Transient growth and optimal excitation of thermohaline variability

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Background & Motivation:

- Transient amplification
- THC stability regimes, potential for large variability in a stable regime

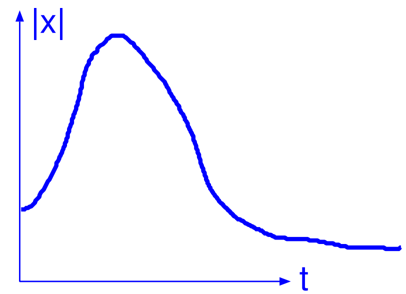
Questions:

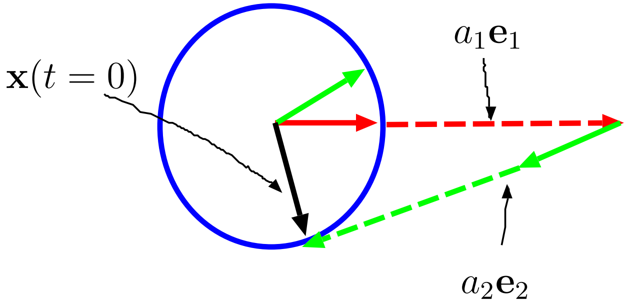
- What are possible physical mechanisms of transient growth of THC anomalies?
- Can transient amplification lead to a significant THC amplification?
- Can it lead to the crossing of the THC stability threshold?

[Tziperman & Ioannou, *JPO, 2002; Zanna and Tziperman, JPO, 2006]*

Preliminaries: Transient amplification

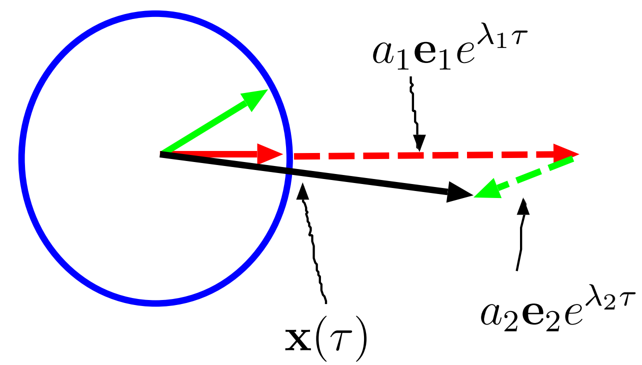
(Farrell, 1988) A damped linear system: $\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x}$, such that $\mathbf{x} \to 0$ as $t \to \infty$ may still have an initial amplification: why? How?





$\leftarrow$ Let $\mathbf{A}e_i = \lambda_i \mathbf{e}_i$; if $\mathbf{A}^T\mathbf{A} \neq \mathbf{A}\mathbf{A}^T$, then $\mathbf{e}_i$ are not orthogonal. Consider the following i.c. in a 2d case

at $t = \tau$, solution is $a_1\mathbf{e}_1 e^{\lambda_1\tau} + a_2\mathbf{e}_2 e^{\lambda_2\tau}$. If $\lambda_2 \ll \lambda_1 \ll 0$ then $a_2\mathbf{e}_2 e^{\lambda_2\tau} \to 0$ quickly, leaving mostly $\mathbf{x}(\tau) \approx a_1\mathbf{e}_1 e^{\lambda_1\tau}$

