Stommel model notes

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1 Outline

- Introduction: THC, NADW, WOCE salinity section figure, a couple of figures from the nydeden Stommel slides file, effect on climate, etc.

- Divide ocean into two boxes, one for high latitude (box 1) and one for the tropics (box 2).

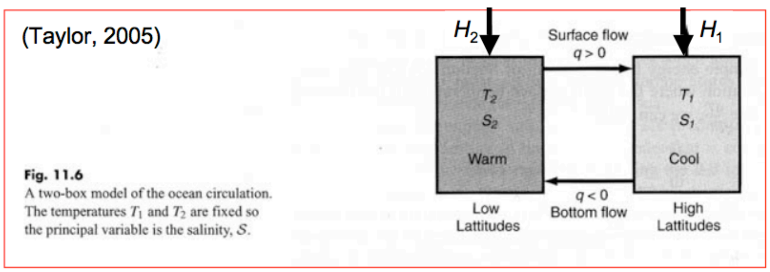

Figure 1: box model schematic.

- Introduce circulation q between the boxes.

- Circulation strength proportional to density difference.

- Density is a function of temperature and salinity, $\rho(T, S) = \rho_0 - \alpha(T - T_0) + \beta(S - S_0)$.

- Consider how evaporation changes salinity in a bucket filled with salt water. Under the assumption that $S \approx S_0 = 35$ppt, introduce virtual salt flux. Details: mass conservation is,

$$\frac{dV}{dt} = -EA$$

where E is the net evaporation rate per unit area and A the area of bucket. Salt conservation is

$$\frac{d}{dt}(SV) = 0$$

which we can expand, using the mass conservation into

$$V\frac{dS}{dt} = -S\frac{dV}{dt} = SEA \approx S_0 EA$$

and we term $S_0 E$ the "virtual salt flux" per unit area. If evalporation is small (in the ocean, 1m/yr for a depth of 4000m), the approximation of using S_0 instead of S is a very good one and accurately describes salinity changes due to evaporation and precipitation. This also means that we do not need to be concerned with actual mass exchanges involved in the evaporation-precipitation process, and can replace them by the total virtual salt flux which is denoted F_s below.

- Need equations for temperature and salinity. Assume temperature is fixed by the atmosphere above each of the boxes, so that when circulation changes only salinity can change. Write a salt conservation equation,

$$\begin{aligned} V_1\frac{dS_1}{dt} &= |q|(S_2 - S_1) - F_s \\ V_2\frac{dS_2}{dt} &= |q|(S_1 - S_2) + F_s \end{aligned}$$

Note the absolute value on the transport, and derive it by considering the equations for both positive and negative transport, and by showing that the combined form is equivalent to the one above with the absolute value.

- Next, $q = K(\rho_1 - \rho_2) = K(-\alpha\Delta T + \beta\Delta S)$.

- Take difference of these two salinity equations and define $\Delta T = T_1 - T_2 < 0$ and $\Delta S = S_1 - S_2$, and assume $V_1 = V_2 = 1$ for simplicity,

$$\begin{aligned} -\frac{d\Delta S}{dt} &= 2|q|\Delta S + 2F_s \\ &= 2K|(-\alpha\Delta T + \beta\Delta S)|\Delta S + 2F_s \end{aligned}$$

- Define $X = \alpha\Delta T < 0$, $Y = \beta\Delta S$; in a steady state, $d\Delta S/dt = 0$, and we can solve the simple quadratic equation for $Y = \beta\Delta S$,

$$|Y - X|Y = -\frac{\beta F_s}{K}$$

However, one has to be careful due to the absolute value, so there are two cases to consider, $q > 0$ and $q < 0$, equivalent to $X > Y$ and $X < Y$. Noting again that $X < 0$, we have in the first case, $Y < X$,

$$Y^2 - XY - \frac{\beta F_s}{K} = 0$$
$$Y = \frac{X}{2} - \frac{1}{2}\left(X^2 + 4\frac{\beta F_s}{K}\right)^{1/2}$$

While in the second, $Y > X$,

$$Y^2 - XY + \frac{\beta F_s}{K} = 0$$
$$Y = \frac{X}{2} \pm \frac{1}{2}\left(X^2 - 4\frac{\beta F_s}{K}\right)^{1/2}$$

Note that the plus solution in the first case has a negative value and is therefore not consistent (because $Y < X$ is not satisfied...), so three solutions.

- Stability analysis shows that one of the solutions is unstable, and two are stable.
- For the results of X as function of the fresh water forcing F_s, see Fig. 4.
- What do the solutions look like for q as function of the salt flux: Fig. 3; Note that solution 1 is northward flow, solution 3 is a weak reversed flow! (a bit of an artifact, it vanishes for more complex models). The model therefore predicts a shutdown of the THC for large fresh water fluxes. Discuss bifurcations, hysteresis.
- Show trailer again now.

