Non normal thermohaline circulation dynamics and predictability in a coupled GCM

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Background & Motivation:

- Linear THC dynamics, transient amplification, stochastic optimals (Farrell 1989)

Questions:

- Can transient amplification lead to a significant THC amplification? **Implications to THC predictability?**
- Physical mechanisms of transient growth of THC anomalies?

Approaches:

- Hierarchy of models, from the GFDL Coupled Model 2.1, via a zonally averaged model, to a 3 box toy model
- Linear inverse modeling (Penland 1989, 1996)

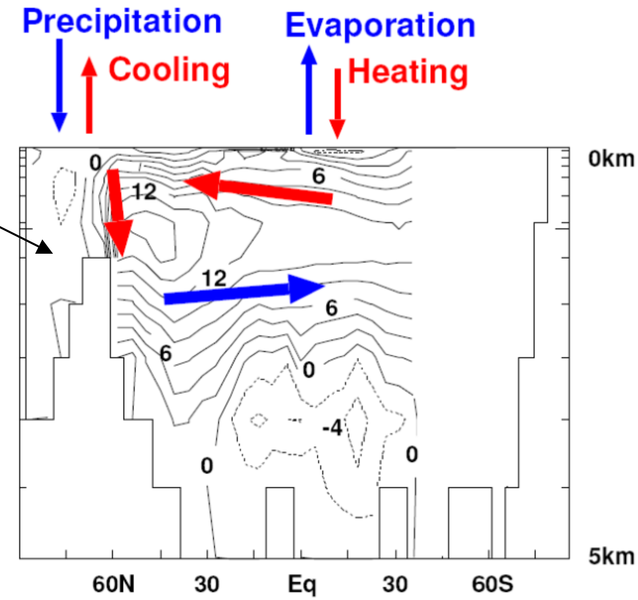
Thanks to Tom Delworth and Keith Dixon. GFDL

[Tziperman & Ioannou, *JPO, 2002; Zanna & Tziperman, JPO 2005; submitted 2007a,b]*

The Thermohaline circulation

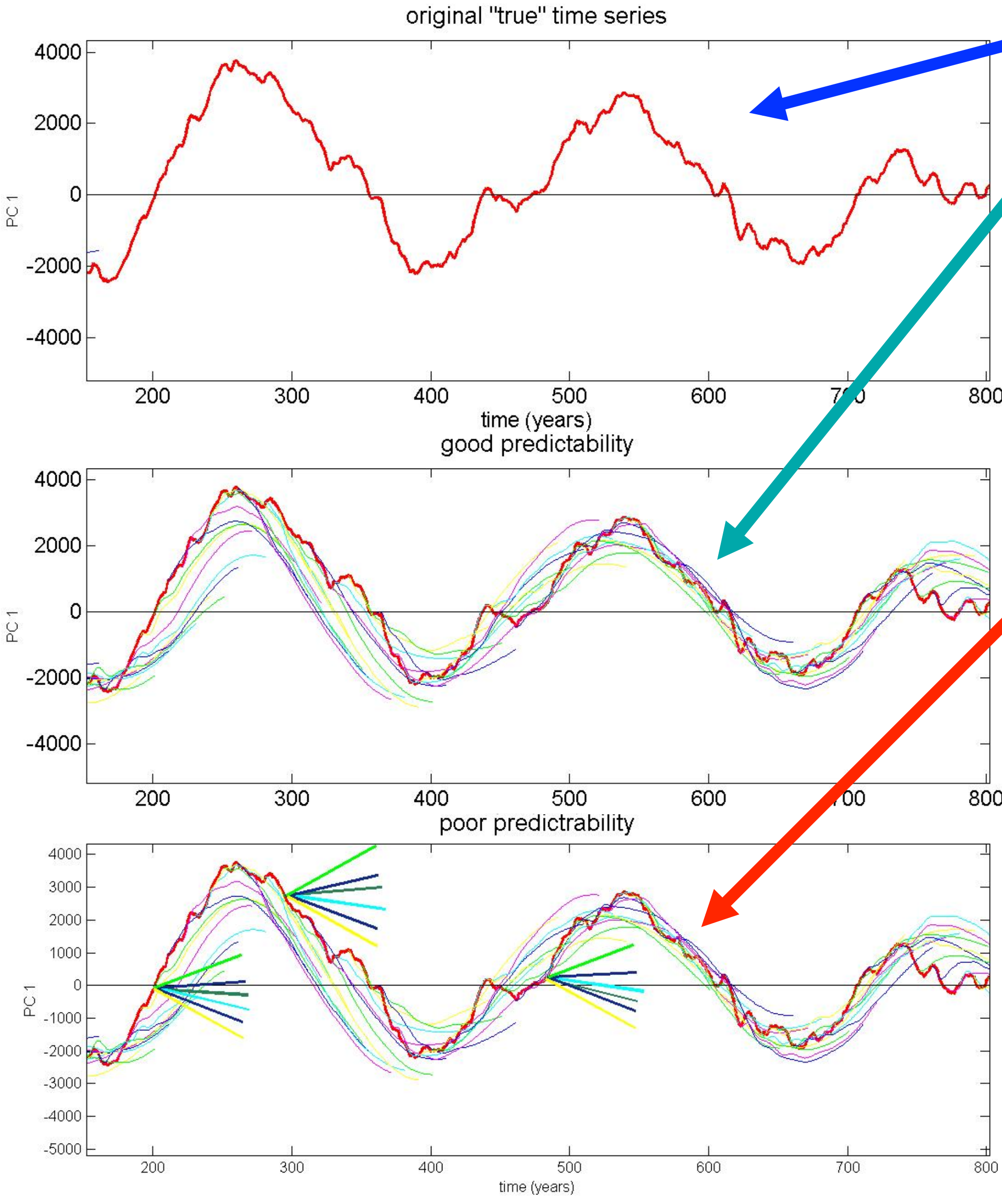
THC schematic

THC cross-section along Atlantic ocean



- Salinity ⇧, Temperature ⇩ → density ⇧
- Driven by north-south temperature gradients due to atmospheric heating & cooling.
- "Braked" by north-south salinity gradient due to precipitation & evaporation.
- present-day THC is "thermally dominant"
- Carries significant heat flux from equator to poles.
- Mass flux: 20Sv = 20 X (combined world rivers)

On the importance of initial conditions when determining predictability



Observed/model time series

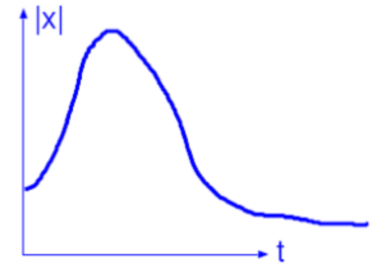
Predictability experiments using some random perturbations to initial conditions → good predictability?

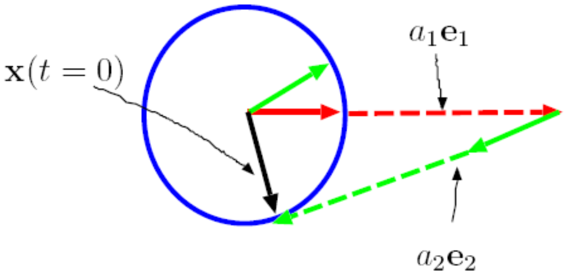
Predictability experiments using some other perturbation to I.C.s → poor predictability!

→ Need I.C.s that result in maximum growth of perturbations to correctly evaluate predictability.

Preliminaries: Transient amplification

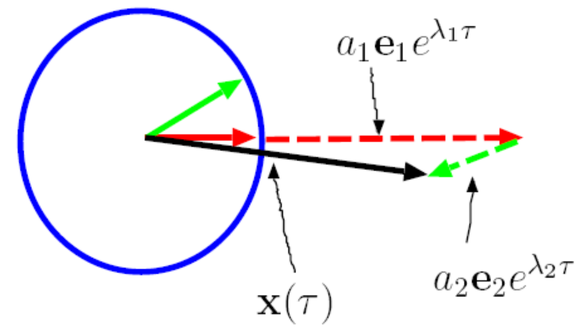
(Farrell, 1988) A damped linear system: $\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x}$, such that $\mathbf{x} \longrightarrow 0$ as $t \longrightarrow \infty$ may still have an initial amplification: why? How?





