A zero-dimensional version of the "Budyko-Sellers" energy-balance model for earth's temperature

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1 Derivation of energy balance equation

An equation for the earth heat budget has the general form

$$\frac{d}{dt}\text{Heat} = \text{net incoming radiation} - \text{outgoing radiation}$$

Suppose the incoming radiation from the sun, averaged over the earth surface, is $S\ watts/m^2$. A portion α of this radiation is reflected by the ice, snow, clouds, land and oceans. This portion depends on the amount of ice and snow cover and therefore on the temperature, $\alpha = \alpha(T)$. So only the net incoming radiation is $S(1-\alpha(T))$. The outgoing radiation is in the form of infrared radiation, or heat, which according to the law of black body radiation is σT^4. Because the earth is not a perfect black body this expression is multiplied by a factor ε, the emissivity, that is smaller than 1.

A globally-averaged energy balance equation for the atmosphere may therefore be written as

$$C\frac{dT}{dt} = S(1-\alpha(T)) - \varepsilon\sigma T^4$$

where the lhs is the heat capacity times the time rate of change of the globally averaged temperature; the rhs includes the incoming radiation S multiplied by one minus the albedo (reflectivity, α), and the outgoing long wave (infrared) radiation. at steady state the lhs vanishes. Assuming the albedo is mostly a function of the amount of ice, it makes sense to assume it has a low value at high temperature (no ice), a high value at very low temperature (earth completely frozen), and some linear variation in between

$$\alpha(T) = \begin{cases} \alpha_1 & \text{if } T < T_1 \\ \alpha_1 + (\alpha_2 - \alpha_1)[T - T_1]/[T_2 - T_1] & \text{if } T_2 > T > T_1 \\ \alpha_2 & \text{if } T > T_2 \end{cases} \tag{1}$$

2 Qualitative solution

Steady state is obtained when the lhs vanishes, so that incoming solar radiation is equal to the outgoing long wave radiation. Fig. 1 shows the incoming radiation, outgoing, and their difference. Note that there are three equilibria (middle panel). Two of them are stable and one unstable. The leftmost one is a snowball solution, the righ-most one is a no-ice solution.

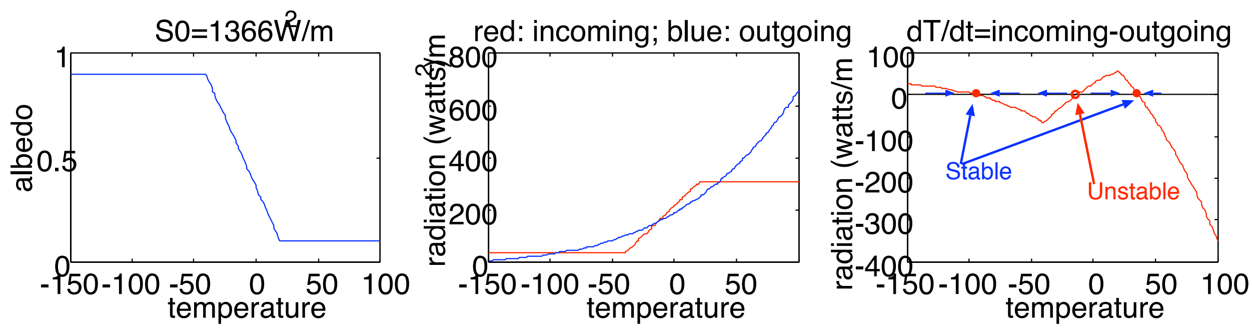

Figure 1: different terms in the energy balance model.

3 Bifurcations and hysteresis

As the strength of the solar input S or the emissivity ε is changed, the two curves in the middle panel of Fig. 1 vertically move with respect to one another. As a result the number of crossing points between them changes. If the blue curve moves down (atmospheric emissivity becomes lower, therefore increasing the greenhouse effect and warming climate, or solar constant S becomes larger), the two lower crossing points representing cold and moderate climates disappear via a saddle node bifurcation, leaving only the warmer climate solution. If it moves up, the two upper crossing points similarly disappear, leaving only the cold (snowball) climate. The number and character of the solutions as function of such a bifurcation parameter may be plotted schematically as Shown in Fig. 2

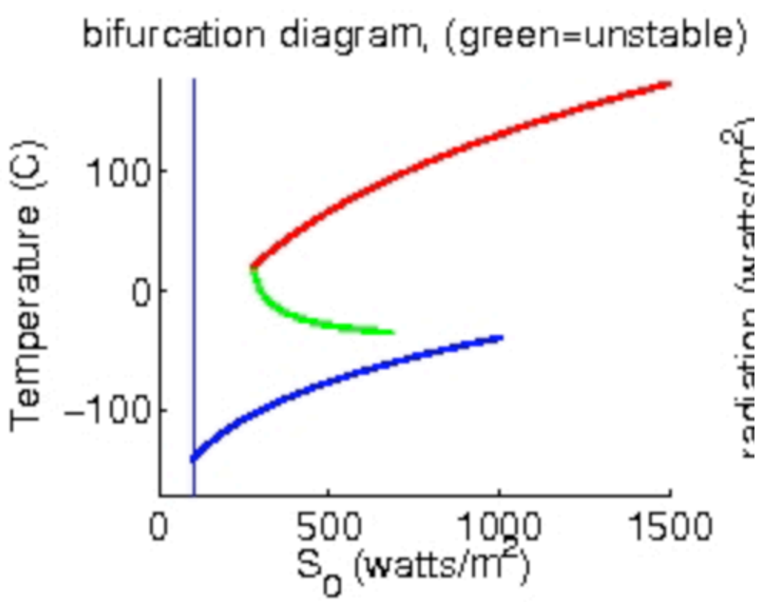

Figure 2: Bifurcation diagram for 0d energy balance model.

Note that a hysteresis is implied here, as discussed in class.