0.3 The Stommel model

Our objective is to understand what might happen to the AMOC in a warmer climate, when a freshening of the northern North Atlantic is expected due to ice melting and increased precipitation. In particular, we are interested in the prospect of an abrupt collapse of AMOC as CO_2 gradually increases

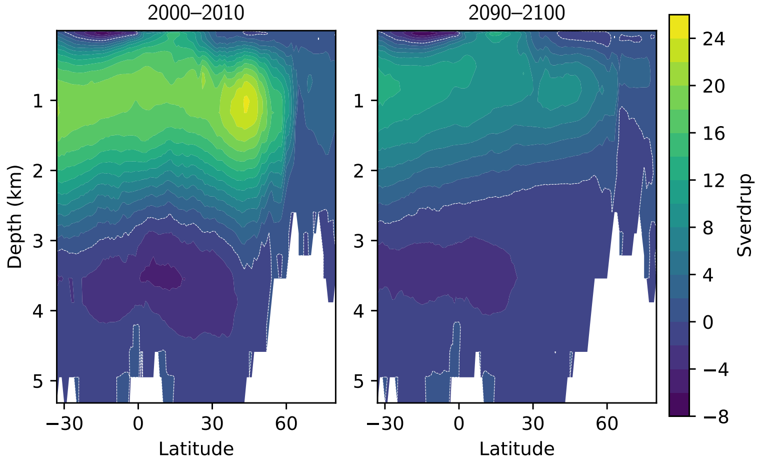

Figure 3: The AMOC streamfunction (Sv) in an RCP8.5 scenario during the first and last decade of the 21st century.

beyond a certain threshold, a scenario often referred to as the crossing of a "tipping point".

Divide the North Atlantic Ocean very crudely into two boxes (Figure 4), one representing the high latitudes (box 1) and the other the sub/tropics (box 2). The boxes are assumed to extend from the surface to the ocean bottom and be well-mixed (clearly a crude approximation, as we know the ocean temperature, salinity and density to vary significantly with depth, box 0.1). The two boxes are connected via a circulation, representing AMOC and denoted by the red and blue arrows in the figure, which transports a volume flux q between the boxes. The AMOC volume transport (m^3/s) is assumed proportional to density difference between the boxes, $q = K(\rho_1 - \rho_2)$, where density is approximated to be a linear function of temperature and salinity, $\rho(T,S) = \rho_0 - \alpha(T - T_0) + \beta(S - S_0)$.

Surface forcing: salinity and evaporation

To see how evaporation and precipitation affect salinity, consider a bucket filled with a volume V of seawater and undergoing evaporation. First, the volume budget of the water in the bucket,

$$\frac{dV}{dt} = -EA,$$

where E is the net evaporation rate per unit area and A the surface area of bucket. Letting the mass of salt per unit mass of seawater be denoted by the salinity S, the total salt in the bucket, not affected by evaporation, is

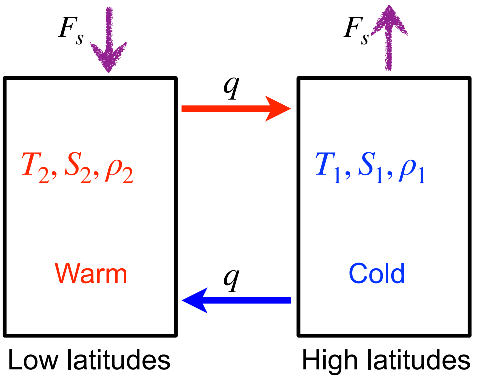

Figure 4: Stommel box model schematic.