$\longleftarrow$ Let $\mathbf{A}e_i = \lambda_i \mathbf{e}_i$; if $\mathbf{A}^T\mathbf{A} \neq \mathbf{A}\mathbf{A}^T$, then $\mathbf{e}_i$ are not orthogonal. Consider the following i.c. in a 2d case

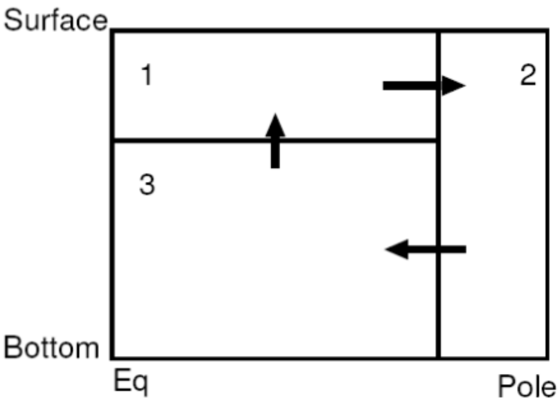
at $t = \tau$, solution is $a_1\mathbf{e}_1 e^{\lambda_1\tau} + a_2\mathbf{e}_2 e^{\lambda_2\tau}$. If $\lambda_2 \ll \lambda_1 \ll 0$ then $a_2\mathbf{e}_2 e^{\lambda_2\tau} \longrightarrow 0$ quickly, leaving mostly $\mathbf{x}(\tau) \approx a_1\mathbf{e}_1 e^{\lambda_1\tau}$



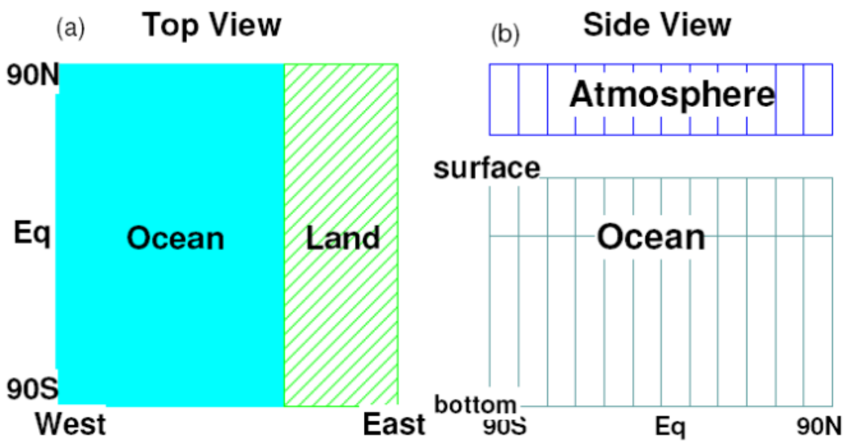
$\Rightarrow$ An initial amplification!! Later, $a_1\mathbf{e}_1 e^{\lambda_1\tau} \longrightarrow 0$ as well, so that $\mathbf{x}(t \longrightarrow \infty) \longrightarrow 0$.

➔ required ingredient for transient amplication:
(i) initial cancellation; (ii) different decaying rates.

Model hierarchy I: simple models, ocean



simple 3-box model under mixed b.c.



Intermediate coupled model: continuous in latitude; 2 ocn & 1 atm levels

Momentum: $\frac{\partial p}{\partial y} = -rv$; $\frac{\partial p}{\partial z} = -\rho g$; ($p$ is pressure, v northward velocity, r friction, ρ density, g gravity) $\Rightarrow$THC: $v = C_u g\left[(\rho_2-\rho_1)D_{upper} + (\rho_3-\rho_4)D_{lower}\right]$

Temperature (T) and Salinity (S) advection-diffusion eqns:

$$\frac{\partial T}{\partial t} + \nabla(T\mathbf{u}) + \kappa\nabla^2 T = H^{air-sea}; \quad \frac{\partial S}{\partial t} + \nabla(S\mathbf{u}) + \kappa\nabla^2 S = (\text{Evap, Precipitation})$$

Convective mixing: when stratification is unstable; density: $\rho = \rho(T,S)$

Model hierarchy II: simple models, atmosphere

•The Atmospheric model is a simple energy balance model (e.g. Nakamura et al 94; Rivin & Tziperman 97) .

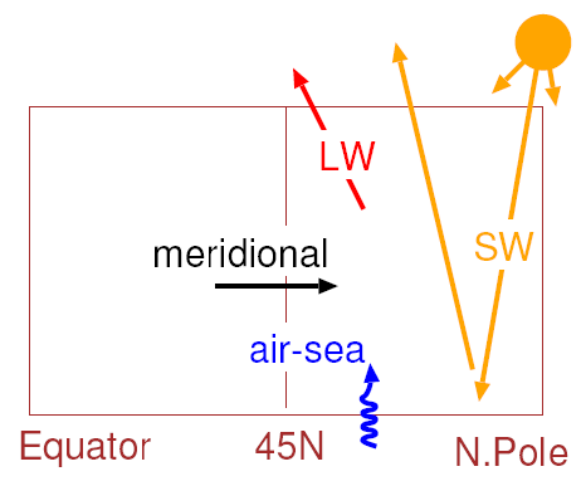
•The temperature at a given box is determine by the balance between the incoming shortwave radiation, outgoing long wave radiation, air-sea fluxes, & meridional atmospheric heat fluxes:

$$\frac{dT^{atm}}{dt} = SW \times (1-\alpha) - LW + \text{air-sea} + \text{meridional fluxes}$$

α: albedo, including contributions from land, land-ice, ocean, sea ice

LW: black body (long wave) radiation to outer space

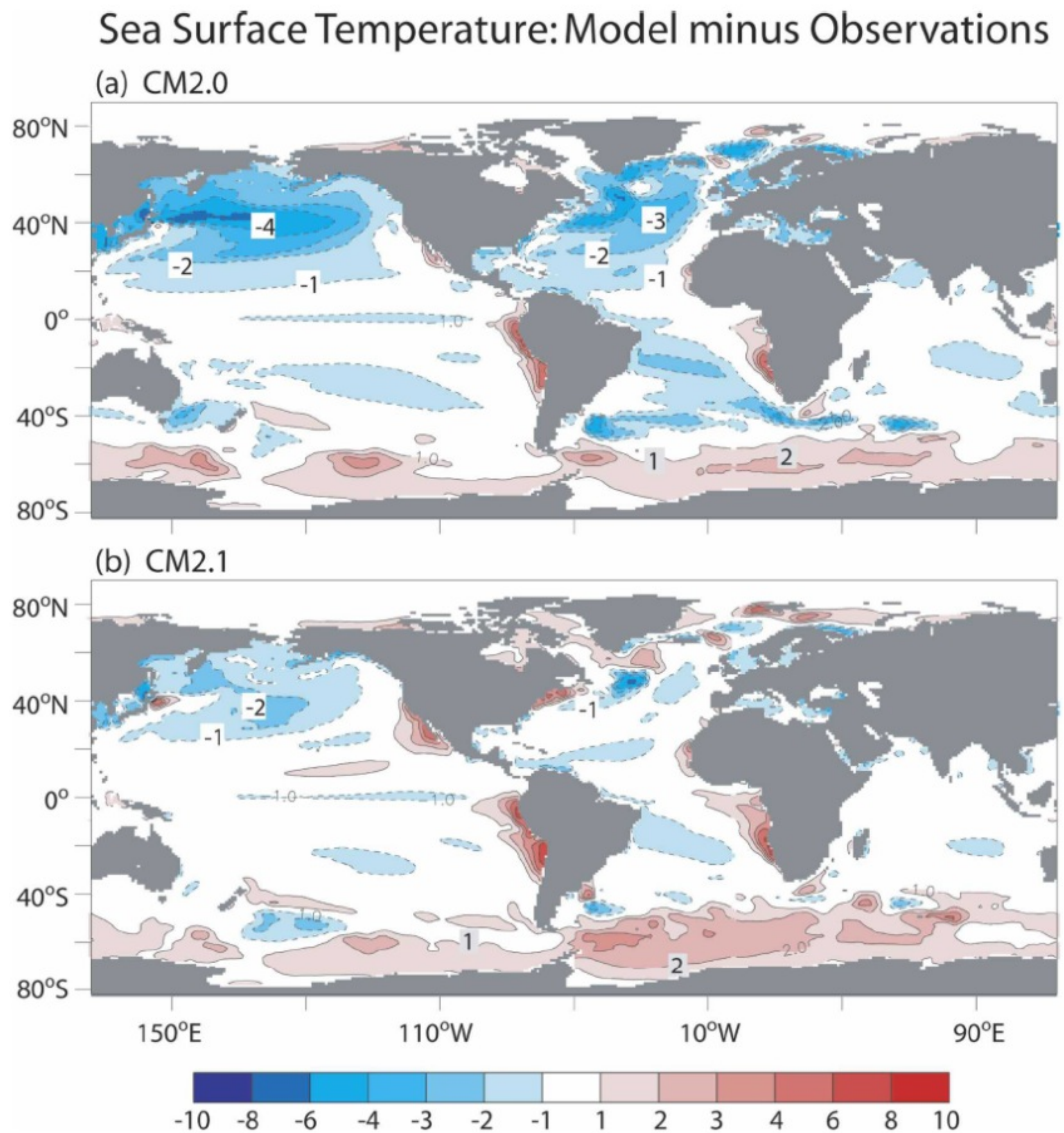
SW: incoming (short wave) solar radiation



Model hierarchy III: GFDL coupled model

Model output used:

- 2000 years of control run from CM 2.1
- Eliminating first 500yrs due to drift, left with 1500 years of annually averaged temperature and salinity fields
- Subsampled every 3rd grid point



Delworth et al, 2006, J. Climate

Transient amplification: terminology

(1) Given a stable linear(ized) dynamical system: $d\mathbf{x}/dt=A\mathbf{x}$, then,

