/2}\text{, and } \tau^{1/2}\text{ is the half-life of radiocarbon, 5730 yr}].\)
As in the previous case, the IG was computed from the first and second moments of the full-model TTD. The concentration of the radiocarbon-like tracer on various depth surfaces was then estimated at 1000 yr.

The percent error after 1000 yr of integration on the 245- and 1000-m depth surfaces is shown in Fig. 14. The error is, in general, much smaller at 245-m depth than it was for the CFC-like tracer. At 1000 m the error is less than 10% in most of the ocean. The reason for the improved prediction skill at longer times is in large part because, by 1000 yr of integration, the TTD has asymptoted to its final value almost everywhere in the upper 1000 m of ocean. This means that differences in the early part of the shape of the TTD and IG, which would have a large impact at 50 yr, say, have been largely integrated out by the time the TTD and IG are fully converged.

c. Identifying regions of large prediction error

There are characteristic TTDs that can be identified that will yield a poor agreement between tracer fields predicted using the full TTD and the IG approximation. The most obvious case is when the TTD is multimodal. In such regions, there will always be a discrepancy between the TTD and the IG, and there will therefore always be a difference in predicted tracer distributions using the two functions. A bimodal TTD that takes a fairly simple form (the sum of two inverse Gaussians) can be fully described with four free parameters; multimodal TTDs need correspondingly more

![Fig. 13](https://example.com/fig13.png)

Fig. 13. (top) Concentration of actual CFC-like tracer at 245-m depth (pmol kg\(^{-1}\)) after 50 yr of model integration. (middle) Absolute difference between actual and predicted tracer concentration (pmol kg\(^{-1}\)) at the same time. (bottom) Percent difference between actual and predicted tracer concentration. The two lower panels have a zero contour to highlight the sign of the difference.
parameters. The predominant region of the ocean in which the model predicts multimodal TTDs is the Southern Ocean. The multiple peaks in the TTD represent multiple arrivals of different water masses; for example, waters formed in the Weddell and Ross Seas have distinct tracer signatures and arrive at a given point in the Southern Ocean at different times. An example of a multimodal TTD at 59°S, 52°W, at 1000-m depth in the Atlantic sector of the Southern Ocean is shown in Fig. 15. The IG computed by the first and second moments is also shown. It is apparent from this figure that prediction of a tracer using both the TTD and the IG at early times (say 50 yr after introduction into the ocean, as was the case for the CFC-like tracer) will then tend to be much larger than the numerator, yielding a product that is a very small number. The exponential term will therefore tend to one in such cases, and the IG will be approximately equal to the first term in Eq. 6 \(\frac{(\Gamma^2/(4\pi^2\Gamma^2))^{1/2}}{t^3} \). Because this term

![Fig. 14 live 4/C](image)

**Fig. 14.** (top) Percent difference between actual and predicted radiocarbon-like tracer at 245-m depth after 1000 yr of integration. (bottom) Percent difference at 1000-m depth at the same time. Both panels have a zero contour to highlight the sign of the difference.

Another general regime in which the IG and TTD tend to yield very different tracer distributions is when the first-moment \( \Gamma \) is much smaller than the second centered moment \( \Delta \). An example of where this is the case is in the southern subtropics of the Indian Ocean. The upper few hundred meters of water are very rapidly ventilated, which gives rise to distributions with a sharp peak at 2 or 3 yr. The tail of the TTD drops off rapidly, but maintains a very small nonzero value for a number of decades (resulting from weak mixing with deeper waters). This leads to a much larger \( \Delta \) than \( \Gamma \). The denominator of the exponential term in Eq. (A1) will then tend to be much larger than the numerator, yielding a product that is a very small number. The exponential term will therefore tend to one in such cases, and the IG will be approximately equal to the first term in Eq. 6 \(\frac{(\Gamma^2/(4\pi^2\Gamma^2))^{1/2}}{t^3} \). Because this term

![Fig. 15](image)

**Fig. 15.** First 1000 yr of model TTD at 59°S, 52°W, at 1000-m depth in the Atlantic sector of the Southern Ocean (multimodal curve) and inverse Gaussian computed using first and second moments of the TTD (unimodal curve).
represents a rapid decay, the IG is generally a poor approximation to the model TTD.