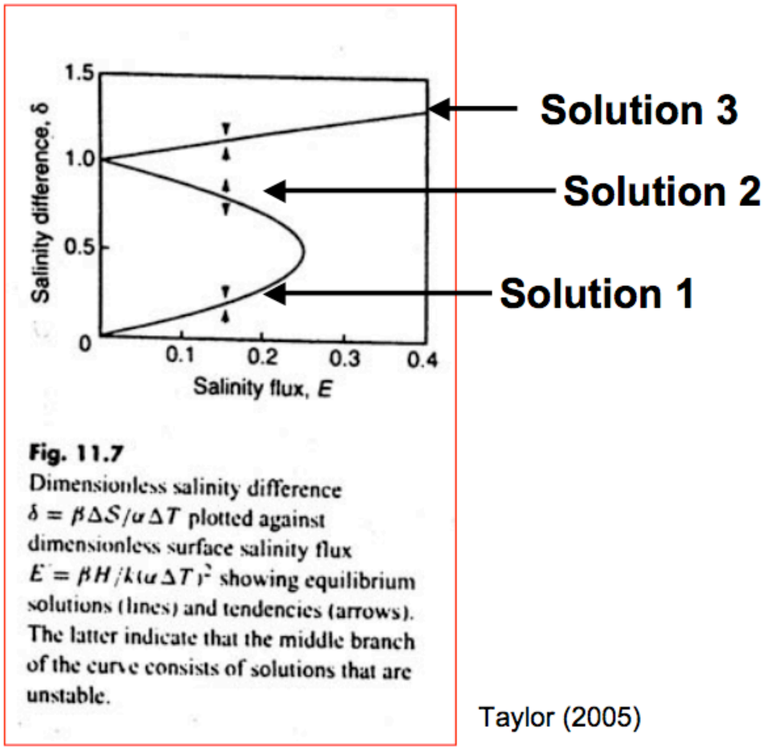

Figure 2: Solutions for Y as function of F_s.

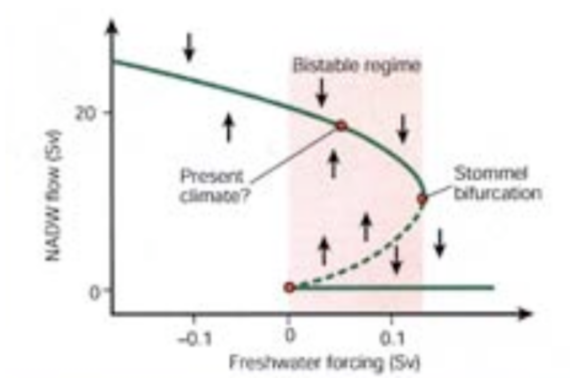

Figure 3: Schematic stability.

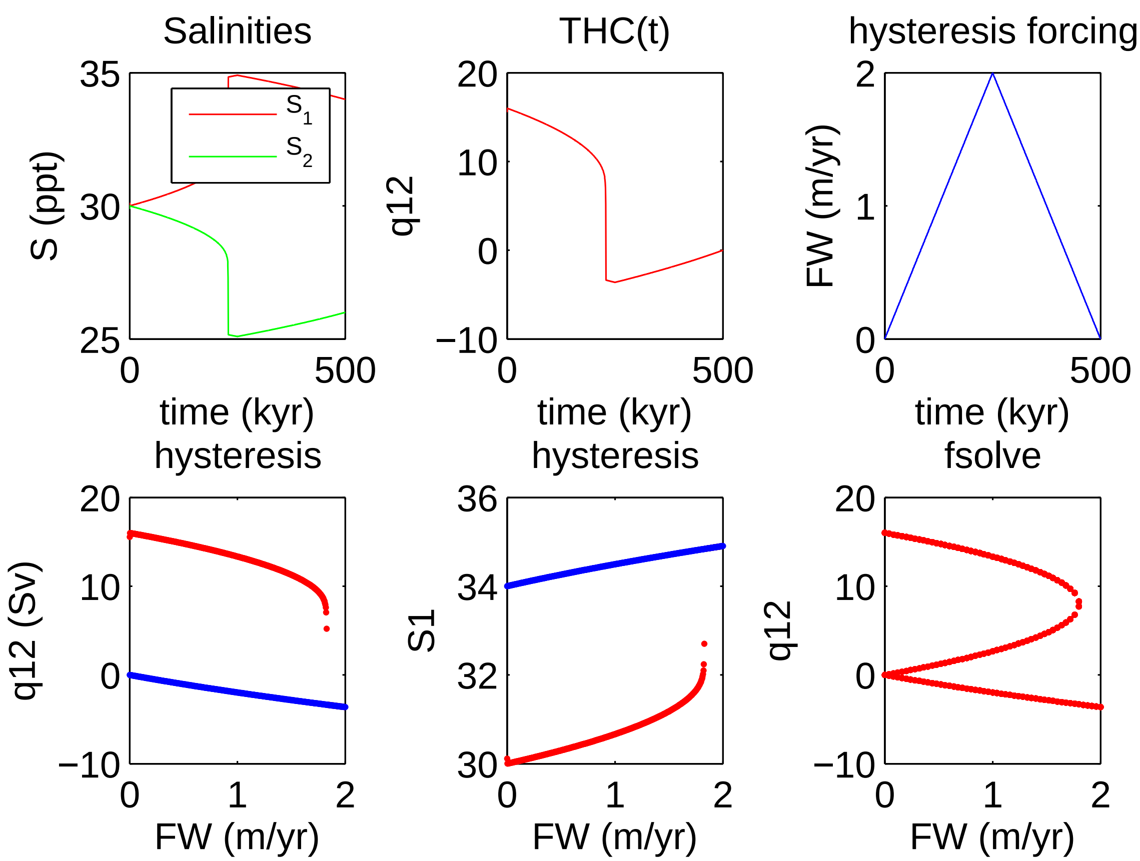

Figure 4: Solution of the 2 box model, including unstable steady states obtained using Fsolve. Code is in Stommel2box.m in the climate workshop directory.