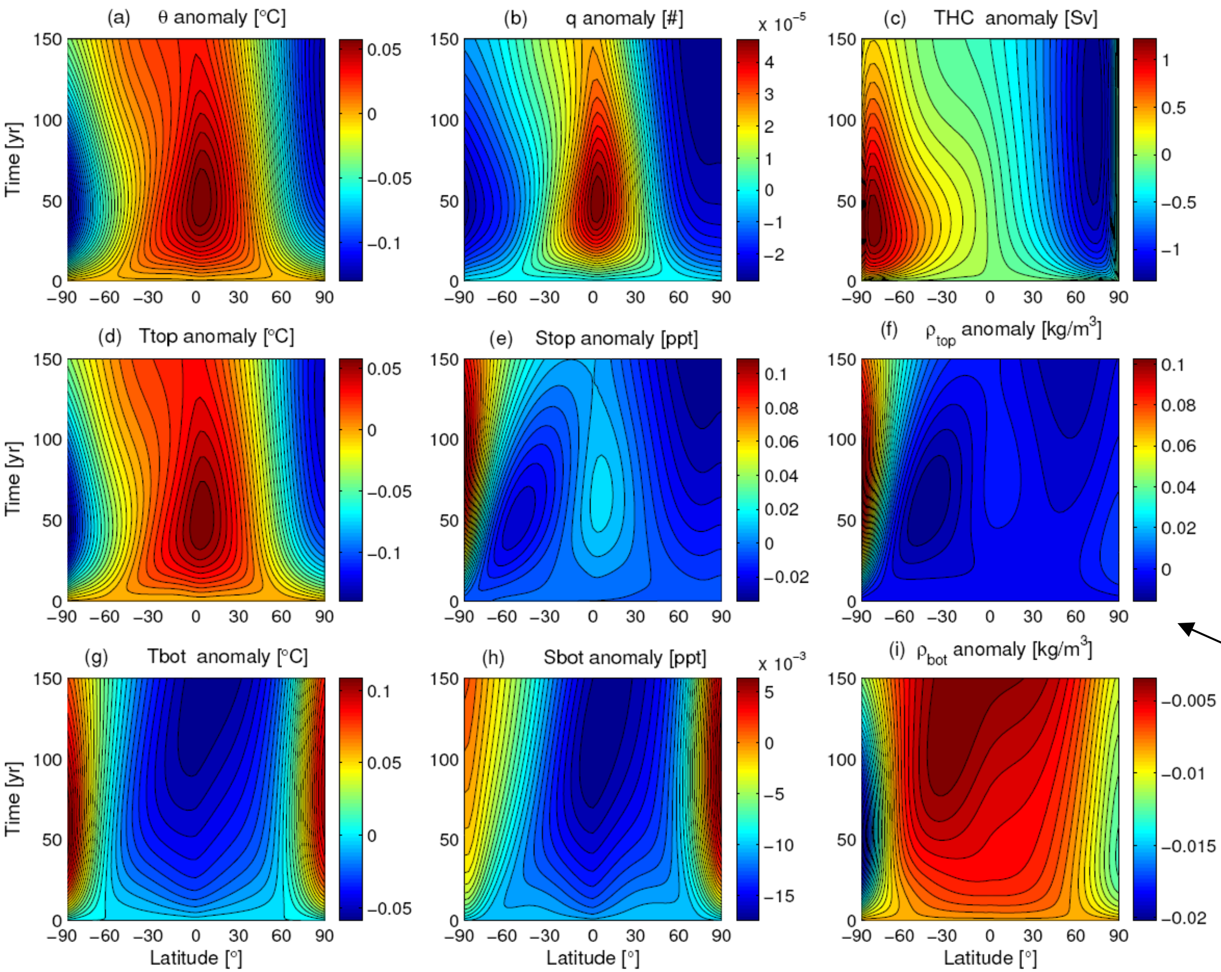
➔ Optimal initial conditions: $\mathbf{x}(t=0)$ that lead to maximum growth of $|\mathbf{x}(t=\tau)|^2$ at time τ.

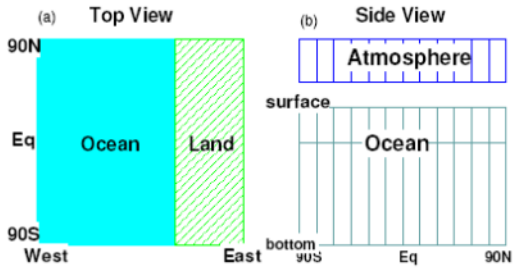
(2) Given a stochastically forced linear system, $d\mathbf{x}/dt=A\mathbf{x}+\xi(t)$, where $\xi(t)=\xi_0\nu(t)$, and $\nu(t)$ is white noise, and ξ_0 represents, say, the spatial structure, then,

➔ Stochastic optimals: The spatial structure of the stochastic forcing ξ_0 that results in maximum variance $\langle\mathbf{x}(t)^T\mathbf{x}(t)\rangle$

These can both be easily calculated from the linear model A

Intermediate model results: optimal THC initial conditions





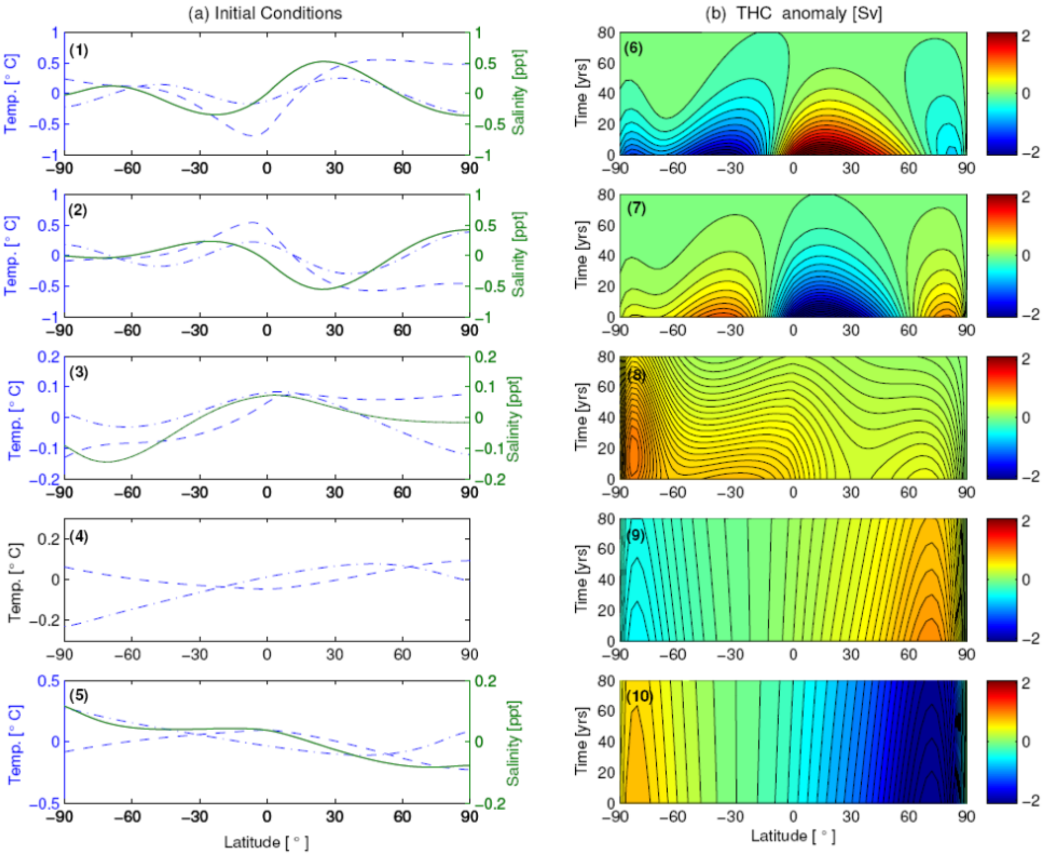
latitude-continuous coupled ocean-atm Model

Optimal initial conditions for THC growth (Zanna & Tziperman, 2005, JPO)

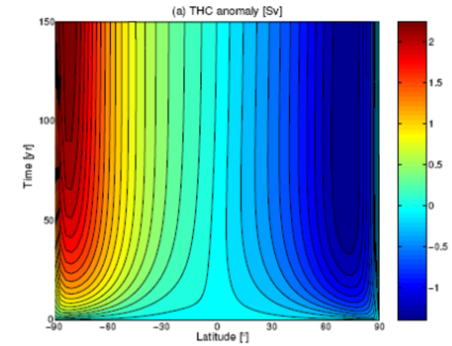
(See also: Stommel box model: Lohmann & Schneider ’99)

Amplification in ~40yr is x20 for salinity & x600 for temperature!

