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GROWTH OF ASTEROIDS AND PLANETESIMALS BY ACCRETION*

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ABSTRACT

The basic theory of the accretion process, while applicable to planets generally, is applied here to the asteroids as examples The approach velocity among early asteroidal particles in the solar nebula is estimated to be of the order 1×10^4 cm/sec. Asteroids could grow to observed radii in this case in 10^8 years, assuming efficient sticking (earliest growth may have been by another, non-accretionary process). The overabundance of the largest asteroids ($d \gtrsim 300$ km) is attributed to two factors: the capture cross-section begins to rise steeply at diameter about 140 km, and the asteroids become stable against mass loss by even the highest-velocity impacts at diameter roughly 350 km. Larger bodies would grow rapidly, accounting for the apparent overabundance of large asteroids It is suggested that the mass distribution reconstructed by Anders, with the three largest asteroids overabundant, was "frozen" when the growth process was interrupted, perhaps by disruption of the solar nebula. Approach velocities have increased since the formation of the planets to the present value of a few km/sec, and consequently asteroids smaller than about 300-km diameter have eroded during most of solar-system history.

In "Survey of Asteroids," Kuiper, Fujita, Gehrels, Groenveld, Kent, van Biesbroeck, and van Houten (1958) determined the magnitude distribution among asteroids and noted that the three largest bodies appeared to be overabundant with respect to the smooth distribution for other sizes. They suggested that the occurrence of these marked a "separate phenomenon," perhaps dividing the asteroids into two classes, "original condensations and collisional fragments."

Anders (1965) has also argued that original "accretions" still exist but that the division between original bodies and fragments occurs not among the largest asteroids, but among those of about 30-km radius ($g \simeq 10$), where a "hump" in the (log-log) radius distribution occurs. This hump is reconstituted by Anders into an initial bell-shaped distribution, not characteristic of a fragmentation process. The situation is illustrated in Figure 1. Anders' conclusions are supported in a study by the writer (in preparation) showing that a certain number of collisions, reasonable on several physical grounds, would transform the bell curve to that which is presently observed.

Two questions remain: why the very largest asteroids, with radii greater than about 150 km (Ceres, Pallas, and Vesta; 390-, 245-, and 200-km radii, respectively), should be overabundant, and why the radii of the smaller ones were apparently optimized at 30 km (with an uncertainty of roughly a factor 1.5).

This paper investigates the accretion process with the aim of answering these two questions. The results can be applied, with modifications, to the general case of accretion of planets, but as indicated below they appear to have special applicability to the asteroids.

The process by which the planets grew is virtually unknown. Hoyle (1946) proposed that a body could grow to at least asteroidal dimensions by condensation out of the supersaturated nebular gas in a period on the order of 10^7 or 10^8 years. It appears likely that condensation did operate, but it is questionable whether bodies grew so large directly through this process. The condensable mass may rather have been tied up in a multitude of smaller particles.

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It is assumed here that accretion was the dominant process at the time the planetesimals reached asteroidal dimensions (tens of kilometers), and it will be shown that this accounts for the overabundance of the largest asteroids.

Accretion, by definition, requires that in each instance of collision, on the average, some fraction of the mass of the projectile is added to the target body. That is, the mass of the projectile exceeds the mass of that portion of the ejecta which escapes the target body altogether. If this were not true, collisions would tend to destroy newly forming planets. Clearly, what happens in a given collision event is dependent on the impact velocity, which (given the target body) depends on the velocity of approach at infinity.