7. Summary and discussion

We have shown that the model-simulated spatial distribution of the global TTD and the mean age can be well reconciled with conventional wisdom on the major oceanic regimes, and that they agree qualitatively with other related modeling studies (England 1995; Primeau 2005). Below the mixed layer, the simulated TTD is extremely long tailed almost everywhere in the ocean, meaning that many thousands of years of integration are required to fully describe the distribution. In the subthermocline Pacific Ocean, the time scale for convergence is on the order of 5000 yr; in the deep Atlantic it is considerably shorter (on the order of 1000 yr) because of more rapid ventilation by the thermohaline circulation.

Selected results have been presented from a suite of 11 regional TTDs. It has been shown that several of regional TTDs can be important contributors to the global TTD at a given location, and that these regional TTDs themselves contain important information about the pathways and time scales of ventilation leading to the TTD at a given point. Often, the contributions from different water masses lead to multimodal behavior (ubiquitous in the Southern Ocean), but can also (somewhat deceptively) sum up to produce a unimodal distribution. Also notable is the fact that each of the regional TTDs are essentially identical (to within a scale factor) after about 2000 yr of model integration, which reflects the time scale of decay for all but the lowest eigenmode of the system.

Because one major application of the TTD method is estimation of the spatial and temporal evolution of a given tracer at every point in the ocean, the important question arises of whether a very simple analytical function (such as the inverse Gaussian; Hall et al. 2002) could describe the TTD throughout much of the ocean. On small time scales (from decades to centuries), the tracer distribution in the subtropical gyres tends to be fairly well approximated by the IG (as compared with the full forward model), while errors tend to be much larger in the low- and high-latitude oceans. On longer time scales, such as the millennial time scales for tracers such as natural radiocarbon, a much greater portion of the TTD/IG is used in the convolution integral, and the errors as a whole tend to be much smaller than those for the shorter-time-scale transient tracers. In interpreting these results, however, two caveats should be borne in mind. First, we simulated highly idealized tracers by convolution of an idealized surface ocean boundary condition with a single global TTD, rather than summing multiple TTDs over many outcropping regions with slightly different tracer concentrations. Investigation of how the predicted tracer distributions change when summing the TTDs obtained from more realistic, spatially varying boundary conditions will be the subject of future work. Second, there is no guarantee that the model-simulated TTD does in fact represent the “true” ocean TTD. The skill of the model in reproducing the observed global ocean CFC field [see Peacock et al. (2004) for a detailed study of the skill of this model in reproducing CFCs in the Indian Ocean] suggests that the model TTDs must be a fair approximation to the real ocean TTDs in the thermocline at least. However, the degree to which the real ocean TTD resembles the model-simulated TTD remains an open question.

There are regions in the ocean where we know that the TTDs from this simulation must be inaccurate because of the coarse resolution of the model. In particular, narrow boundary currents and flows controlled by complex topography simply cannot be adequately resolved using a 3° grid, resulting in potentially large errors where the TTD evolution is determined primarily by advective time scales. One notable example of this is the Atlantic Deep Western Boundary Current, which plays an important role in ventilating the deep ocean and has an unrealistically small speed in this simulation. As seen in the lower panel of Fig. 9, the peak of the distribution resulting from northern Atlantic ventilation occurs after about 65 yr. Because the mean age is the first moment of this time series (i.e., weighted by \( t \)), a very different estimate would be obtained if the peak was significantly earlier.

To begin investigating such questions related to resolution, we have performed a 50-yr eddy-permitting (0.4°) simulation of the global, Arctic/Northern Antarctic, Southern, and Northern Pacific TTDs (M. Maltrud and S. Peacock 2004, unpublished manuscript). In many regions of the ocean, especially where currents are not strong, the agreement between the low- and high-resolution TTDs is quite good. In the DWBC at 40°S, however, the ventilated water from the northern Atlantic peaks at about 30 yr (as opposed to 65 yr in the coarse run), yielding a substantially lower estimate of mean age. The eddy and interannual variability in the eddy-permitting model also make it harder to identify multimodal distributions, which should be remembered when trying to estimate TTDs for the real ocean. Further comparisons of resolution effects will be pursued in the future.