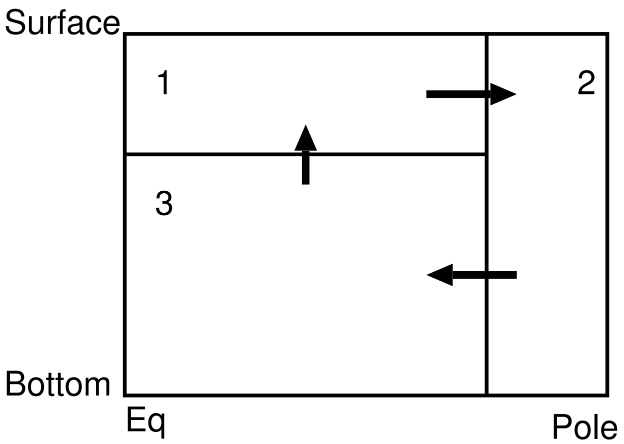


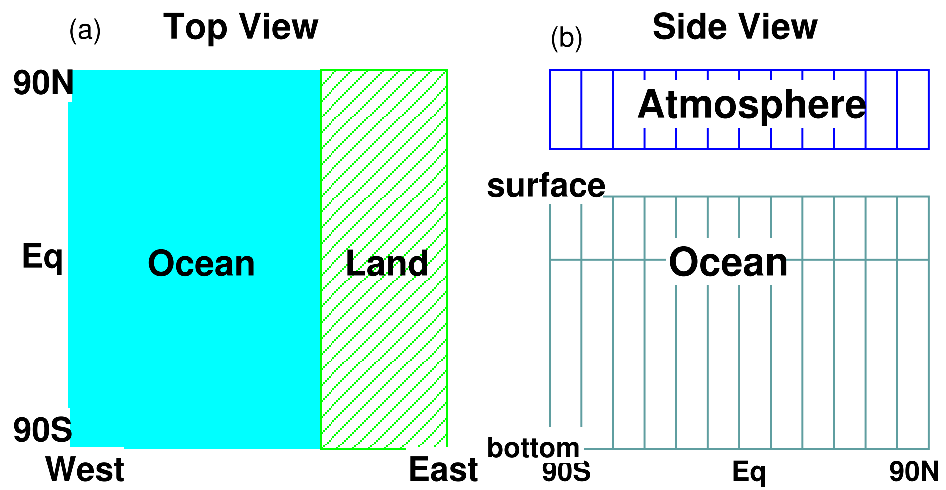
$\Rightarrow$ An initial amplification!! Later, $a_1\mathbf{e}_1 e^{\lambda_1\tau} \to 0$ as well, so that $\mathbf{x}(t \to \infty) \to 0$.

$\Rightarrow$ required ingredient for transient amplification:

(i) initial cancellation; (ii) different decaying rates.

The models: ocean component





simple 3-box model under mixed b.c.

Intermediate coupled model: continuous in latitude; 2 ocn & 1 atm levels

Momentum: $\frac{\partial p}{\partial y} = -rv; \quad \frac{\partial p}{\partial z} = -\rho g$; ($p$ is pressure, v northward velocity, r friction, ρ density, g gravity) $\Rightarrow$THC: $v = C_u g\left[(\rho_2 - \rho_1)D_{upper} + (\rho_3 - \rho_4)D_{lower}\right]$

Temperature (T) and Salinity (S) advection-diffusion eqns:

$$\frac{\partial T}{\partial t} + \nabla(T\mathbf{u}) + \kappa\nabla^2 T = H^{air-sea}; \quad \frac{\partial S}{\partial t} + \nabla(S\mathbf{u}) + \kappa\nabla^2 S = (\text{Evap, Precipitation})$$

Convective mixing: when stratification is unstable; density: $\rho = \rho(T,S)$

The models: atmospheric component

The Atmospheric model is a simple energy balance model (Nakamura et al 94; Rivin & Tziperman 97) .

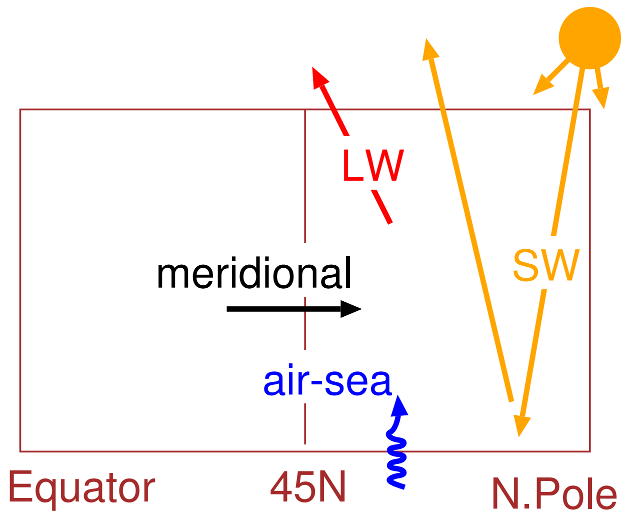
The temperature at a given box is determine by the balance between the incoming shortwave radiation, outgoing long wave radiation, air-sea fluxes, and meridional atmospheric heat fluxes.