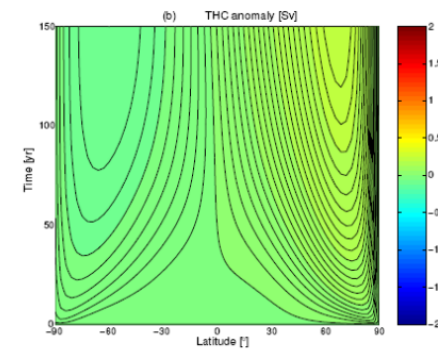
Intermediate model results: Mechanism of transient growth of THC



without $\bar{v}\nabla(S', T')$:



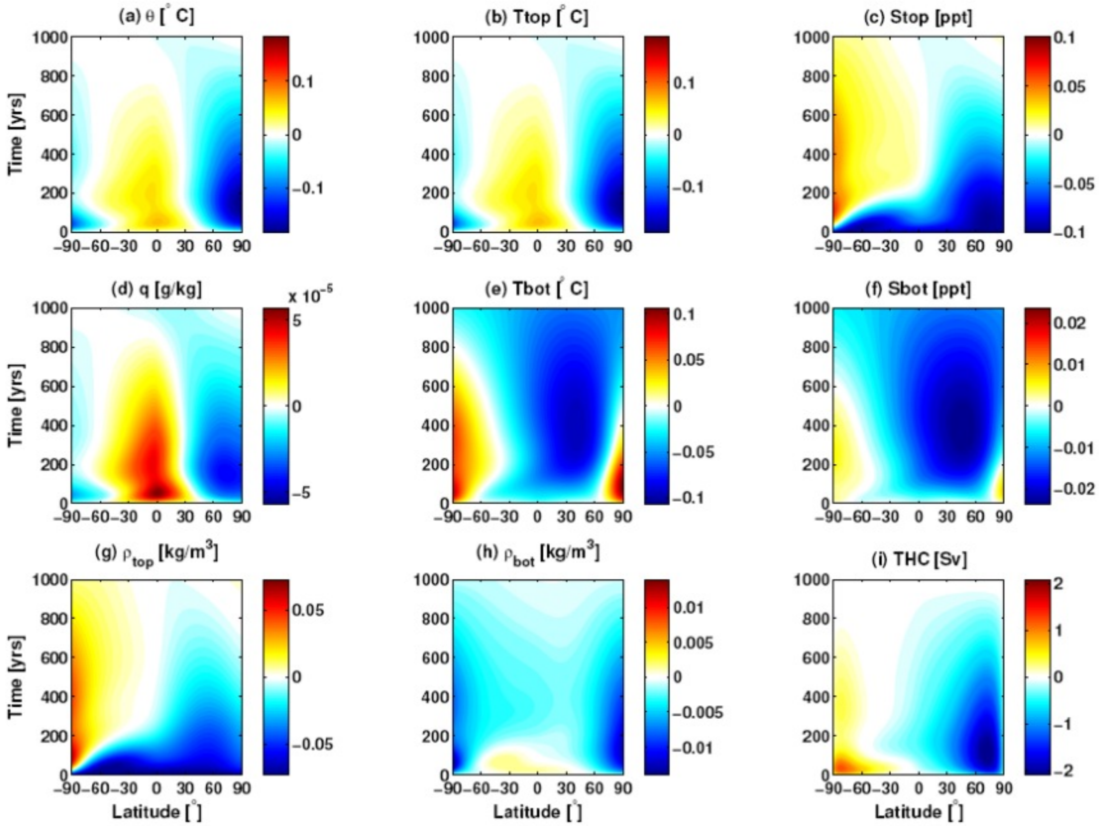
without $v'\nabla(\bar{S}, \bar{T})$:

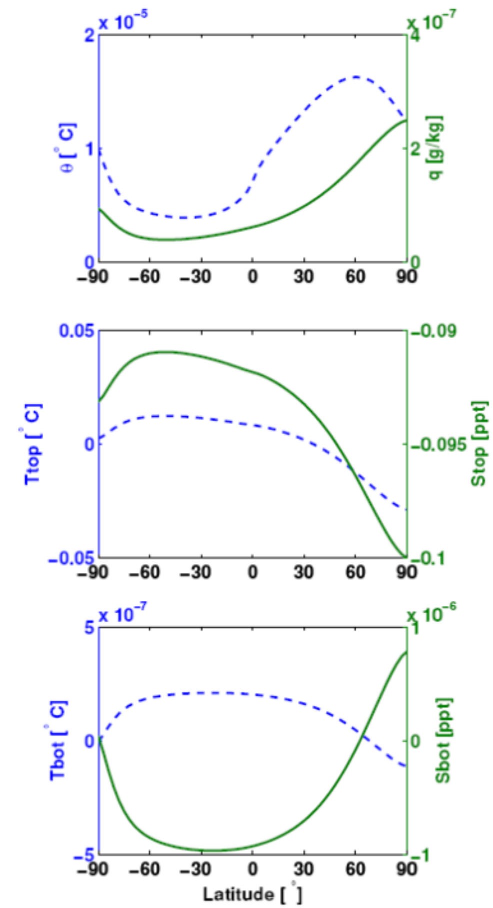


5 large scale damped (23, 25, 87, 281, 784 yrs; some oscillatory) modes interact ➔ transient amplification; Growth due to $v'\nabla(\bar{S}, \bar{T})$ decay due to $\bar{v}\nabla(S', T')$

Unlike THC instability, amplification is ***not*** due to advective instability feedback ($v'\nabla\bar{S}$)

Intermediate model results: optimal ***surface*** excitation of THC anomalies

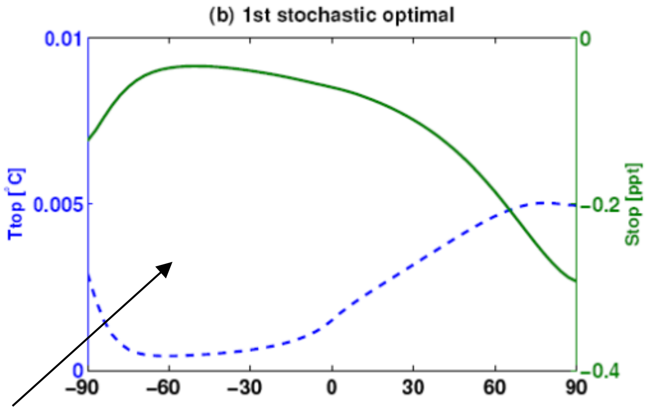


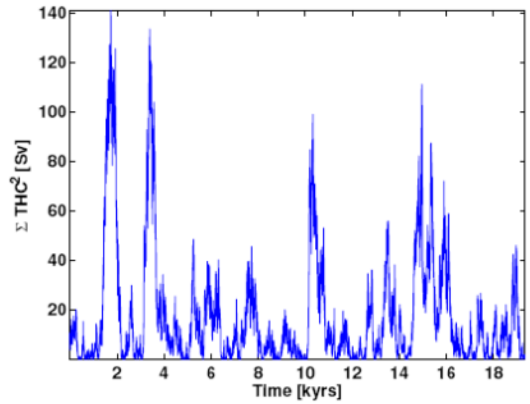


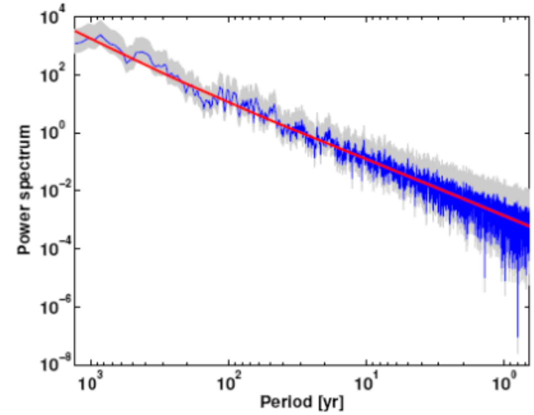
Linearized model evolution from optimal *surface* i.c., resulting in a maximum amplification of the THC after 121 years.

Optimal *surface* i.c. leading to max THC amplification after 121 yrs

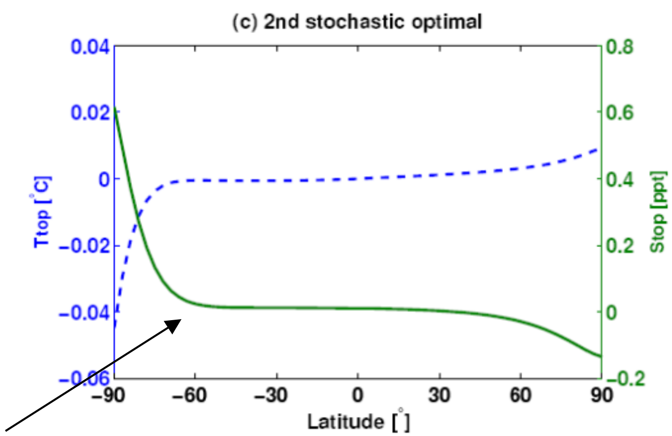
Intermediate model results: optimal surface ***stochastic optimals***

