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Ocean Circulation Theory

With 167 Figures

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a geostrophic zonal flow to match the source input. Since π is independent of θ, it follows that when $S = 0$ the zonal velocity independent of θ. Fluid can proceed across the sector only as shown in Fig. 7.2.3d. After reaching the western boundary it can travel along the western boundary to the radius of the sink, and retracing its path in azimuth, it leaves the sector by flowing eastward at constant radius to the sink. This is perhaps the most bizarre of all the cases. The constraints placed on the motion by the potential vorticity dynamics in the interior and the absence of an eastern boundary layer forces the flow into an immensely indirect and intricate path to flow from source to sink.

It is even now astonishing to realize that these dynamical ideas achieved full confirmation in the experiments of Stommel, Arons, and Faller (1958), and the reader is referred to their original paper both for a complete discussion of the experimental evidence and for the original sense of wonder at the power of the constraint imposed by the ambient potential vorticity gradient which yields such remarkable flow patterns.

With this experimental confirmation in hand Stommel and Arons were emboldened to apply their ideas to planetary motions in the ocean's abyss.

7.3 Stommel-Arons Theory: Abyssal Flow on the Sphere

Stommel and Arons (1960 a,b) extended the dynamical ideas of the previous section to describe a theory for the abyssal motion on a spherical earth. They envisioned the entire ocean as a two-layer model with localized sinking at both poles from the upper to lower layer and a widespread return flow from the lower layer, which represents the abyss, into the upper layer, which is meant to represent the thermocline driven by the wind. The return flow has a vertical velocity, w_0, specified over the area of the ocean basin at the interface between the two layers, i.e., at the top of the abyssal layer. This distributed sink acting on the abyss provides the driving mechanism for the interior flow. As in the analysis of the Stommel, Arons, and Faller experiment, western boundary currents are added as needed to satisfy mass balance considerations.

In this form the abyss is represented as a single layer, and therefore at each geographical location the abyss moves in a single direction. As we have noted above, the evidence from tracer and hydrographic data implies that motion in the abyss is strongly baroclinic, with interleaving of water masses moving in quite different directions. The Stommel-Arons model is clearly unable to deal with this vertical structure and should be considered as a model only for the vertical average of the abyssal flow. The authors themselves were concerned about this restriction. (I am indebted to Arnold Arons for a fascinating and enlightening conversation about the historical development of the Stommel-Arons theory.) However, they considered that the model presented an important first step to which refinements could be added later. Similarly, the model treats the abyss as a layer of fluid with a flat bottom. Topography is ignored, and, again, Stommel and Arons were clear that they expected the pronounced bottom topography of the abyssal ocean basins to cause "deviations" from the simple theory which they presented. In spite of these reservations the theory still exists today as the fundamental building block of our understanding of the motion of the deep water.

For the layer representing the abyss, the motion is geostrophic. Thus in spherical coordinates:

$$fu = -\frac{1}{R}\frac{\partial P}{\partial \theta}, \qquad fv = \frac{1}{R\cos\theta}\frac{\partial P}{\partial \phi}. \tag{7.3.1 a,b}$$

where the variables have their usual meanings, except that we introduce $P = p/\rho$ where ρ is the constant density of the abyssal layer in the model. R is the earth's radius and ϕ and θ are longitude and latitude, respectively, and u and v are the eastward and northward velocities. The continuity equation for the incompressible fluid of the model abyss is:

$$\frac{1}{R\cos\theta}\frac{\partial(v\cos\theta)}{\partial\theta} + \frac{1}{R\cos\theta}\frac{\partial u}{\partial\phi} + \frac{\partial w}{\partial z} = 0. \tag{7.3.2}$$

Eliminating the pressure from (7.3.1a,b) and using (7.3.2) yields the Sverdrup vorticity equation:

$$\beta v = f\frac{\partial w}{\partial z}. \tag{7.3.3}$$

Note that since the scales are planetary in extent, f and β are variable, i.e.:

$$f = 2\Omega\sin\theta, \quad \beta = \frac{2\Omega\cos\theta}{R}. \tag{7.3.4 a,b}$$

The abyss is a single layer of fluid which, in the absence of motion is idealized as having a uniform thickness H. At the top of this layer a vertical velocity, w_0, is specified which drives the interior flow. As discussed in Section 7.1, this velocity is identified with the flow entrained into the base of the thermocline, w_∞, owing to the balance achieved there between diffusion and upward advection of temperature. It is important to keep in mind that in this theory this vertical velocity is not due to thermal anomalies induced only by the motion in the abyss itself but rather is due primarily to dynamics in the adjacent thermocline.