equal to ρSV where ρ is the water density. Assuming the density is nearly constant, the salt conservation statement is, therefore,

$$\frac{d}{dt}(SV) = 0,$$

which we can expand, using the mass conservation, into

$$V\frac{dS}{dt} = -S\frac{dV}{dt} = SEA \approx S_0EA.$$

Salinity in the ocean varies mostly between 34 and 36 parts per thousand (1 ppt roughly representing kg salt per meter cube of seawater), and $S_0 = 35$ ppt denotes an averaged reference salinity. We term S_0E the “virtual salt flux” per unit area. If evaporation is small (a typical order of magnitude in the ocean is 1 m/yr, compare with an average ocean depth of 4000 m), the changes to the salinity will be small and the above approximation of using S_0 instead of S is a very good one, and accurately describes salinity changes due to evaporation and precipitation. This also means that we do not need to be concerned with actual water volume exchanges involved in the evaporation-precipitation process, and can replace them by the virtual salt flux which is denoted F_s in Figure 4 and in the derivation below, also referred to as the “fresh water forcing”.

Salt budget equations for the Stommel model

We assume that ocean temperature is set by interaction with the atmosphere, which is cold in polar areas and warmer in subtropical areas. We therefore let the ocean temperatures T_1 and T_2 be fixed to a first approximation, so that circulation changes occur only due to ocean salinity changes. The only unknowns, therefore, are the salinities of the two boxes and the circulation q. Write the salt budget equation for box 1, first assuming that $q > 0$, representing a salt transport of qS_2 into box 1 and a transport qS_1 out of box 1 to box 2, as well as a freshening effect induced by excess precipitation in the high latitude box 1, represented by a $-F_s$ contribution to the budget,

$$V_1 \frac{dS_1}{dt} = qS_2 - qS_1 - F_s.$$

If $q < 0$, we have salt transport of $(-q)S_2$ into box 1 and a transport $(-q)S_1$ out of box 1 to box 2,

$$V_1 \frac{dS_1}{dt} = (-q)S_2 - (-q)S_1 - F_s.$$

Together, this may be combined into a single form, which we write for both boxes as,

$$\begin{aligned} V_1 \frac{dS_1}{dt} &= |q|(S_2 - S_1) - F_s \\ V_2 \frac{dS_2}{dt} &= |q|(S_1 - S_2) + F_s. \end{aligned} \tag{1}$$

Note the absolute value of the transport appearing here, combining the above two cases of positive and negative AMOC volume transport q. The fresh water forcing is assumed positive, $F_s > 0$, corresponding to a freshening of the polar box and to making the tropical box saltier.

Solution

Take the difference of the two salt budget equations (1) and define $\Delta T = T_1 - T_2 < 0$ and $\Delta S = S_1 - S_2$, and let the box volumes be the same, $V_1 = V_2 \equiv V$, for simplicity. The transport may then be written as,

$$q = K(\rho_1 - \rho_2) = K(-\alpha\Delta T + \beta\Delta S),$$

and the equation for the salinity difference becomes,

$$-V \frac{d\Delta S}{dt} = 2|q|\Delta S + 2F_s$$

or,

$$-V \frac{d\Delta S}{dt} = 2K|(-\alpha\Delta T + \beta\Delta S)|\Delta S + 2F_s. \tag{2}$$

Define a rescaled known temperature difference variable, $X = \alpha\Delta T < 0$, and an unknown rescaled salinity difference, $Y = \beta\Delta S$. In a steady state, $d\Delta S/dt = 0$, so that (2) leads to a simple quadratic equation for $Y = \beta\Delta S$,

$$|Y - X|Y = -\frac{\beta F_s}{K}.$$

Because of the absolute value, there are two cases to consider, $q > 0$ and $q < 0$, equivalent to $X > Y$ and $X < Y$. Noting again that $X < 0$, we have in the first case, $Y < X$,

$$\begin{aligned} Y^2 - XY - \frac{\beta F_s}{K} &= 0 \\ Y = \frac{X}{2} - \frac{1}{2}\left(X^2 + 4\frac{\beta F_s}{K}\right)^{1/2}&. \end{aligned} \tag{3}$$

While in the second, $Y > X$,

$$\begin{aligned} Y^2 - XY + \frac{\beta F_s}{K} &= 0 \\ Y = \frac{X}{2} \pm \frac{1}{2}\left(X^2 - 4\frac{\beta F_s}{K}\right)^{1/2}&. \end{aligned} \tag{4}$$

Note that the plus solution in the first case is negative and is therefore not consistent with the assumption $Y < X$ used to obtain that solution, so there are no more than three solutions for a given value of the fresh water forcing F_s.