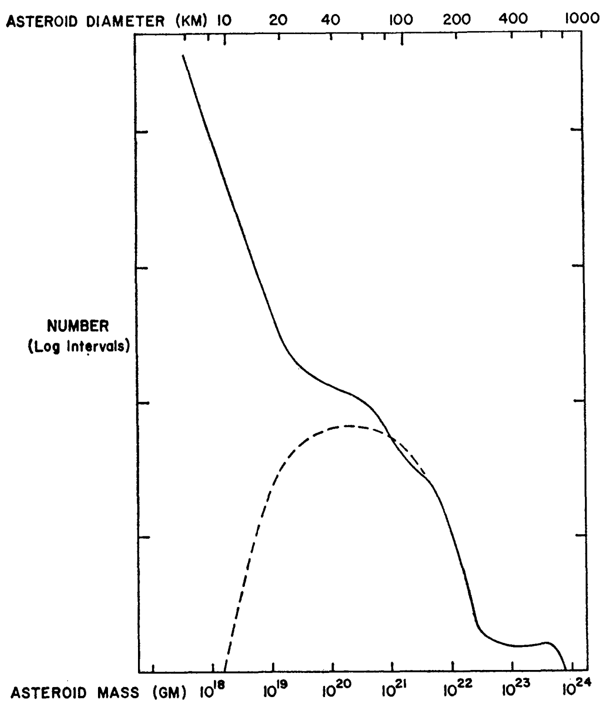

FIG. 1.—Schematic size distribution of asteroids. Solid line is observed (after Kuiper *et al.* 1958) and dashed line is the reconstructed original distribution (after Anders 1965).

The first unknown to be considered here, therefore, is the relative approach velocities of particles in the early solar nebula. Today, Earth and the Moon are being struck by asteroidal debris at velocities averaging about 1.5×10^6 cm/sec and by cometary debris at velocities several times as great. This material is perturbed from the outer regions of the solar system toward the terrestrial planets. The Moon suffers a net loss of mass when struck by such particles, according to the theory of Öpik (1961).

For interasteroidal collision, Piotrowski (1953) has derived a value of 5×10^5 cm/sec in the present asteroid belt. This value is suspected to be on the high side (G. P. Kuiper and E. Roemer 1966, private communications). It includes the effect of eccentric orbits produced both by perturbations and by previous fragmentation by collision.

Interacting particles in the early solar nebula, on the other hand, must have been on much more circular orbits. The density in the asteroidal part of the nebula, in the range 10^{-10} to 10^{-9} g/cm^3 (derived from the "augmented masses" of the planets), suggests that small planetesimal particles would achieve nearly circular orbits. The circularity of present-day planetary orbits is presumably a result of this. Assuming an eccentricity of 0.05 (a characteristic value for the planets) for the pre-asteroidal particles, approach velocities are found to be $\sim 5 \times 10^4$ cm/sec. One could argue for a still lower value in

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view of the appreciable density; mean free paths in the gas were much less than orbital dimensions, suggesting rapid approach to circular orbits for the initial very small particles. Hence we may expect that approach velocities were less than 5×10^4 cm/sec.

In the density range quoted, mean free path considerations suggest an uncertainty in the validity of Stokes's law, but if it is valid, velocities of small particles ($r \lessgtr 1$ cm) with respect to the nebular gas could be considerably further reduced, again producing circular orbits and low relative velocities.

Several other arguments bear on the approach velocity. If we are to assume that accretion occurred at all, the impact velocity must have been lower than some critical value at which the mass of the escaping ejecta equals the projectile mass. Öpik (1958) suggests that little crushing of rock targets occurs at velocities less than about 1.9×10^4 cm/sec. Further, conservation of energy predicts that accretion occurs as long as the impact velocity does not exceed by too great a factor the escape velocity. We have assumed that asteroids larger than tens of kilometers grew by the accretion process. Escape velocity for a body of $R = 10^6$ cm is 1.4×10^3 cm/sec, suggesting that the approach velocity must be less than $O(10^4$ cm/sec) for the theory to be consistent.