Since the density in the layer is constant, the horizontal geostrophic motion is depth independent and therefore the vertical integral of (7.3.3) over the depth H yields:

$$\beta v = \frac{f w_0}{H} \tag{7.3.5}$$

or:

$$v = \tan\theta \frac{w_0 R}{H}. \tag{7.3.6}$$

The meridional velocity is always *poleward* as long as w_0 is positive. Thus in the southern hemisphere the flow is southward while it must be northward in the northern hemisphere. As long as w_0 is finite at the equator, it follows that v *must vanish on the equator*. There can therefore be no interior flow across the equator. Any flow crossing the equator must occur in regions of different dynamics where the geostrophic Sverdrup balance is upset, i.e., in the western boundary current.

From (7.3.1b) and the condition that P be constant on the eastern boundary of the ocean basin at $\phi = \phi_e$:

$$P = -\frac{2\Omega R^2}{H} \int_{\phi}^{\phi_e} w_0(\phi, \theta) \sin^2\theta \, d\phi'. \tag{7.3.7}$$

The zonal velocity follows from (7.3.1a):

$$u = \frac{1}{\sin\theta} \frac{R}{H} \frac{\partial}{\partial\theta} \left[\int_{\phi}^{\phi_e} w_0(\phi, \theta) \sin^2\theta \, d\phi' \right] \tag{7.3.8}$$

in which form it is valid even for eastern boundaries which are not meridians, i.e., for which $\phi_e = \phi_e(\theta)$.

If, for simplicity, the velocity w_0 is taken to be independent of horizontal position, the algebra is considerably simplified. For a barotropic model such as this there is no substantial qualitative effect in considering w_0 constant, although when baroclinic effects are considered, as in Section 7.7, the geographical distribution of w_0 becomes important. For now, however, we take w_0 as a constant. Thus, for example:

$$u = 2w_0 \cos\theta \frac{R}{H} (\phi_e - \phi) \tag{7.3.9}$$

so that the zonal flow is always eastward.

Consider as a first example, the ocean basin composed of the spherical sector shown in Fig. 7.3.1. It stretches from the equator to the pole, and its meridional boundaries are at $\phi = \phi_e$ on the eastern and $\phi = \phi_w$ on the western boundary. A source of fluid meant to represent the sinking of cold water at the pole in the North Atlantic is shown at the apex of the sector and has a magnitude S_0. The streamlines of the interior flow are already known since, as in the experiment described in the previous section, the interior flow is completely independent of the position of the sources.

The mass balance for the subsector north of an arbitrary latitude θ, shown by the dotted line in the figure, is found by balancing the sum of the interior mass flux, the western boundary current's transport, and the source's contribution, S_0, all into the sector, against the flux out of the abyss due to