Analysis: multiple equilibria, tipping point, hysteresis

Multiple equilibria. Given the solution for $Y = \beta\Delta S$ as function of the fresh water forcing F_s (eqns 3 and 4, plotted in Figure 5a), we can calculate the salinity difference ΔS and the circulation q as function of the fresh water forcing F_s, as shown in Figure 5b. The number of solutions varies from 1 to 3 depending on the value of the fresh water forcing. Note that the solution shown by the green line in Figure 5b represents a northward flow, while that shown in red is a weak, reversed flow that is the only possible solution for large fresh water forcing. The results are already remarkable at this point: a given fresh water forcing can lead to three different solutions for the overturning circulation. As we will see shortly, the existence of such "multiple equilibria" is the first ingredient needed for tipping points to occur.

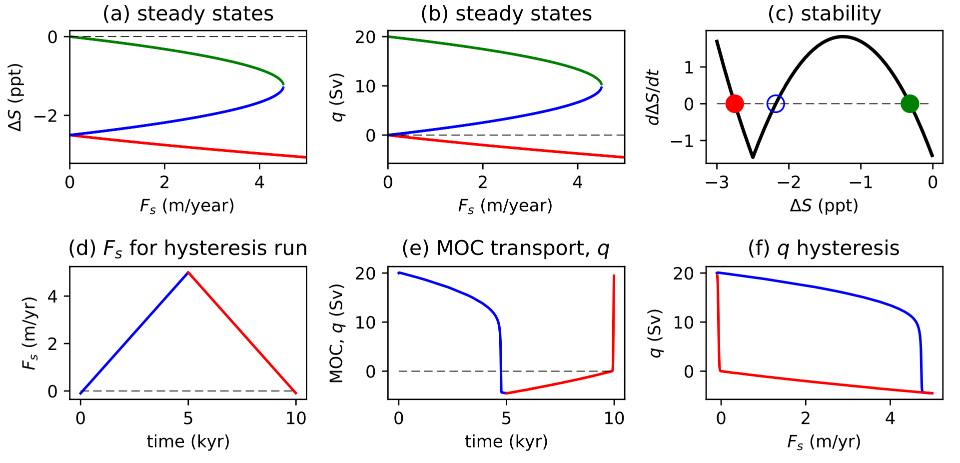

Figure 5: Solution of the 2-box model: (a, b) Steady states of salinity difference and MOC as function of fresh water forcing F_s. (c) Stability analysis: $d\Delta S/dt$ as function of ΔS for $F_s = 2$ m/s, from eqn (2). (d) Time-dependent fresh water forcing used for the hysteresis run. (e, f) Results of the hysteresis run.

Stability. It can also be seen that the intermediate solution (blue in Figure 5a,b,c) is unstable: a small perturbation from that steady state would result in the circulation transitioning to one of the other two solutions that exist for the same fresh water forcing. To see this, consider $d\Delta S/dt$ plotted as function of ΔS in Figure 5c using equation (2). If the solution is exactly at the steady state denoted by the empty blue circle, $d\Delta S/dt = 0$ and the steady state is maintained. However, suppose that the solution deviates a bit to the right (salinity difference between the two boxes increases due to some random weather event affecting precipitation and evaporation). At that point $d\Delta S/dt > 0$ and the salinity difference keeps increasing and getting away from this state. A similar growing deviation occurs if the solution deviates to the left of the steady state marked by the empty blue circle in panel c or the blue line in panels a,b, and this solution is therefore referred to as being *unstable*. If we start near one of the stable solutions (filled red and green circles in Figure 5c), a small increase in ΔS leads to $d\Delta S/dt < 0$, while a small decrease in ΔS leads to $d\Delta S/dt > 0$. Thus, in both cases the deviation decreases back toward these two steady states, and they are therefore referred to as *stable* steady states.