The simulation described in this paper represents a single realization in a large parameter space. Because of the prohibitive cost, the degree to which changes in
parameterizations, such as the vertical and horizontal mixing, can change the shape of the distribution has not been explored. However, it is expected that even with an extremely small amount of vertical mixing, for example, the distributions would still be very long tailed. Thus, the fundamental structure of the TTDs and the spatial patterns observed in the model are expected to have relevance to the TTDs in the actual ocean.

Further exploration of model sensitivities can be made more efficient with recent advances in acceleration and offline transport methods. Obtaining a steady-state ocean circulation can be achieved more quickly using acceleration methods [see Danabasoglu (2004) for a study using essentially the same model setup as is used here]. Once a steady ocean state is available, offline matrix methods, such as those described by Primeau (2005) and Khatiwala et al. (2005), can be used for passive tracer studies. Although these methods only provide an approximation to the solution that would be obtained using a full forward model (but at a small fraction of the computational cost), they provide a very useful complement to simulations of the sort described here.

Acknowledgments. This work was supported by the Climate Change Prediction Program of the U.S. Department of Energy Office of Science. The authors thank Timothy Hall for useful suggestions and two anonymous reviewers for their help in improving the manuscript.

APPENDIX

Model Validation

Extensive effort has been expended to ensure that the version of POP used in this study can maintain a thermohaline circulation (THC) and does not exhibit significant drift in deep-ocean properties when integrated over millennial time scales. The lack of drift in the model fields is important for being able to obtain the TTD as a response to a single pulse in the surface boundary condition. For the first 600 yr, the drift in globally averaged temperature (Fig. A1, upper panel) is on the order of 0.015°C per 100 yr after 1500 yr of integration the drift is on the order of 10^-6°C per 100 yr, (i.e., effectively zero). After an initial adjustment on a time scale of decades (which occurred during the 500-yr spin-up phase of the model, before introduction of the tracers), the strength of the THC has a similarly small drift (Fig. A1, lower panel), indicating that the model has reached a steady state. Although it appears that we perhaps should have spun up the model for another 1000 yr before introducing the tracers, we believe that the results presented here would not be significantly different because the overturning only
changes by a small fraction of a Sverdrup (1 Sv = 10^6 m^3 s^-1) over this time.

The magnitude of the overturning has a maximum of around 18 Sv in the North Atlantic, and a few Sverdrups of Antarctic Bottom Water (AABW) are present at a depth of around 4 km (Fig. A2). This is in good agreement with observational-based estimates of the strength of the THC (Schmitz 1995). In the North Atlantic, the model realistically convects in the Labrador and Irminger Seas (Fig. A3), whereas the convection south of the Iceland–Scotland–Faroe Ridge system is suspect. The problem of accurately simulating the overflow waters is common to all coarse-resolution z-coordinate GCMs, which do not employ partial bottom cells. In the Southern Ocean, the model convects in the Weddell and Ross Seas. It should be noted that the mixed layer depths of around 200 m in Fig. A3, which lie to the southwest of Australia and Chile, are the signature of shallow subduction (Luyten et al. 1983), not deep convection.

A comparison of modeled versus observed CFC fields in the Indian Ocean (Peacock et al. 2004) shows that this configuration of POP does a remarkably good job at simulating the decadal-scale ventilation processes in the upper ocean. The model has been shown to very accurately predict the spatial distribution and penetration depth of CFCs, which have entered the ocean over the past 50 yr or so. It is therefore to be expected that time scales of upper-ocean ventilation in the model are a fairly close approximation to reality.
Fig. A4. The global average volume-weighted ideal age in the model is shown as a function of time by the dashed line; the first moment of the average TTD (defined as the average mean age) is shown by the gray line; the first moment (mean age) predicted by Haine and Hall (2002) [based on Eq. (7)] is shown by the solid black line.