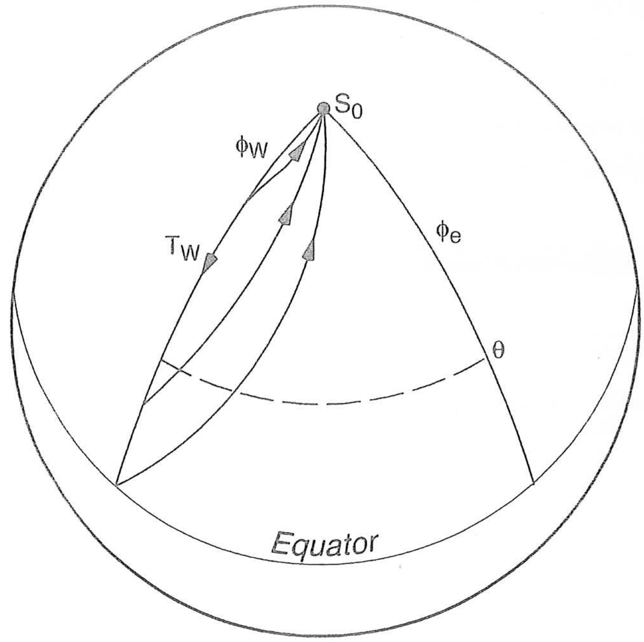

Fig. 7.3.1. Circulation in a spherical sector driven by the source S_0 in the apex of the basin and uniform upwelling at the upper surface of the abyss. To determine the transport in the western boundary current a mass balance is made for the area north of the arbitrarily chosen latitude θ

the vertical motion through the interface into the thermocline. Thus the northward flux of the interior velocity T_I is, using (7.3.6):

$$T_I = \int_{\phi_w}^{\phi_e} vH \cos\theta\, R\, d\phi' = w_0 R^2 \sin\theta(\phi_e - \phi_w) \qquad (7.3.10)$$

while the fluid entering the thermocline and leaving the abyss in that subsector is:

$$T_{\text{out}} = \int_{\theta}^{\pi/2} \int_{\phi_w}^{\phi_e} w_0 R^2 \cos\theta\, d\phi\, d\theta' = w_0 R^2 (\phi_e - \phi_w)[1 - \sin\theta]. \qquad (7.3.11)$$

If T_w is the *northward* transport of the western boundary current, and S_0 is the source strength, the mass balance implies that:

$$T_w + T_I + S_0 = T_{\text{out}} \qquad (7.3.12)$$

or, using (7.3.10) and (7.3.11):

$$T_w = -2w_0 R^2 (\phi_e - \phi_w) \sin\theta + \left[w_0 R^2 (\phi_e - \phi_w) - S_0\right]. \qquad (7.3.13)$$

If the mass is balanced over this northern hemispheric sector:

$$S_0 = \int_0^{\pi/2}\int_{\phi_w}^{\phi_e} w_0 R^2 \cos\theta\, d\phi d\theta = w_0 R^2 (\phi_e - \phi_w). \qquad (7.3.14)$$

In this case, where the sector is isolated from the rest of the ocean, the two terms in the square bracket in (7.3.13) exactly balance and:

$$T_w = -2S_0 \sin\theta. \tag{7.3.15}$$

The western boundary layer transport is southward for all $\theta \geq 0$. At the pole the transport is exactly twice the source strength since the boundary current must carry southward the fluid issuing from the source as well as all of theinterior flow impinging on the apex. This interior flux, which comprises a basinwide recirculation, has a northward mass flux at the apex equal to the source strength. Thus at the apex half the mass flux of the boundary layer comes from the source and half from the recirculating interior flow. The interior is fed from the western boundary current as the current flows southward, as indicated in Fig. 7.3.1. The transport of the western boundary current falls to zero as the current approaches the equator. During its flow southward the western boundary current loses a total mass flux equal to $2S_0$. One half of this flux is recirculated in the interior and ends up at the apex to return through the western boundary current. The other half is lost through the interface to the thermocline and is directly replaced in the western boundary layer at the apex by the source.

Of course there is no reason why the sector need be isolated from the other hemisphere. That is, there is no a priori reason why the balance in (7.3.14) need apply if there are sources or sinks of fluid in the other hemisphere. The interior flow is still zero at the equator but the transport of the western boundary current at the equator, from (7.3.13) would be:

$$T_w(\theta = 0) = w_0 R^2[\phi_e - \phi_w] - S_0. \tag{7.3.16}$$

If the upwelling to the thermocline is greater than the source strength at the pole, there must be an import of fluid across the equator from the south, and T_w is positive. If the source strength is larger than the upwelling, the reverse is true, and the boundary current flows across the equator from the northern to southern hemispheres. This is likely to be the situation in the North Atlantic.