Tipping points and hysteresis. The existence of multiple equilibria for a given fresh water forcing value F_s leads to both the possibility of abrupt changes as the forcing F_s changes gradually, as well as to possibly irreversible changes to the circulation as CO_2 increases. To see this, suppose the circulation is at a steady state corresponding to a point on the upper (green) solution for q in Figure 5b, and that we gradually and very slowly increase the fresh water forcing such that the solution for the overturning circulation is always at equilibrium with the fresh water forcing. The circulation solution then moves to the right along the green line and thus weakens, until the fresh water forcing amplitude is at the critical value of $F_c \approx 4.2$ m/s at which the system switches from three solutions to only one. That one remaining solution is the very weak (reversed) circulation on the red curve in Figure 5b, so the circulation must abruptly switch to that solution, i.e., collapse!

The lower three panels of Figure 5 demonstrate this abrupt collapse scenario. Suppose that in a global warming scenario, a gradual increase of CO_2, leads to a gradual increase in precipitation/melting and thus in the fresh water forcing F_s as function of time, as shown by the blue curve in Figure 5d. Figure 5e shows the transport as function of time for this forcing, showing with a blue line a gradual decrease and then a collapse just before year 5000. This abrupt collapse, corresponding to the switch between the two equilibria solutions, denotes the occurrence of a tipping point. If the fresh water forcing is now made gradually weaker in time (red curve in Figure 5d, say because CO_2 values are finally gradually decreasing...), the circulation strengthens again (red curve in Figure 5e) gradually at first. Even as the fresh water forcing is reduced below the critical value F_c, the circulation does not recover (does not jump to the green line) if we decrease F_s by a small amount. Recovery happens only when $F_s = 0$ (red curve in Figure 5f). The different evolution of the solution for increasing and decreasing forcing, expressed as the loop showing the transport as function of the fresh water forcing in Figure 5f is termed "hysteresis". The existence of multiple stable and unstable solutions and the resulting hysteresis are all a result of the nonlinear nature of equation (2) for the salinity difference between the two boxes.

Remarkably, full-complexity ocean climate models show the same abrupt changes to AMOC and a corresponding hysteresis loop when an appropriate scenario of increasing and then decreasing fresh water forcing is applied. The only difference is that AMOC seems to weaken and possibly vanishes at high fresh water forcing values in more realistic climate models, rather than reverse as in the box model. Furthermore, it was suggested that the present-day circulation may be close to the threshold that leads to such an abrupt irreversible collapse. On the other hand, note that the collapse of AMOC in a full complexity coupled ocean-atmosphere climate model seen in Figure 2 shows a gradual decline rather than an abrupt transition at some point. It is possible that the CO_2 change in this scenario was too fast to allow the tipping point to be clearly expressed, or that this model does not show a tipping point for AMOC, demonstrating the uncertainty in this prediction.

The existence of tipping points has been proposed in other climate components, from clouds to ice sheets, underling the possibility that a gradual CO_2 change may lead to abrupt changes and thus surprises whose precise timing is difficult to predict.

References:

The box model used here is that of:
* Marotzke, J. (1990). Instabilities and multiple equilibria of the thermoha- line circulation. PhD thesis, Berlin Instit Meereskunde, Kiel.
* Marotzke, J. (2000). Abrupt climate change and thermohaline circulation: Mechanisms and predictability. Proc. Natl. Acad. Sci. U.S.A., 97:1347– 1350.

The original Stommel reference is
* Stommel, H. (1961). Thermohaline convection with two stable regimes of flow. Tellus, 13:224–230.