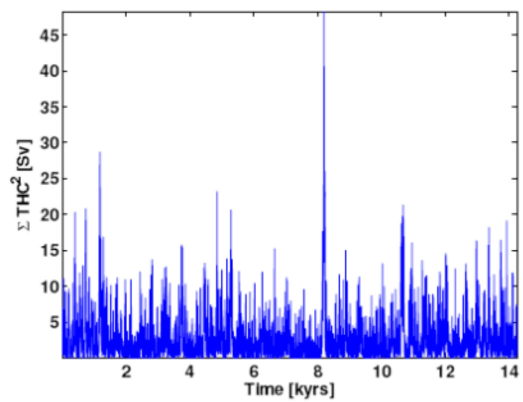


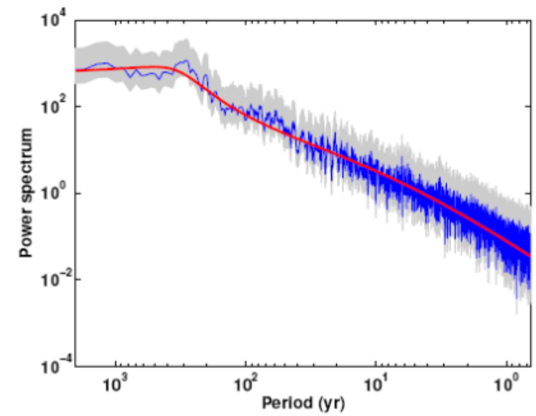




1st stochastic optimal, time series & spectrum of model response





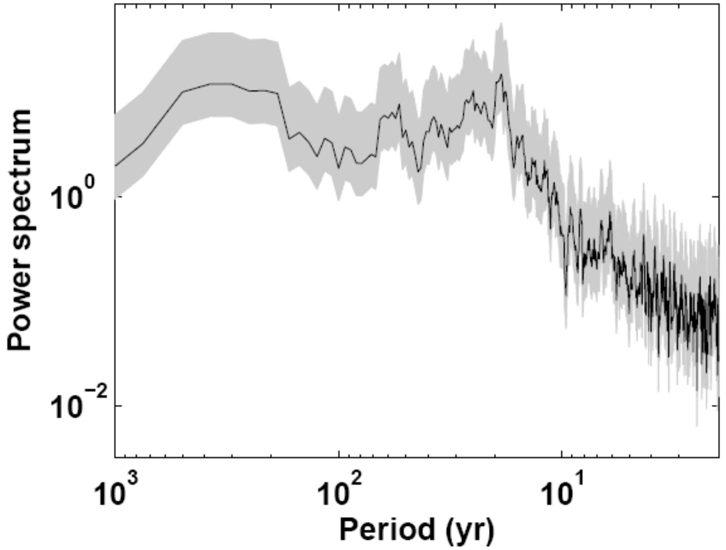


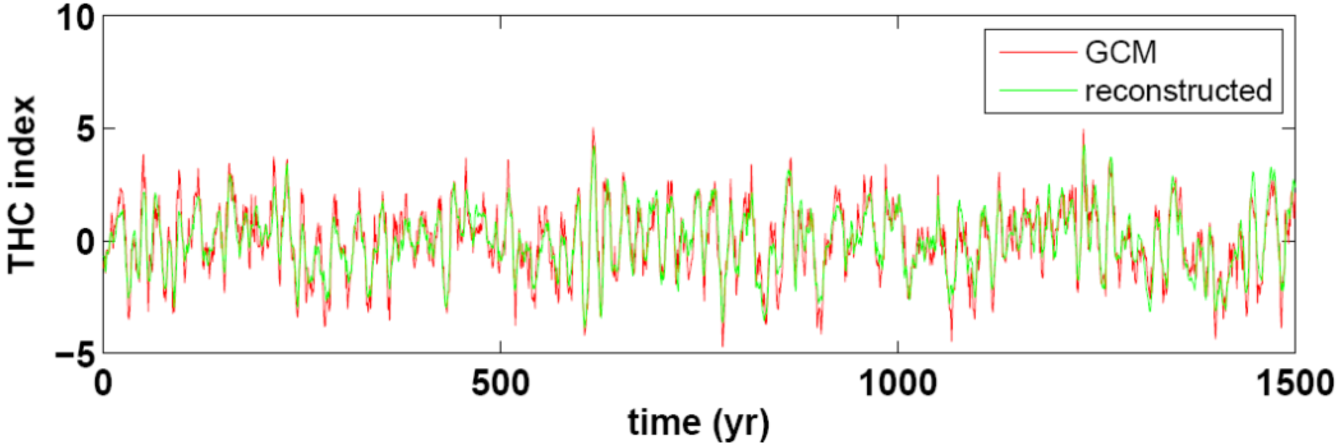
2nd stochastic optimal, time series & spectrum of model response

[Zanna & Tziperman, 2007, Submitted, JPO]

Non normal THC dynamics in GFDL's coupled GCM

- CM2.1 (Delworth 2006; Delworth et al. 2006; Gnanadesikan et al. 2006; Griffies and Coauthors 2005; Stouffer et al. 2006; Wittenberg et al. 2006)
- Displays THC variability with two time scales: ~20 & ~300 years
- The challenge: we don't have the linear equations available to us!