On the other hand, if we consider a similar sector in which there is no source at the apex, all the water must be imported from the southern hemisphere. In such a case S_0 would be zero and T_w is given by:

$$T_w = w_0 R^2(\phi_e - \phi_w)[1 - 2\sin\theta] \tag{7.3.17}$$

instead of (7.3.15). This might be the case in a simple model of the North Pacific which is thought to have no local sources of deep water (Warren 1981). The interior flow is the same as in the previous example. However, now at the equator the boundary current is positive as the current delivers the necessary flow to the northern hemisphere sector. As the current flows northward, its mass is exhausted as the flow through the upper surface of the abyss into the thermocline draws fluid from the boundary layer. In fact at 30°N the transport vanishes. North of this latitude the boundary current actually moves *southward,* driven by the cyclonic recirculation of the interior flow.

This important example emphasizes the fact that the direction of the interior flow and the boundary layer flow can be directly opposed to that which might be naively inferred from the knowledge of the source position and the characteristics of the water at any location with respect to the source. Thus, for example, the water in the western boundary current north of 30°N has its source in the southern hemisphere. However, because of the strongly recirculating character of the flow it is moving southward towards the source. A flux equal to $w_0R^2(\phi_e - \phi_w)$ enters the northern hemisphere across the equator in the western boundary current and enters the interior south of 30°N. Half of this flux is absorbed into the thermocline in the area south of 30°N. At 30°N, where $T_w = 0$, the other half of the original mass flux, i.e. $0.5w_0R^2(\phi_e - \phi_w)$ enters the northern portion of the sector through the interior and upwells into the thermocline. However, a flux equal to the full flow across the equator reaches the apex of the sector, i.e., a flux of $w_0R^2(\phi_e - \phi_w)$. This flux is fed by the recirculation in the western boundary current in the region north of 30° which carries at the apex a flux, $w_0R^2(\phi_e - \phi_w)$, southward. Hence the maximum flux in the boundary current at the pole is *double* the transport that entered the region north of 30°. The recirculation is twice as strong as the flow involved in the mass circuit from the source in the western boundary layer at the equator to the distributed sink in the northern portion of the gyre. The ability of the source/sink flow to create strong recirculations tends to decouple the local flow direction from the origin of the source water. This is true both for the interior and the boundary layer.

Consider now an ocean basin bounded between the latitudes θ_s and θ_n and the longitudes, as before, ϕ_e and ϕ_w. Again, let w_0 be geographically uniform so that (7.3.10) applies. Suppose that a source of strength S_n is placed in the northern sector at the northern boundary, and a source S_s is placed at the southern boundary of the basin. The longitude of each of the sources is arbitrary. The situation is shown in Fig. 7.3.2 and presents a unifying and generalizing arrangement of the previous two examples.

The total mass flux upwelling in the basin is:

$$\begin{aligned}\int_{\phi_w}^{\phi_e}\int_{\theta_s}^{\theta_n} w_0R^2\cos\theta\, d\theta\, d\phi &= w_0R^2(\phi_e - \phi_w)\left[\sin\theta_n - \sin\theta_s\right]\\ &= S_s + S_n \qquad (7.3.18)\end{aligned}$$

which relates the upwelling amplitude to the *total* source strength.

Consider the upwelling into the thermocline in the subregion north of the latitude θ. The flux leaving the abyss is given by the first line in (7.3.18) with θ_s replaced with θ. The mass balance for this region consists of balancing this sink against the fluid that enters in the western boundary layer, the fluid delivered by the source S_n and the interior flow (7.3.6) integrated across the latitude θ. When the balance is made the result requires a western boundary layer transport:

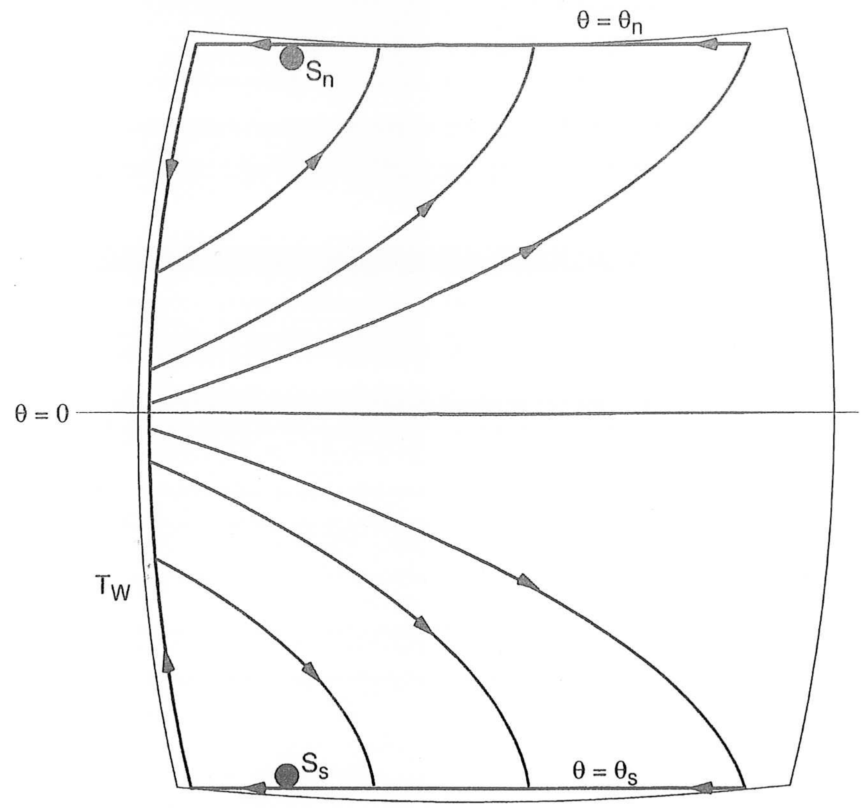

Fig. 7.3.2. Enclosed ocean basin whose northern and southern boundaries are θ_n and θ_s, respectively. The basin typically straddles the equator. A source of strength S_n lies at the northern boundary, and a source S_s is placed at the southern boundary. The inferred circulation is as shown for the case where the two sources are equal, and where $\theta_n = -\theta_s$. Note the northern and southern boundary layers required to close the circulation provoked by the interior flow

$$T_w = w_0 R^2(\phi_e - \phi_w)\left[\sin\theta_n - 2\sin\theta\right] - S_n. \tag{7.3.19}$$

The upwelling velocity is related to the sum of the source terms, i.e.:

$$S_s + S_n = w_0 R^2(\phi_e - \phi_w)\left[\sin\theta_n - \sin\theta_s\right]. \tag{7.3.20}$$

When, for example, S_n is zero the entire deep water source is in the south. Again, this might be considered a model of the situation in the Pacific, which lacks a deep water source in the northern hemisphere. In this case:

$$T_w = w_0 R^2(\phi_e - \phi_w)\left[\sin\theta_n - 2\sin\theta\right] \tag{7.3.21}$$

which vanishes where:

$$\theta = \sin^{-1}\left\{\frac{\sin\theta_n}{2}\right\}. \tag{7.3.22}$$

Thus the position of the point where the western boundary layer flow changes direction is a function only of the latitude of the position of the northern boundary of the basin. If θ_n is 70° instead of 90° the position of the stagnation point in the western boundary layer moves only slightly, i.e., to 28°N from 30°N. If θ_n becomes small on the other hand (e.g., the Indian Ocean), the latitude where the boundary current changes direction becomes $1/2\theta_n$. If, instead, the two sources are equal, i.e., if $S_n = S_s = S$:

$$T_w = w_0 R^2(\phi_e - \phi_w)\left\{\frac{\sin\theta_n + \sin\theta_s}{2} - 2\sin\theta\right\}. \tag{7.3.23}$$

If the southern boundary of the region lies in the southern hemisphere, $\sin\theta_s < 0$. If the domain is symmetric, i.e., if $\theta_s = -\theta_n$ the first term in the final bracket of (7.3.23) ~~will~~ vanishes, and the boundary-layer transport vanishes only on the equator. In this case, where both the source strength and the geometry of the two hemispheres are perfect mirror images of each other, there is no interchange across the equator in either the interior or boundary layer. The situation is shown in Fig. 7.3.2. Note the need in general for northern and southern boundary layers required to close the flow produced by the circulation in the interior.