$$\frac{dT^{atm}}{dt} = SW \times (1 - \alpha) - LW + \text{air-sea} + \text{meridional fluxes}$$

α: albedo, including contributions from land, land-ice, ocean, sea ice

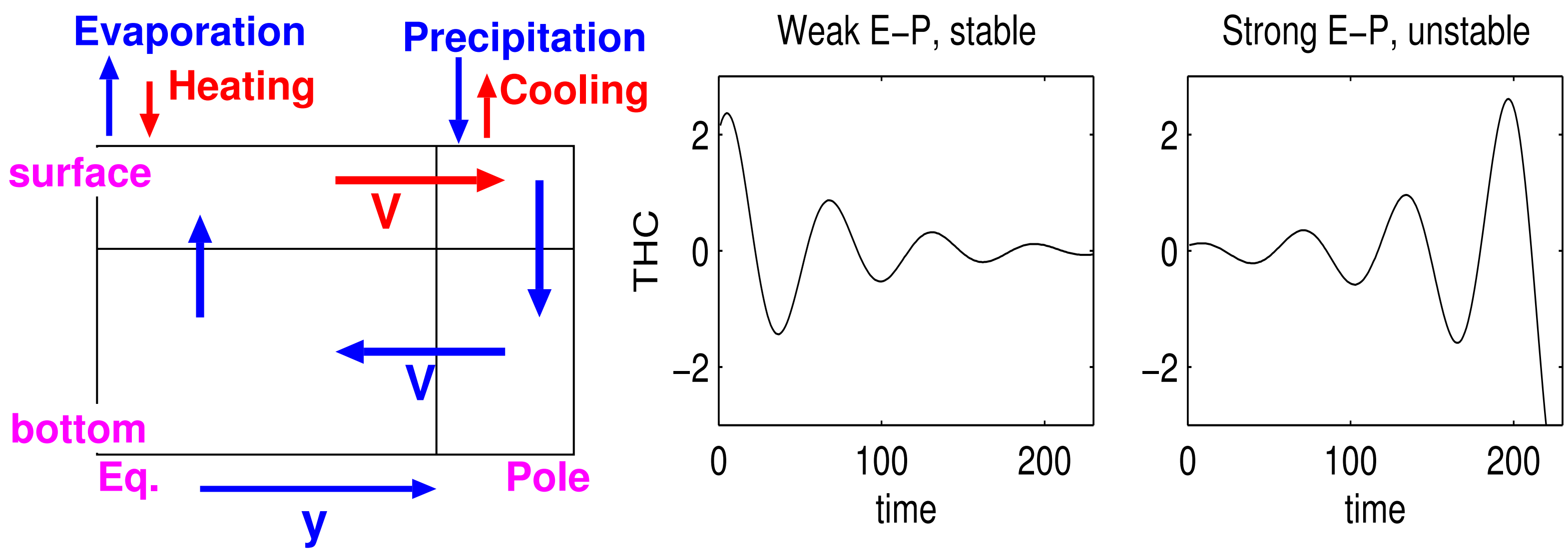
LW: black body (long wave) radiation to outer space

SW: incoming (short wave) solar radiation

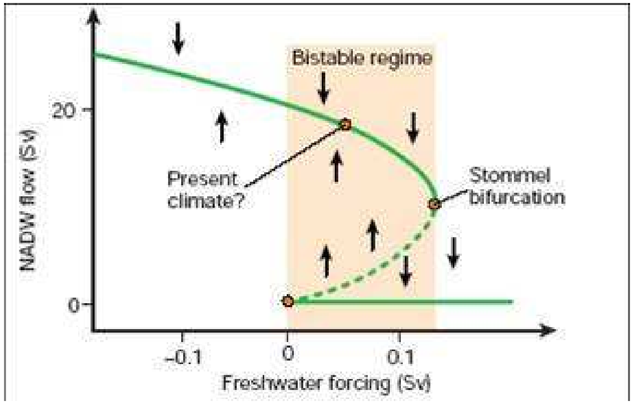


THC stability regimes

Stability regimes under mixed b.c.:



THC is “driven” by temperature gradients & “braked” by salinity gradients.



Can a transient amplification of the “right” small THC anomaly result in large amplification and destabilization/ collapse of THC?