A *lower* limit can be placed on the approach velocity if it assumed that accretion was effective. The process must have gone nearly to completion in less than 10^8 years. The theory of accretion (e.g., Kuiper 1951, p. 369) shows that, assuming 100 per cent sticking efficiency for the impact process.

$$v_\infty \simeq \frac{4R}{t} \frac{\rho}{\rho_a}, \tag{1}$$

where v_∞ is the approach velocity, R the planetesimal radius, ρ its density, ρ_a the density of accretable matter in the nebula, and t the time. Kuiper (1953) estimates that ρ_a is the order 3×10^{-3} times the nebular density. Hence, with $\rho_a \leq 3 \times 10^{-12}$ g/cm^3, ρ assumed (following Anders 1965) to be 3.6 g/cm^3 (the density of hypersthene chondrites), $R = 3 \times 10^6$ cm, $t \leq 10^8$ yr, and 100 per cent sticking efficiency, we have the lower limit $v_\infty \geq 4 \times 10^3$ cm/sec. Kuiper (1953) estimated $v_\infty = 2.2 \times 10^4$ cm/sec, using $t = 10^7$ yr and a higher value of ρ_a. Anders (1967, private communication) notes that a higher value of ρ_a may be preferable but that the model here is still applicable.

As a solution to the problem of velocity, it is assumed that at the time of formation of multikilometer, accreting bodies, typical approach velocities were of the order 4×10^3 to 2×10^4 cm/sec corresponding to eccentricities of 0.004 to 0.020 in the region of the asteroids. (We are not constrained to assume that accretion accounted for the earliest, smallest bodies. They may have grown by another process, e.g., condensation.)

The calculation based on equation (1) demonstrates that appreciable growth by accretion can occur under reasonable conditions and on a reasonable time scale. We will now consider the evolution of the accretion process itself.

As the growth proceeds beyond diameters of the order 10 km, the asteroid or planetesimal will develop an appreciable gravity field and there will be a dramatic increase in the effective cross-section. This may be seen through conservation of energy and angular momentum, giving the result

$$S^2 = R^2 \left(1 + \frac{8\pi G R^2 \rho}{3 v_\infty^2}\right), \tag{2}$$

where S is the effective capture radius of the planetesimal, R the geometric radius, ρ the planetesimal density, and v_∞ the unperturbed approach velocity at infinity. The important point is that when R reaches a certain critical value

$$R_{\text{crit}} = v_\infty \left(\frac{3}{8\pi G \rho}\right)^{1/2} = 7.05 \times 10^2 v_\infty \text{ cgs}, \tag{3}$$

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the cross-section will suddenly begin to increase as R^4, rather than as the geometric cross-section. In equation (3), the density is again assumed to be 3.6, following Anders (1965).

What happens at this point depends on whether accretion continues, i.e., whether mass loss or mass gain is the result of collisions. In spite of the fact that some kind of growth proceeded at least to this point (otherwise the asteroids would not exist at all), it is not obvious that accretion would continue now, because now the body has its own appreciable gravity, and the impact velocity,

$$v_I = \sqrt{\left(v_\infty^2 + \frac{2GM}{R}\right)} = \sqrt{\left(v_\infty^2 + \tfrac{8}{3}\pi G R^2 \rho\right)}, \quad (4)$$

also begins to increase more rapidly. Impact velocities will now exceed escape velocities, and there is no guarantee that ejecta velocities will not increase so as to produce mass loss with further impacts. All that is guaranteed is that at the critical radius, the flux of impacting particles seen by an observer on the planetesimal will begin to increase more rapidly than before.

If mass loss did begin to occur, the growth process would have been abruptly terminated at this radius. Could such a "steady-state" hypothesis explain the optimum radius of 3×10^6 cm found by Anders? That is, was the "optimum radius" simply the critical radius? Equation (3) in this case gives an approach velocity $v_\infty = 4 \times 10^3$ cm/sec, which agrees quite well with our expectations. The idea is consistent to this point, but it is difficult to show that mass loss would indeed have started under these conditions. In fact, two objections exist: (1) The impact velocity is considerably less than Öpik's crushing velocity of 1.9×10^4 cm/sec (although the figure would be less for a loosely accreted surface: the figures are 3.8×10^4 cm/sec for aluminum and 5.1×10^4 cm/sec for nickel-iron); the problem still requires experimental work. (2) For two bodies of radii $R = 3$ and 6×10^6 cm, escape velocities would be 4.3 and 8.5×10^3 cm/sec, respectively, and impact velocities only about 6.0 and 9.3×10^3 cm/sec, allowing very little excess kinetic energy for ejection and net loss of mass.