When the sources and each of the hemispheric basins are mirror images, the two hemispheres are isolated from each other. When the symmetry is broken, either by unequal source strengths or by dissimilar basin geometries in the two hemispheres, there results a flux across the equator in the western boundary current, and the two hemispheres are in communication. This lack of symmetry is certainly the more realistic situation and therefore we expect to see cross-equatorward transport in the western boundary layer in all basins.

A further illuminating example presented by Stommel and Arons in their original paper is shown in Fig. 7.3.3. A hemispheric sector is partly blocked by a barrier at the longitude ϕ^* which stretches from the apex to a latitude θ^*. The barrier is meant to represent a continent, and it is of interest to see its effect on the deep circulation driven by a source, S_0, placed to the south of the continent. The source can be thought of either as the actual location of production of deep water or as the point of entry of fluid from the western boundary layer into the basin.

The interior *meridional* flow is still given everywhere by (7.3.6). Consider the region north of $\theta = \theta^*$. There are two subsectors, one on each side of the barrier. The total upwelling into the thermocline for the entire region north of θ is given by (7.3.11). For the subsector to the right of the continent and in the region north of the latitude $\theta = \theta^*$, the total mass balance for the region $\phi \geq \phi^*$ is:

$$T_w^* + w_0 R^2(\phi_e - \phi^*)\sin\theta = w_0 R^2(\phi_e - \phi^*)[1 - \sin\theta] \tag{7.3.24}$$

so that the boundary layer transport, T_w^*, on the eastern side of the continent is given by:

$$T_w^* = w_0 R^2(\phi_e - \phi^*)[1 - 2\sin\theta] \tag{7.3.25}$$

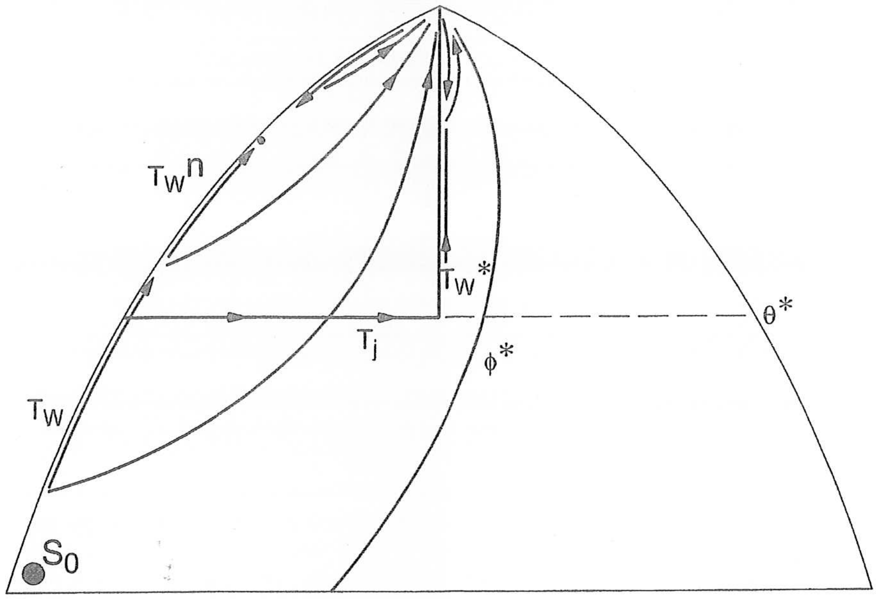

Fig. 7.3.3. Ocean sector is split by a barrier at longitude ϕ^* which extends from the apex at 90°N southward to a latitude θ^*. The presence of the barrier produces a splitting of the western boundary layer transport in the southern region into two tributary currents, one on each western boundary in the region north of θ^* joined by a zonal jet with transport T_j

which is the same as (7.3.17) with ϕ_w replaced by ϕ^*. Thus, the circulation in the eastern subbasin is exactly the same as if the western subbasin did not exist except that the transport of the western boundary layer is reduced because the size of the interior domain and hence the recirculating interior transport is reduced.