Linearized dynamics

Linearize about steady state $T_i = \bar{T}_i + T_i'$

$$\frac{d\mathbf{P}}{dt} = \mathbf{A}\mathbf{P}$$
$$\mathbf{P} \equiv [T_1', T_2', T_3', S_1', S_2', S_3']$$

$\mathbf{A}$ is 6×6 for 3-box model, 366$\times$366 for intermediate model. eigenmodes in damped oscillatory regime are a combination of:

• rapidly decaying ($\sim 1-2$ years) SST modes,
• oscillatory (complex) modes ($\sim$ a few decades to centuries),
• slowly decaying modes (~ 100s years); involving mostly deep T & S.
• a zero eigenvalue, corresponds to a constant shift in salinity of all boxes

So, we have the differential decay rates required for transient growth, we now need to figure out an initial cancellation mechanism.

Formalism: Transient amplification & optimal i.c.

Find optimal i.c. $\mathbf{P}(t=0)$ maximizing THC $t=\tau$, $U(t=\tau)^2$

$$
\begin{aligned}
J(\tau) &\equiv \mathbf{P}(t=\tau)^T \mathbf{X}\mathbf{P}(t=\tau) = (\mathbf{B}\mathbf{P}_0)^T \mathbf{X}(\mathbf{B}\mathbf{P}_0), \\
X_{ij} &= R_i R_j; \qquad \mathbf{R}\cdot\mathbf{P} = U \\
\mathbf{R} &= u_0[\delta\alpha, -\alpha, (1-\delta)\alpha, -\delta\beta, \beta, -(1-\delta)\beta].
\end{aligned}
$$

where $\mathbf{P}(t) = \mathbf{B}(t,s)\mathbf{P}_0$. Also, divided THC into T,S components:

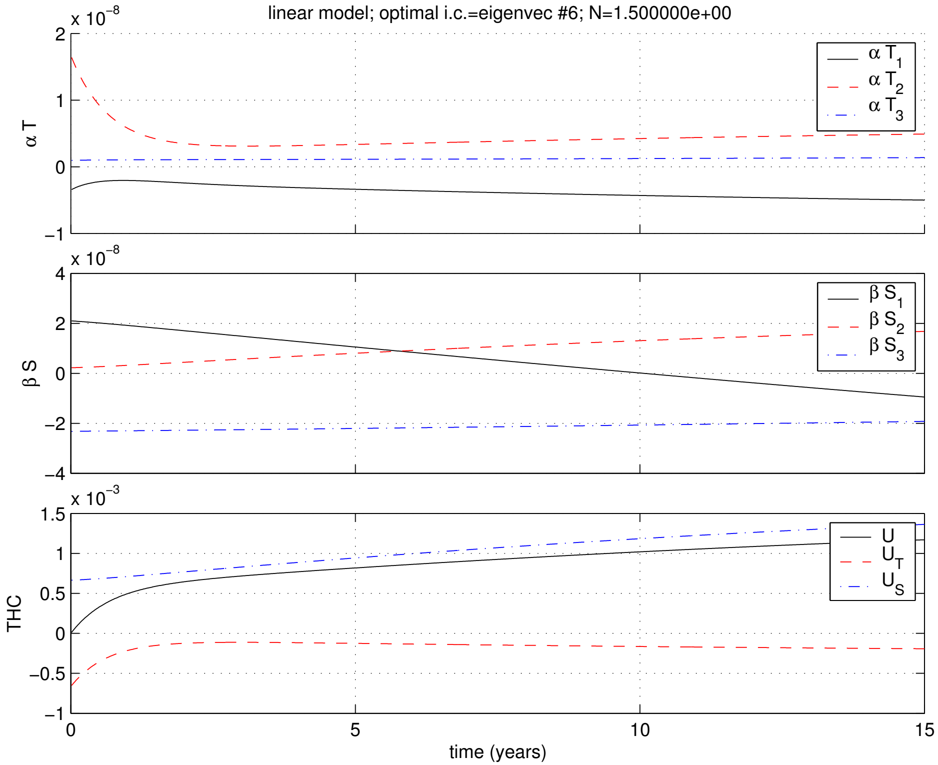
$$
\begin{aligned}
U_T &= \mathbf{R}_T\cdot\mathbf{P}; \qquad \mathbf{R}_T = u_0[\delta\alpha, -\alpha, (1-\delta)\alpha, 0, 0, 0] \\
U_S &= \mathbf{R}_S\cdot\mathbf{P}; \qquad \mathbf{R}_S = u_0[0, 0, 0, -\delta\beta, \beta, -(1-\delta)\beta]
\end{aligned}
$$