Fig. A5. Zonally averaged (top) mean age and (middle) ideal age in years for the Pacific Ocean after 5000 yr of model integration. The contour interval is 500 yr. (bottom) Difference between the ideal and mean age at this time, with a contour interval of 10 yr.
To compare our simulation with previous studies that focus on ventilation age, the ideal age (England 1995) was also carried as a tracer. This is defined as

\[
\frac{\partial \tau_{id}(r, t)}{\partial t} + L[\tau_{id}(r, t)] = 1; \quad \tau_{id}(r, 0) = 0, \quad (A1)
\]

where \(\tau_{id}(r, t)\) is the ideal age, and \(L\) is, again, the general linear transport operator. \(\tau_{id}(r, t)\) is incremented every model step by the value of the time step and is reset to zero in the top model level every time step. Hall and Haine (2002) have recently demonstrated that the following simple relationship holds between the ideal age and the first moment of the TTD:

\[
\Gamma(r, t) = \tau_{id}(r, t) - t \frac{\partial}{\partial t} \tau_{id}(r, t), \quad (A2)
\]

where \(\Gamma(r, t)\) is the mean age [Eq. (4), and we have added a time variable because we are interested in the time evolution] and \(\tau_{id}(r, t)\) is the ideal age. These authors have also shown that the ideal age always converges faster than does the mean age. This is validated in Fig. A4, which shows that the simulated first moment is almost identical to that computed using the simulated ideal age and Eq. (A2) for the first 2000 yr or so of the simulation. After this time, the simulated first moment is slightly greater than the computed mean age, and slightly overshoots the ideal age at about 4500 yr. Again, we suspect that the reason for this divergence between the simulated and theoretical mean age is because of our 1-yr approximation of a delta function noted in section 4.

Equation (A2) implies that the mean age is identical to the ideal age after the ideal age has converged [i.e., \((\partial/\partial t)\tau_{id}(r, t) = 0\)]. This equivalence can be seen from Fig. A5, where after sufficient integration time, ideal age is essentially indistinguishable from mean age. Figure A5 shows both the zonally averaged mean age [as computed from Eq. (4)] and the zonally averaged ideal age [as computed from Eq. (A1)] in the Pacific Ocean after 5000 yr of model integration. It is clear that the differences are very small, with discrepancies in absolute age being at the most a few decades (e.g., in the deep Arctic and abyssal Southern Pacific). This also highlights the importance of doing such a comparison only after both have converged, because the two fields are completely different at, for example, a time of 1000 yr.

Our results can be directly compared with those of England (1995), a study in which ideal age was simulated in an ocean GCM with a somewhat coarser resolution than that used in the model described here. The ages reported in the England (1995) study for the Pacific basin (his Fig. 3c) are roughly 30% smaller than those obtained from our simulation (our Fig. A5), though the general pattern is not too different. Similar behavior is observed for the other ocean basins. The older ages found in the present study are likely related to smaller mixing, because of the use of the GM parameterization and KPP. Comparison of mean age with Primeau (2005) (his Figs. 7 and 8) shows considerable agreement. We typically have significantly older ages in the thermocline, which is likely because of the POP having a much finer vertical resolution in the top 1000 m.

REFERENCES

Cook, P., and D. Solomon, 1997: Recent advances in dating young groundwater: Chlorofluorocarbon, \(^{3}H\)/\(^{3}He\) and \(^{86}Kr\). J. Hydrol., 191, 245–265.


Gruber, N., J. Sarmiento, and T. Stocker, 1996: An improved parameterization and KPP. Comparison of mean age with Primeau (2005) (his Figs. 7 and 8) shows considerable agreement. We typically have significantly older ages in the thermocline, which is likely because of the POP having a much finer vertical resolution in the top 1000 m.

REFERENCES

Cook, P., and D. Solomon, 1997: Recent advances in dating young groundwater: Chlorofluorocarbon, \(^{3}H\)/\(^{3}He\) and \(^{86}Kr\). J. Hydrol., 191, 245–265.