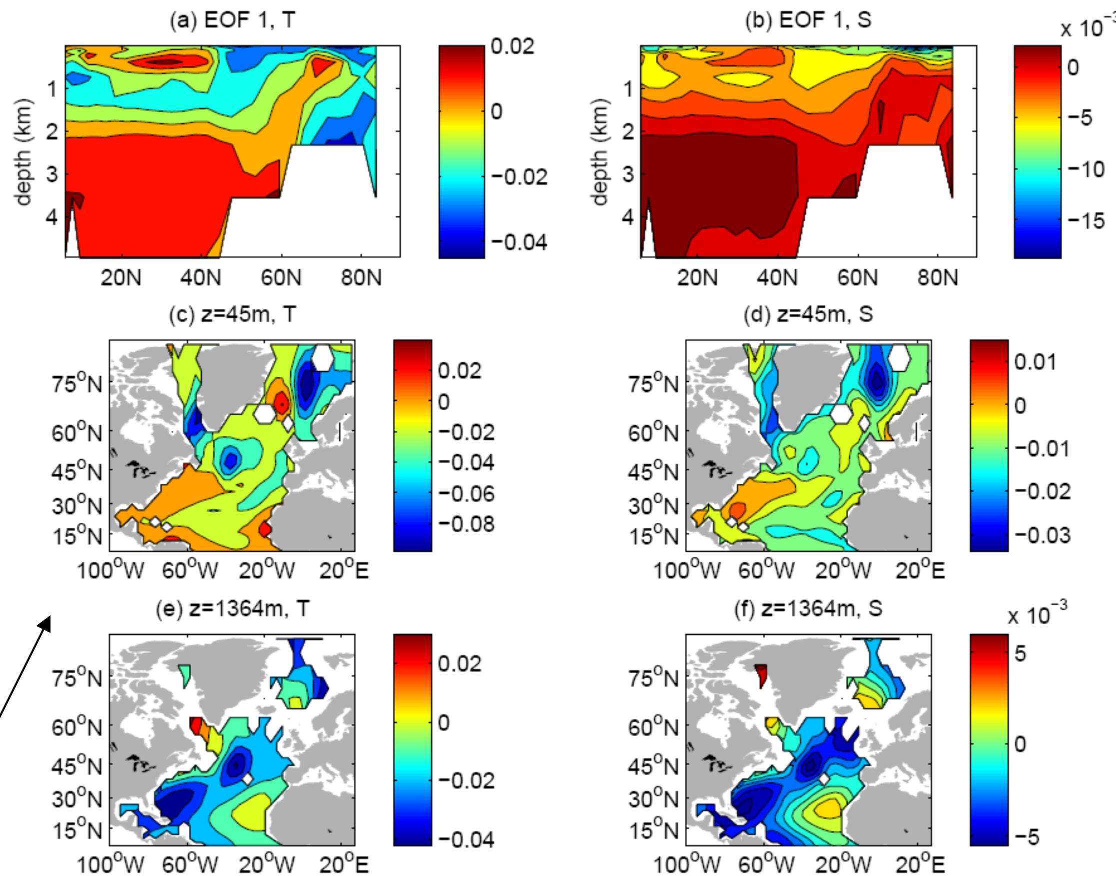




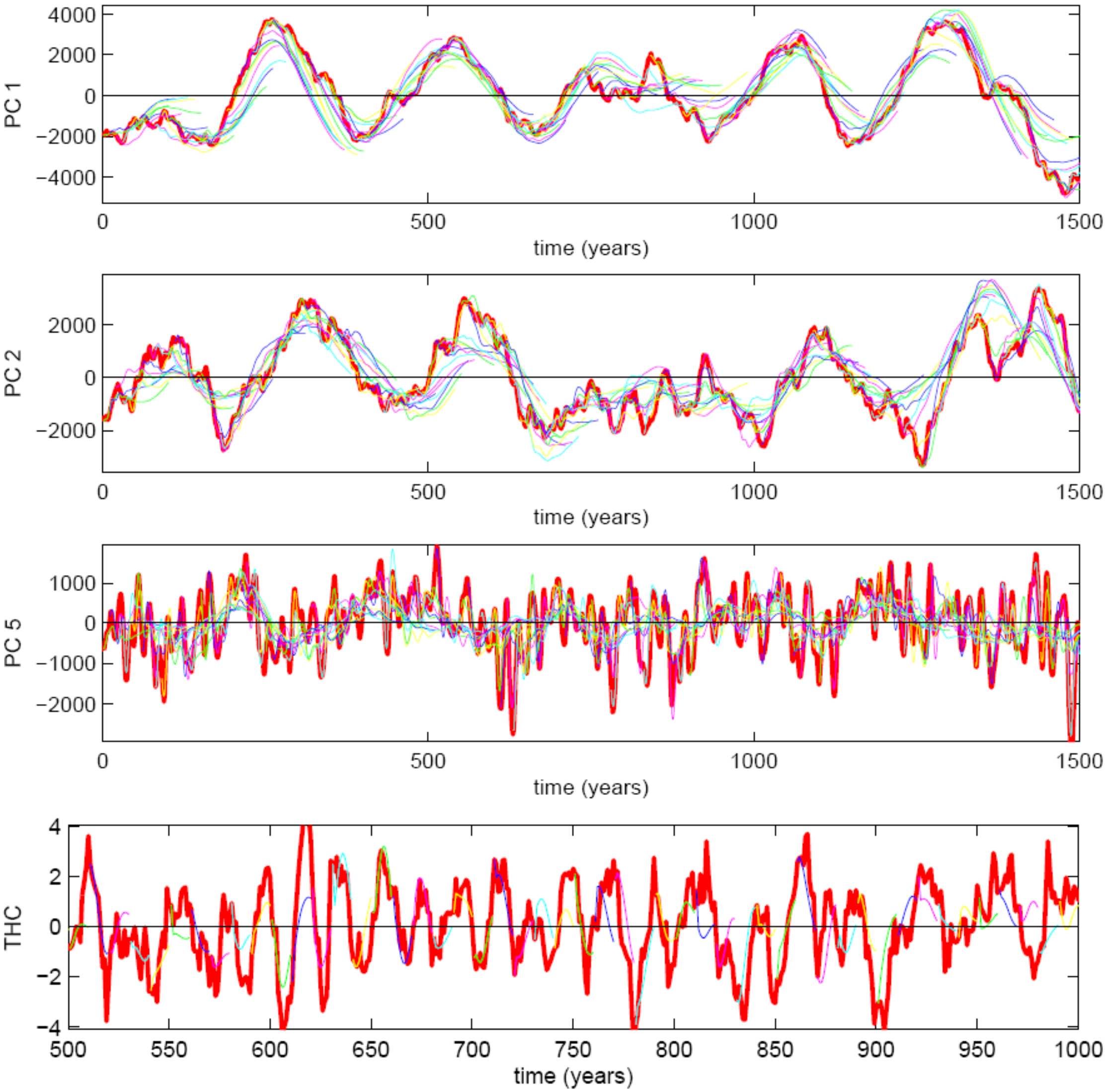
Transient THC amplification in GFDL's coupled GCM: approach

- Analyze 1500 years of control run from GFDL CM2.1
- Calculate 20-30 3D EOFs; let $\mathbf{P}_n(t)$=vector w/ PC amplitudes
- Fit a linear propagator **B** to the PC time series: $\mathbf{P}_{n+\tau} = \mathrm{B}(\tau)\mathbf{P}_n$
- Analyze non normal T,S dynamics of $\mathbf{B}(\tau)$ (a 25x25 matrix),
- PCs and THC are correlated ➔ study THC dynamics as well
- Find optimal initial conditions, plot them in PC space and in T,S space.

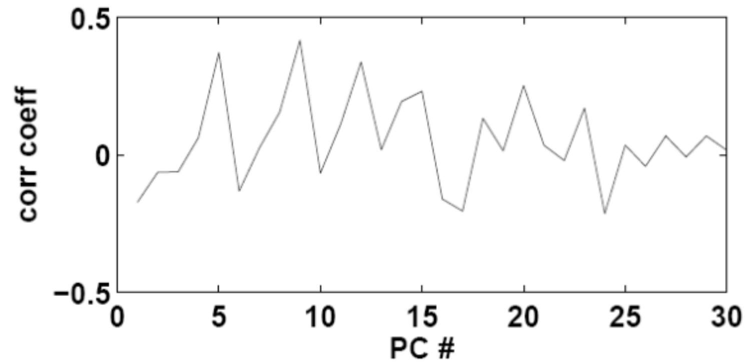
First EOF:



Transient THC amplification: GFDL coupled GCM
Quality of fit of linear dynamics

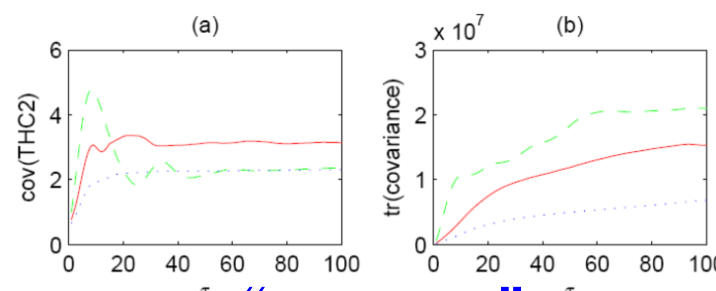


$$\mathbf{P}_{n+\tau} = \mathrm{B}(\tau)\mathbf{P}_n$$



correlation of PCs & THC

$$\rightarrow \quad THC(t) \equiv \mathbf{R}_{THC}^{T}\mathbf{PC}(t)$$

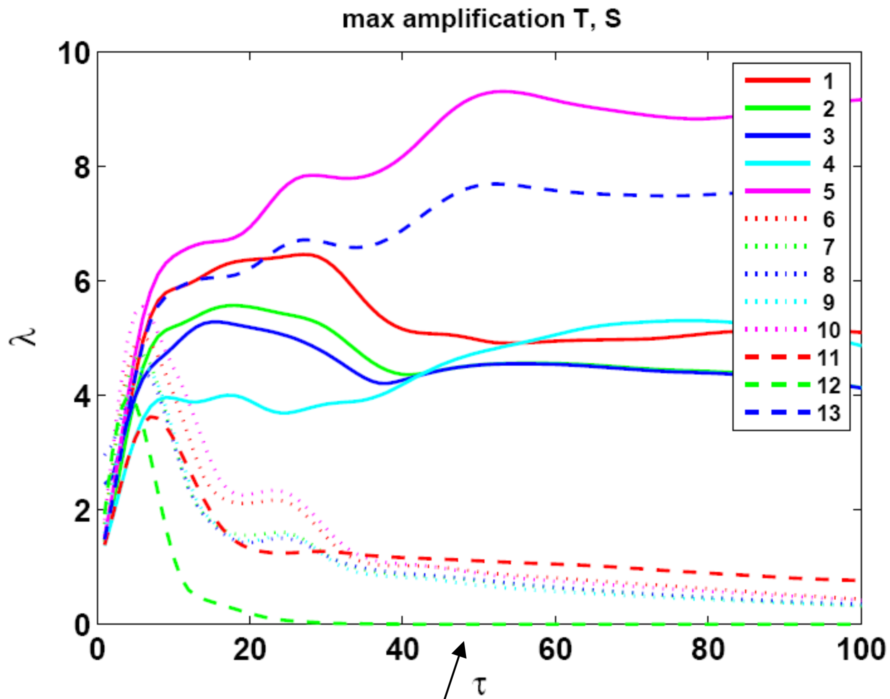


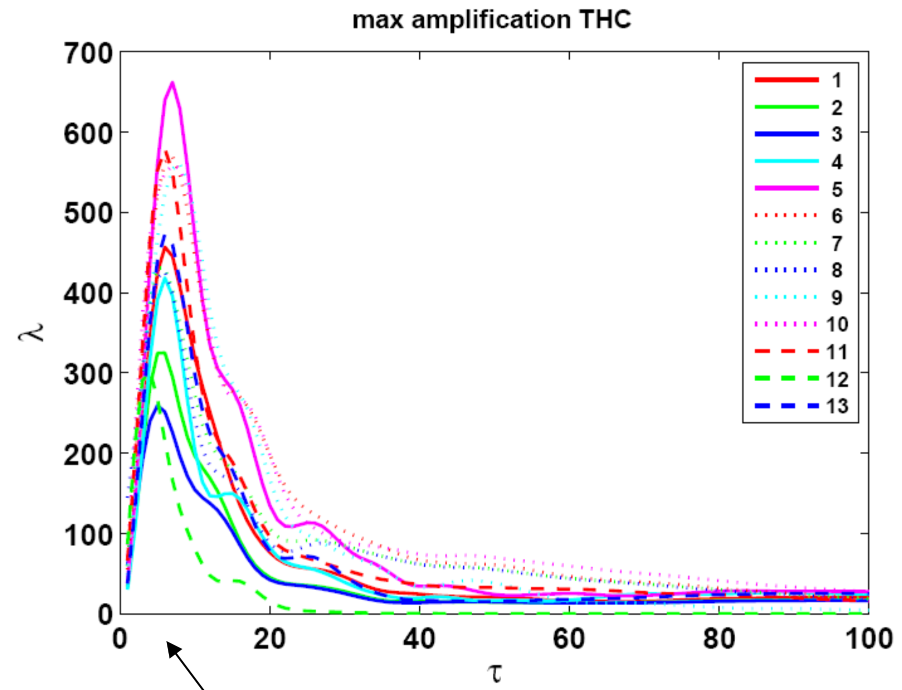
“Tau test”