Note: norm kernel $\mathbf{X}$ is singular (rank=1). Now find $\mathbf{P}_0$ to maximize $\mathbf{P}_0^T\mathbf{B}^T\mathbf{X}\mathbf{B}\mathbf{P}_0 + \lambda\mathbf{P}_0^T\mathbf{Y}\mathbf{P}_0; \Rightarrow$ optimal i.c.s are eigenmodes of $\mathbf{B}(\tau,0)^T\mathbf{X}\mathbf{B}(\tau,0)\mathbf{e} = \lambda\mathbf{Y}\mathbf{e}$.

For $\mathbf{Y} = \mathbf{X}$, eigenproblem is singular $\Rightarrow$ regularize $\mathbf{X}$ by adding to it $\varepsilon\mathbf{I}$.

Results: Two different physical mechanisms of transient THC amplification.

Transient amplification, 4-box model: mechanism 1



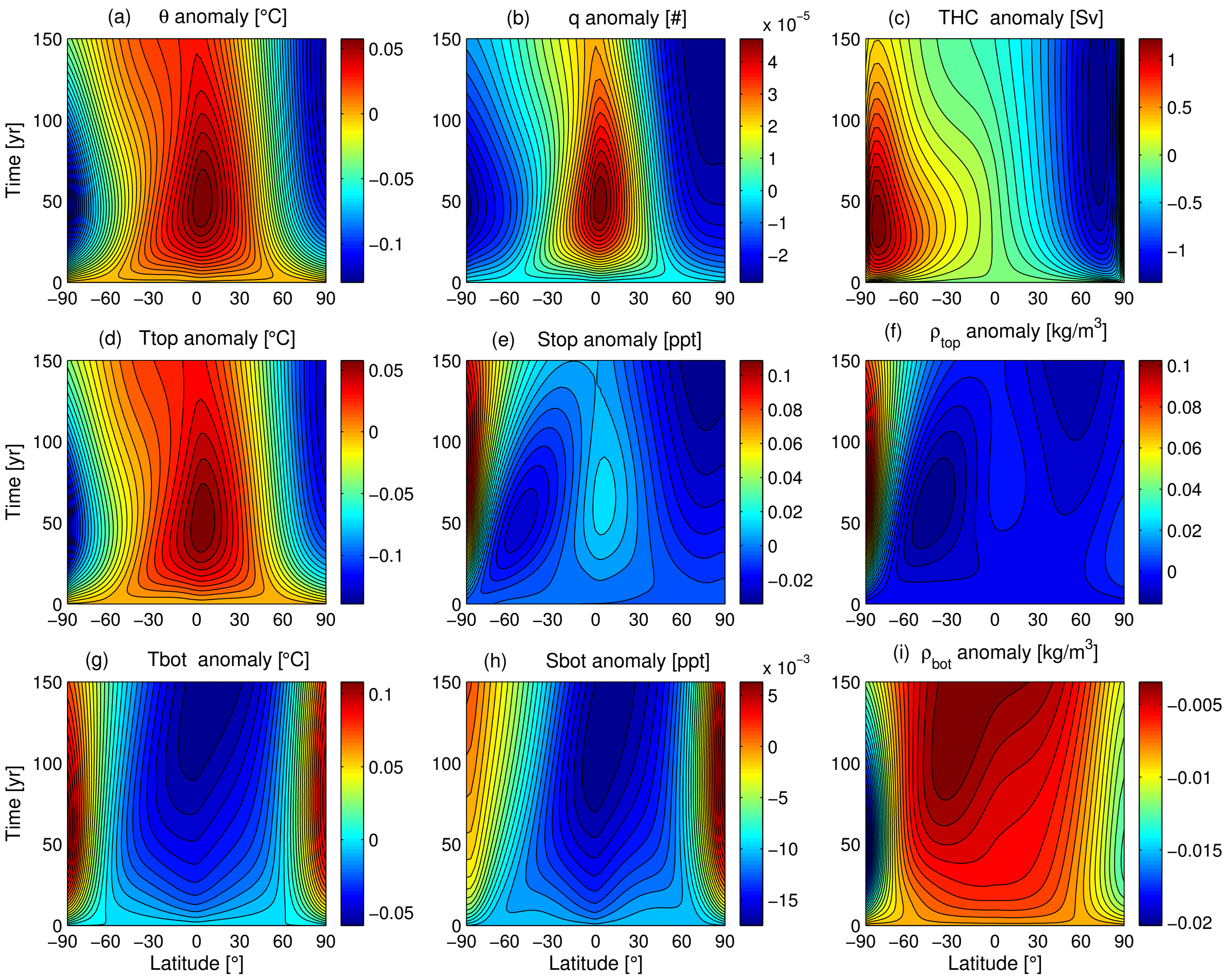
Linearized model results, optimization time $\tau = 2$ yrs $\Rightarrow$ rapid THC growth during first 2 yr:

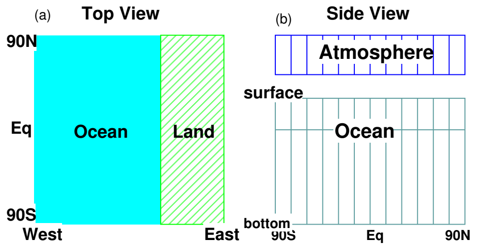
Upper panel: αT_i; middle panel: βS_i; lower panel: THC (U, U_T, U_S)

The mechanism: optimal i.c.s have $U_T = -U_S$, so that $U(t=0) = 0$. U_T depends on SST, $\Rightarrow$ decays rapidly; U_S decays much slower $\Rightarrow$
$U(t) = U_S + U_T \ \longrightarrow \ U_S(t=0)$

Implications: $U(0) = 0$, so amplification $U(\tau)/U(0) = \infty$. Yet, actual maximal THC is limited by amplitude of initial S'. $\Rightarrow$ small amplitude initial perturbation cannot lead to a large amplitude transient amplification that would bring the THC below the stability threshold.

Intermediate model results: optimal THC initial conditions



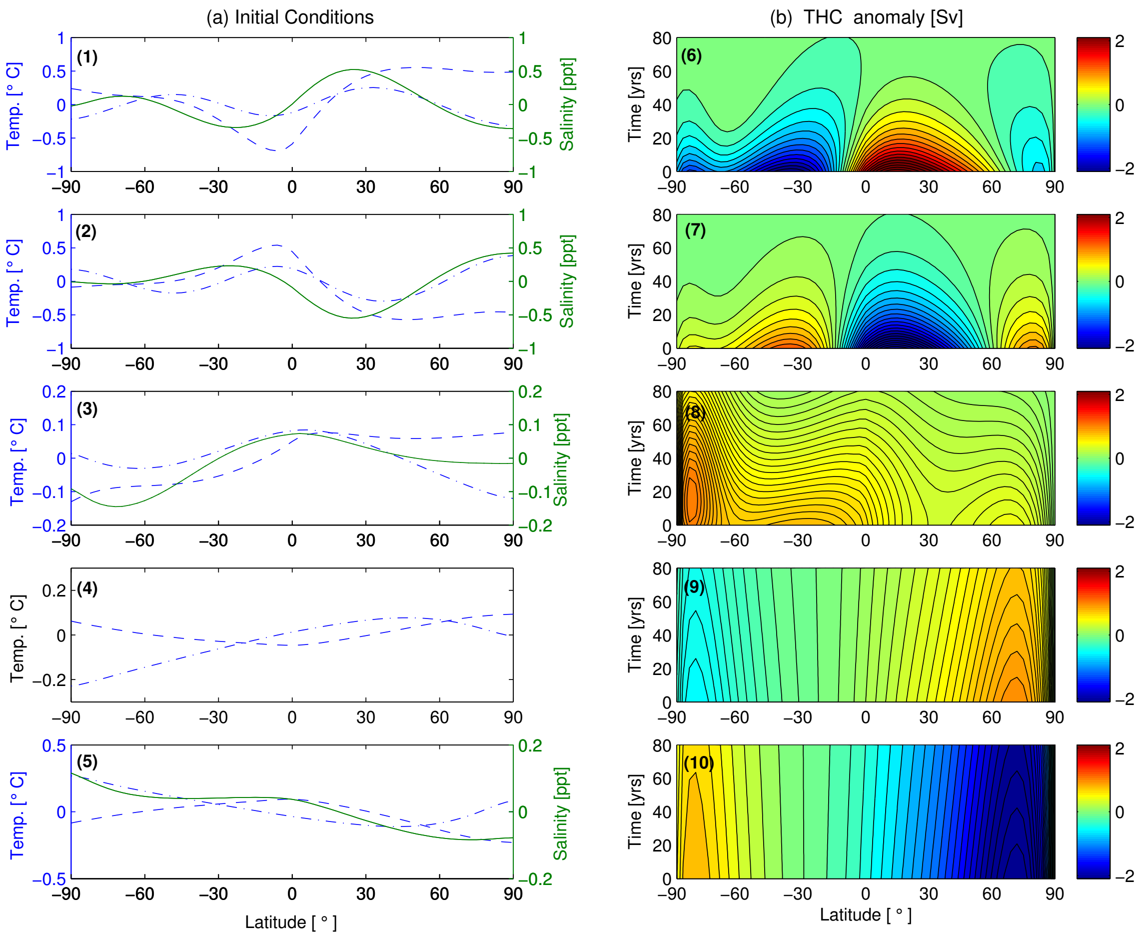


latitude-continuous coupled ocean-atm model

← Optimal initial conditions for THC (Zanna and Tziperman, 2005, in press, JPO)

Amplification factor now is 20 for salinity and 600 for temperature! much more exciting...

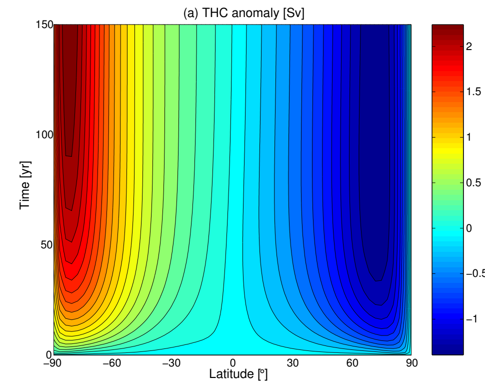
Intermediate model: mechanism of transient growth



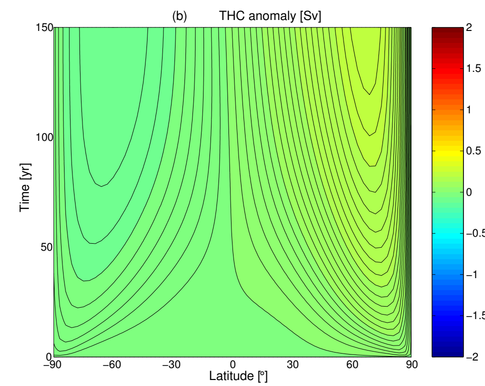
5 large scale damped (23,25,87,281,784 yrs; some oscillatory) modes interact to create the transient amplification

Growth due to $v'\nabla(\bar{S},\bar{T})$, decay due to $\bar{v}\nabla(S',T')$.

without $\bar{v}\nabla(S',T')$:



without $v'\nabla(\bar{S},\bar{T})$:



Unlike THC instability, amplification is **not** due to advective instability feedback ($v'\nabla\bar{S}$).

Conclusions: transient amplification of THC

- 4-box model physical mechanism of transient growth: Initial cancellation of thermally-driven (U_T) and salinity-driven (U_S) THC components: $U_T(t=0) = -U_S(t=0)$; zero initial THC anomaly; thermal component damped by atm; THC grows rapidly to $U \to U_S(t=0)$.
  $\Rightarrow$ max amplification limited by amplitude of initial salinity anomaly
- Simple intermediate coupled ocean-atm model, continuous in latitude: very significant transient amplification ($\times 20$ for Salinity, $\times 600$ for temperature) with a growth time scale of 40 years.
- Optimal stochastic atmospheric forcing induces low-frequency variability by exciting mostly the salinity modes.

The End

A THC modeler realizing that no catastrophic transient amplification can occure in a 4-box model.

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...... and having realized the maximum THC amplitude in the intermediate model is not limited by the initial salinity anomaly as it is in the simple box model.

Next: transient amplification of the THC in an ocean GCM using the MITgcm+adjoint (in progress)