For the western subbasin the meridional flow is still given by (7.3.6). However, the zonal velocity is now given by (7.3.9) with ϕ_e replaced by ϕ^*, which forms the eastern boundary of this subbasin. If we now repeat the analysis that leads to (7.3.17) for the western subbasin, we obtain for the transport of the western boundary current of the *western* basin in the region *north* of θ^*:

$$T_w^n = w_0 R^2(\phi^* - \phi_e)[1 - 2\sin\theta]. \qquad (7.3.26)$$

If (7.3.25) and (7.3.26) are compared to (7.3.17) we see that the *sum* of the two boundary-layer transports on the two western boundaries of the subbasins north of θ^* exactly equals the boundary-layer transport on the western side of the full basin in the absence of the continent. The introduction of the continent has split the western boundary layer transport into two tributary currents whose combined transport matches smoothly to the transport of the single western boundary current which exists in the region south of θ^*, and which is still given by (7.3.17). This implies that the western boundary layer transport at the western boundary of the full sector suffers an abrupt decline as the current passes the latitude of the southern tip of the continent. The western boundary passes the latitude of the southern tip of the continent. The western boundary layer transport falls from the value given by (7.3.17) to that given by (7.3.26). Since the interior meridional transport is continuous across the latitude θ^*, this implies that the drop in transport is effected by a narrow jet, as shown in Fig. 7.3.3, which carries the difference between the transports given by (7.3.17) and (7.3.26), and which becomes the western boundary layer transport at the southern tip of the continent. The eastward transport, T_j, in the zonal jet connecting the two western boundary currents is:

$$\begin{aligned} T_j &= T_w(\theta^*) - T_w^n(\theta^*) = T_w^*(\theta^*) \\ &= w_0 R^2(\phi^* - \phi_w)\left[1 - 2\sin\theta^*\right]. \end{aligned} \tag{7.3.27}$$

Note that the latitudes where the boundary layer transport vanishes are the same in each subdomain if θ^* lies south of 30°N. The presence of a barrier splitting the basin, as would the presence of a continent such as South America, produces a zonal jet linking the boundary currents of the two western boundaries at the latitude where one of them terminates at the tip of the continent.

Since the problem is linear, combinations of the examples described in this section can be used to describe a complex picture of the total circulation of the abyss of the global ocean. The first attempt to do so in a systematic way was due to Stommel (1958) in a classic "letter to the editors" in which he boldly outlined a scheme for the abyssal circulation based on the ideas described here. The schematic circulation that he proposed is shown in Fig. 7.3.4. The numbers