Predicting PC1,2,5 & THC using B

→B is good approx to GCM

Transient THC amplification: GFDL coupled GCM results: **Amplification of T,S & THC**





$\lambda = J(\tau)/J(0)$

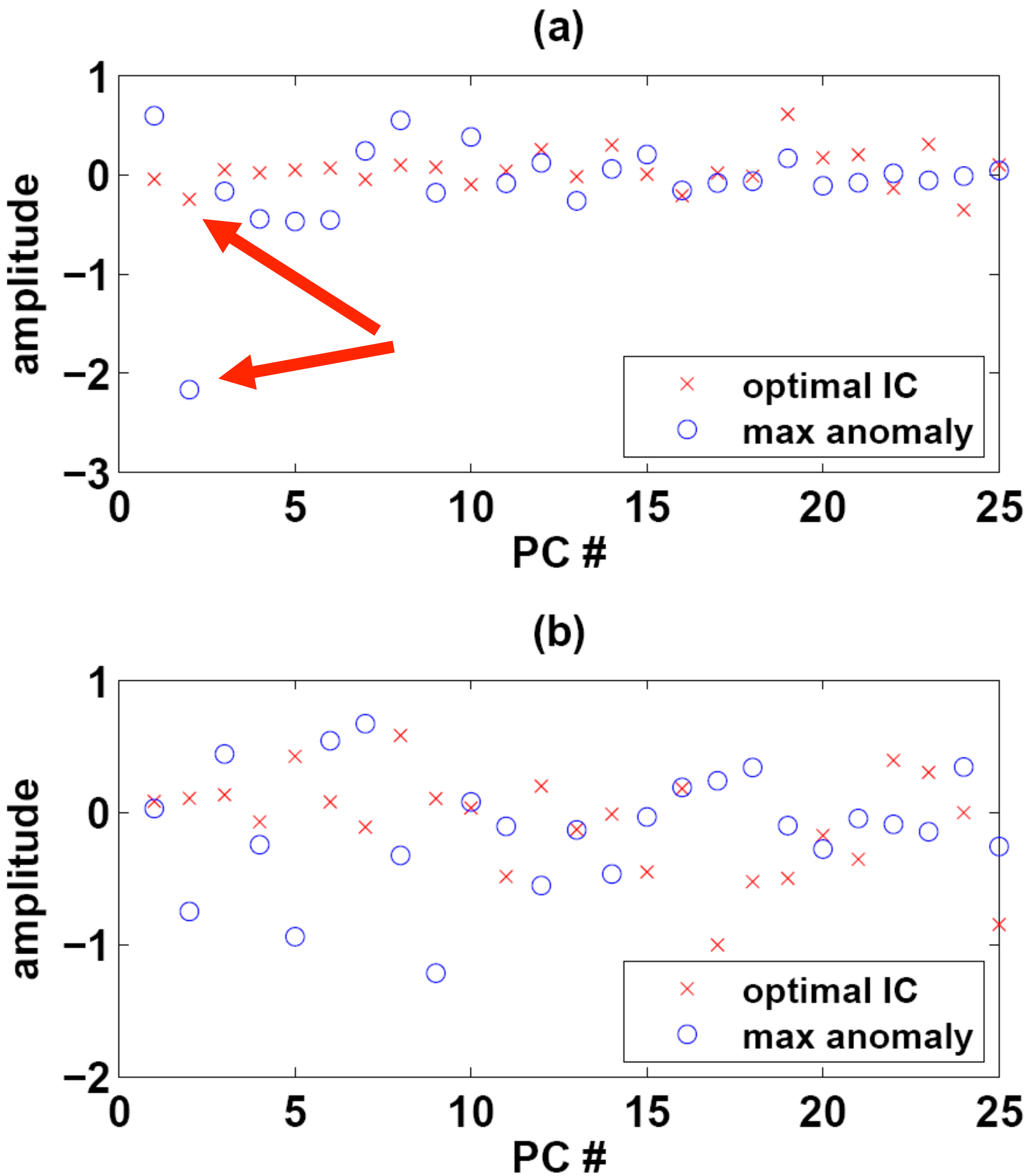
where $J(t) = \Sigma T^2 + S^2$

& T,S are non dimensionalized by e.g. $W_i = \sigma_i / (\Delta V_i / V_0)$

λ= THC(t)/THC(0)

➔Significant non normal transient amplification of T,S & THC. **Rapid growth during first 8 years, when most loss of predictability occurs.**

Transient THC amplification: GFDL coupled GCM results: Principle Components (PC) space

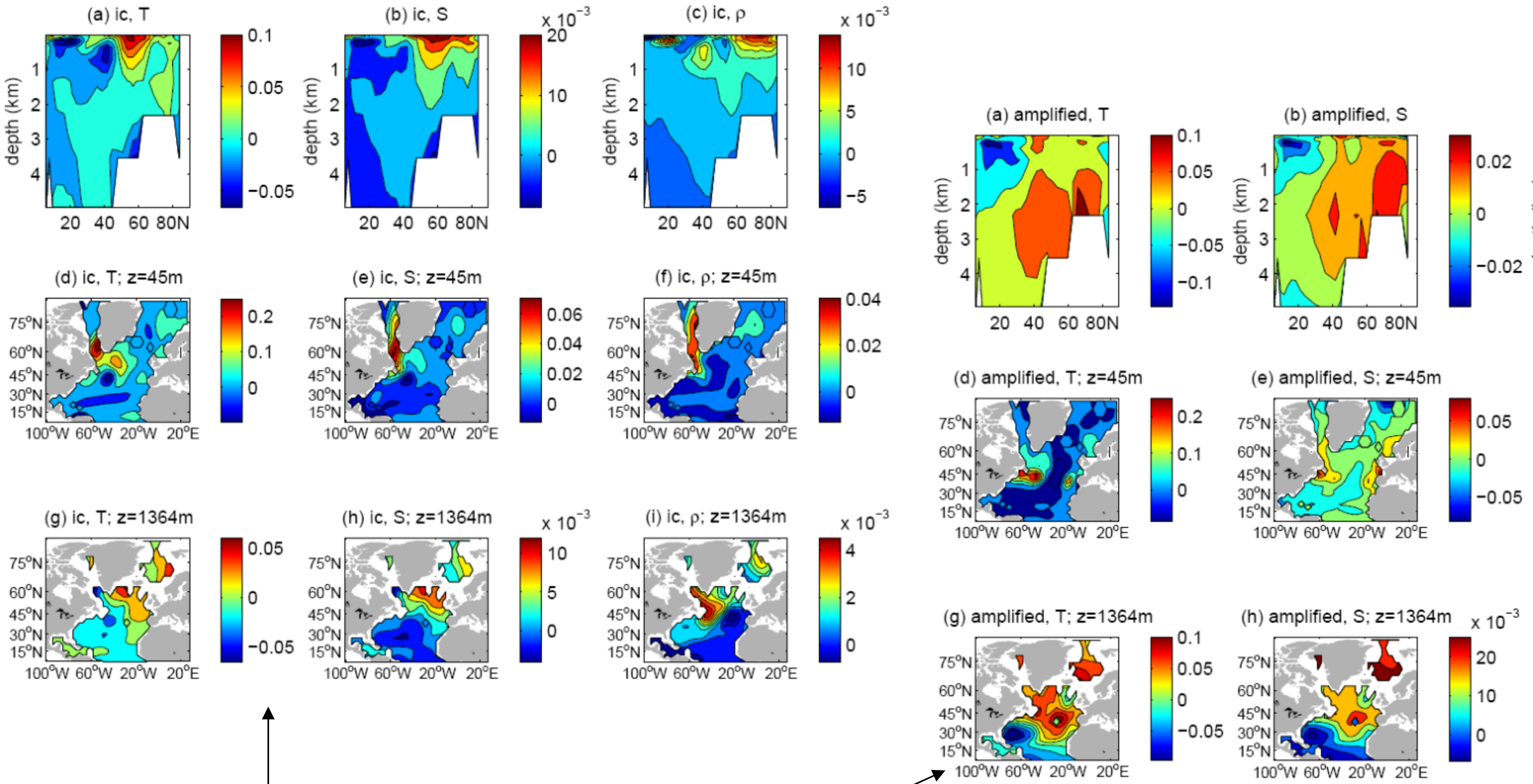


Initial conditions and maximum amplified state in PC space for:

(a) Amplification of sum of squares of T,S

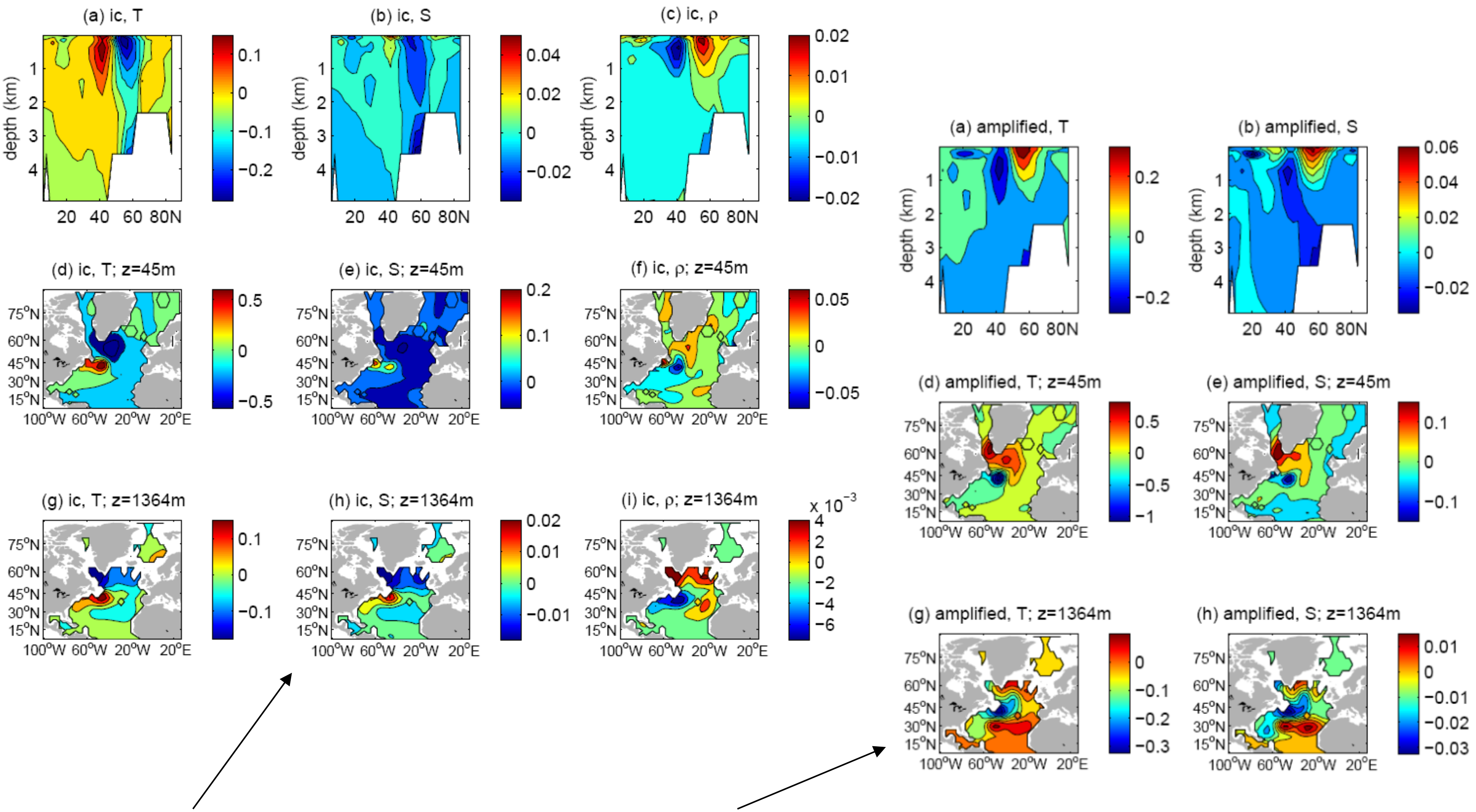
(b) Amplification of THC

Transient THC amplification: GFDL coupled GCM results: physical space



Initial conditions and maximum amplified state in physical space for: Amplification of sum of squares of T,S

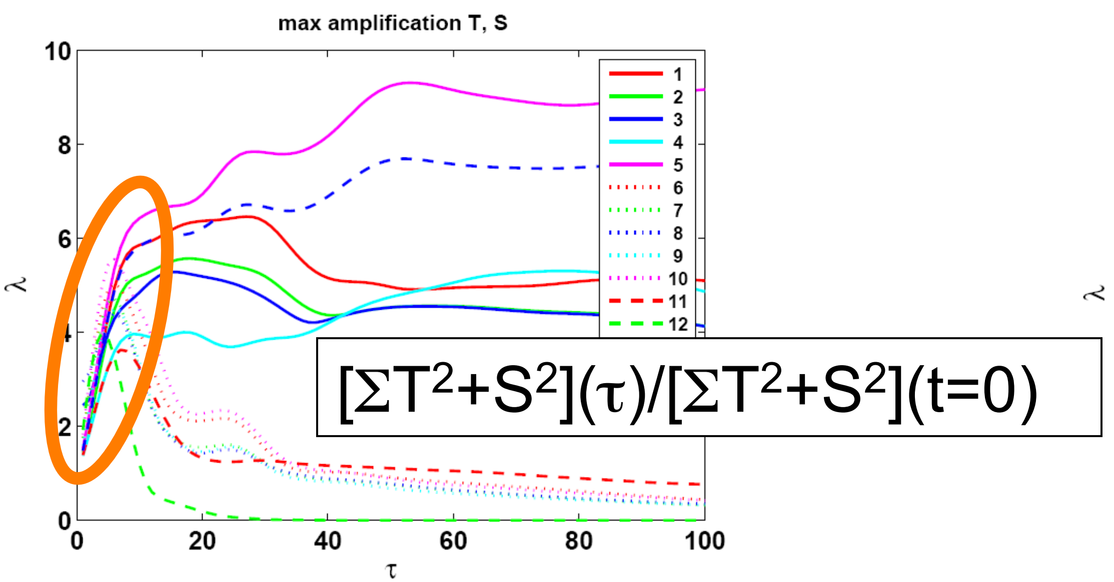
Transient THC amplification: GFDL coupled GCM results: physical space

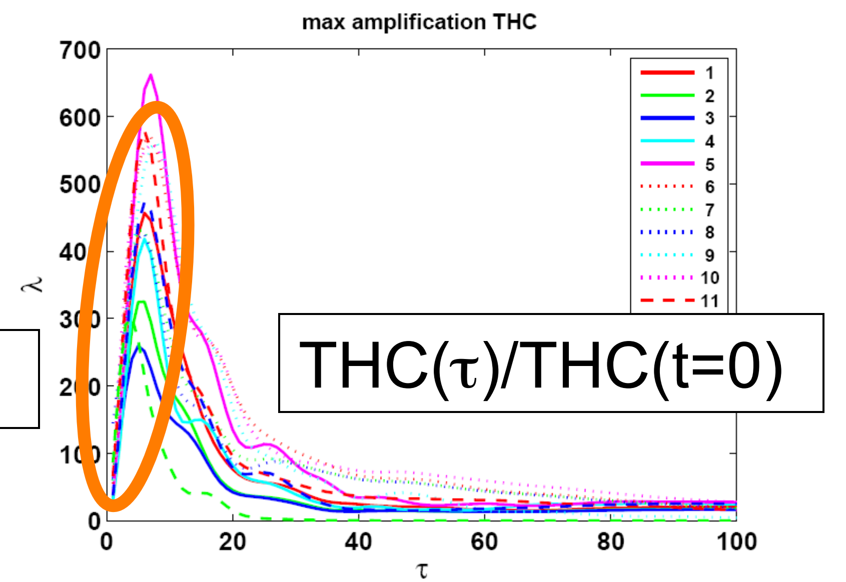


Initial conditions and maximum amplified state in physical space for: Amplification of THC index

Transient amplification in GFDL model: **implications to THC predictability**

- The rapid growth of T,S & THC anomalies during first ~8yrs ➔ small random perturbations to i.c. will rapidly grow & cause loss of predictability skill within about this time.
- Using arbitrary I.C. (e.g. random atmospheric state), not optimal I.C.s, may provide misleading optimistic THC predictability skill.
- The optimal initial conditions can be used in a GCM to evaluate predictability using ensemble runs, as in NWP.





Transient THC amplification: GFDL coupled GCM results: sensitivity to assumptions

- Examine robustness of results to the assumptions used to obtain the linear fit.
- Main sensitivities are to weighting of T,S by $W_i = \sigma_i/(\Delta V_i/V_0)$ and to number of EOFs used.
- More EOFs ➔ more transient amplification.
- But: we are limited by limited length of control run. With 30 EOFs, **B**(τ) is 30x30=900 elements which need to be calculated using 1500 equations (= number of years)

case parameters:	1	2	3	4	5	6	7	8	9	10	11	12	13
num of EOFs N	25	25	25	20	30	25	25	25	20	30	25	25	30
lead time τ	1	2	3	1	1	1	2	3	1	1	1	1	2
weight by *std*	y	y	y	y	y	n	n	n	n	n	y	n	y
weight by *dv*	y	y	y	y	y	y	y	y	y	y	n	n	y

➔Bottom line, robust amplification, significant non normal dynamics in GFDL model.

Conclusions

1. Thermohaline circulation (THC) dynamics was shown to be non normal in a Hierarchy of models, from 3 box models, zonally averaged ones, to GFDL CM2.1.
2. Non normality results in rapid growth of T,S & THC anomalies and loss of THC predictability skill within 8 years in GFDL model.
3. The optimal initial conditions calculated using the approach used here can be used in a GCM to evaluate predictability using ensemble runs, as done routinely & operationally in Numerical Weather Prediction.
4. Other approaches to estimating THC predictability skill may provide misleading seemingly optimistic results.

[Tziperman & Ioannou, *JPO, 2002; Zanna & Tziperman, JPO 2005; submitted 2007a,b]*