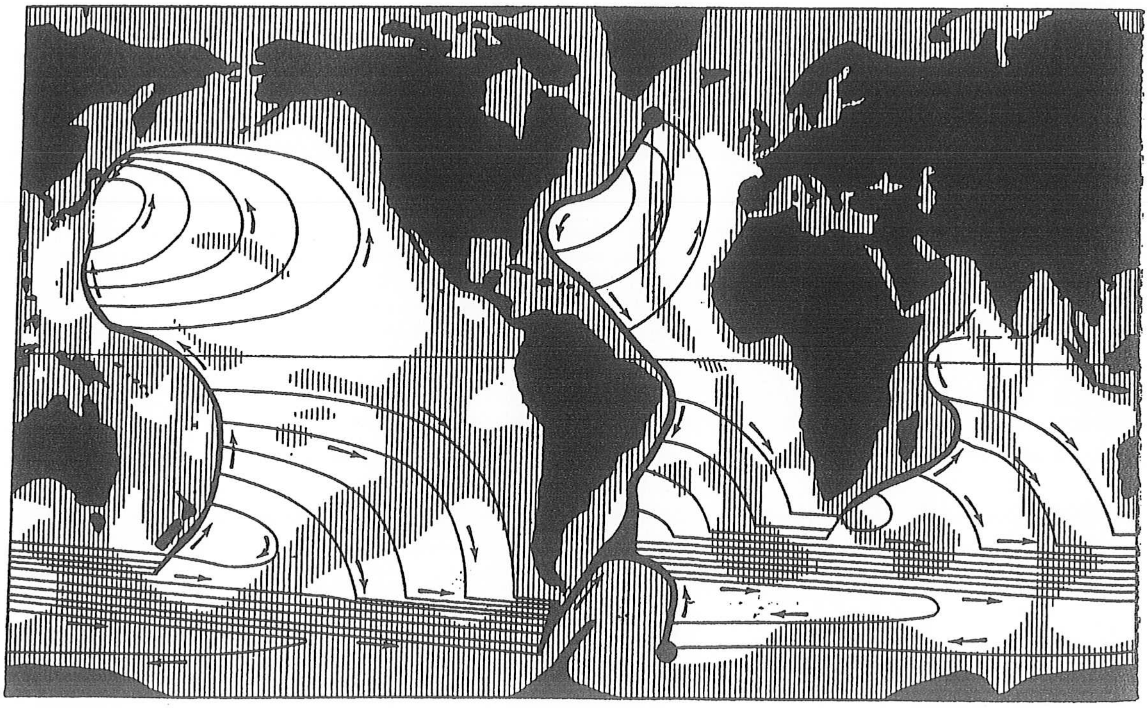

Fig. 7.3.4. Abyssal circulation proposed by Stommel (1958)

associated with circulation were refined in a later paper (Stommel and Arons 1960b), but the pattern of flow remains similar to that shown in the figure (see also Kuo and Veronis 1970).

The original figure of Stommel's is shown because it highlights an important point. The patterns of flow described in the theory assume that the ocean floor is flat. As noted earlier, the presence of the great oceanic ridge systems is expected to disturb the low patterns considerably. The ridge system is indicated in Stommel's figure by the shaded areas, but the streamlines are drawn as if the barriers to the deep flow presented by the ridges were not present. The flow in the figure moves with serene indifference to the presence of the topographic barriers. In Section 7.5 we take up the question of the effect of the ridges and their gaps on the Stommel-Arons circulation. However, first we consider in more detail the circulation associated with isolated sources and sinks within the domain of the basin.

7.4 Dipole Circulation Associated with Isolated Sources

In Section 7.2 we described an experimental situation in a rotating pie-shaped sector in which a source and sink are placed at different radii on the eastern boundary of the sector, and argued on the basis of mass balance considerations that the flow of mass from source to sink takes place in purely zonal jets which are connected to the western boundary current. This picture, in which the flow moves zonally *from* the source and then *to* the sink, is only a partial picture of the circulation. In this section we reexamine the situation more carefully within the context of the Stommel-Arons theory on the sphere.

Consider the situation shown in Fig. 7.4.1. A source of finite extent has a total strength S_+ and feeds the abyss by pumping fluid downward in the zone indicated by the upper circle. South of the source lies a sink with an equal and opposite total strength, i.e., $S_- = -S_+$. Outside of these zones the vertical pumping velocity w_0 is zero. What is the circulation produced by such a distribution?

The meridional velocity is given by (7.3.6) and is zero except in the regions of the source and sink. In the region of the source, where $w_0 < 0$, the meridional flow (assuming the source is in the northern hemisphere) is southward while under the sink where $w_0 > 0$, the meridional flow is northward as indicated in the figure. Elsewhere the interior flow can only be zonal and given by (7.3.8) while the pressure anomaly is given by (7.3.7). Note that the pressure disturbance and zonal velocity are both zero east of the source and sink. The pressure anomaly is also zero west of the source and sink *outside* the latitude bands containing the source and sink. This has important implications. If the zonal flow from the source to the western boundary were unidirectional, for example, strictly westward, the pressure west of the source would have to increase monotonically across the latitude interval of the source