Therefore, it appears reasonable to assume that some other mechanism resulted in the optimum size found by Anders. Perhaps the growth process slowed markedly at this point because the accretable material in the region of the asteroids was nearly exhausted or because the solar nebula itself was beginning to dissipate. Equation (1), with the best estimates of parameters (indicated above: $v_\infty = 1 \times 10^4$ cm/sec, $\rho_a = 10^{-12}$ g/cm^3, and $R = 3 \times 10^6$ cm) gives $t = 1.1 \times 10^8$ yr, a reasonable estimate of the duration of the solar nebula and the growth process. Kuiper (1953) points out that the growth may, in addition, have continued after the dispersal of the gaseous nebula as the last particles spiralled in toward the Sun under the influence of the Poynting-Robertson effect.

If the growth process was terminated in this way, we might expect some characteristic size to have been reached by most of the bodies, which started growing at the same time. This may in part explain Anders' (1965) optimum size, $R = 3 \times 10^6$ cm. Whether Anders' reconstructed curve is correct *in detail* is irrelevant; the important point is simply that a peak in the mass distribution occurred, and survives to this day as the inflection point found by Kuiper *et al.* (1958). Had the growth been terminated sooner, the optimum size would have been smaller and no Ceres-size objects would have formed; had the growth not been terminated, Ceres-size objects would have grown rapidly at the expense of the others and one planet ($M \sim 10^{24}$ g) would result as happened in other regions of the solar system. It is difficult to find any other explanation of the optimum size (e.g, the "steady-state" explanation of two paragraphs above), since larger planetesimals have both greater capture cross-sections and greater stability.

Finally, we consider these largest asteroids in more detail. Among the bodies that grew larger than $R = 3 \times 10^6$ cm before the growth was impeded must have been some that

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had reached the critical radius R_{crit}. By use of our best estimate of the approach velocity, $v_\infty = 1 \times 10^4$ cm/sec, equation (3) gives

$$R_{crit} = 7 \times 10^6 \text{ cm} . \tag{5}$$

Escape velocity in this case is 1×10^4 cm/sec and impact velocity 1.4×10^4 cm/sec. For the same reasons as before, it does not appear that mass loss would set in here. Instead, since these large bodies now have a capture cross-section increasing as R^4 instead of R^2, growth is accelerated.

A second effect also acts soon to accelerate growth. At all stages of growth, there must have been a spread on velocities of particles striking the planetesimals. Fast particles striking with velocities of the order 10^5 cm/sec would cause mass loss in all planetesimals so far discussed, and thus act as a brake on the growth process. Furthermore, as the solar nebula dispersed, the effective mean approach velocity would have increased as orbital eccentricities increased, causing more high-velocity particles to appear. Present-day collision velocities among asteroids are estimated to be a few km/sec. Experiments by Gault, Shoemaker, and Moore (1963) with different projectiles impacting at velocities near 6 km/sec show that the fastest portion of the ejecta, with total mass just equal to the projectile mass, has a lower-velocity limit of about 2.5×10^4 cm/sec. Gault (1967, private communication) believes this result holds over a considerable range of impact velocities, and it is thought to be valid in the range we now consider. This is the escape velocity for a body of some critical radius, found to be

$$R'_{crit} = 1.76 \times 10^7 \text{ cm} . \tag{6}$$

Larger bodies would be stable against high-velocity collisions with particles, and would grow rapidly at the expense of the smaller bodies.

The two different critical radii given by equations (5) and (6), corresponding to diameters 140 and 350 km, fall close to the lower limit of the gap in the mass distribution, which is taken as observational confirmation that at least one of these effects was significant in the growth of the asteroids.

Therefore, it is concluded that the three largest asteroids, of diameters 785, 490, and 400 km, are overabundant with respect to other asteroids because (1) their accretion capture cross-sections grew large; and/or (2) they were able to gain mass in even the high-velocity collisions, in which their smaller companions lost mass. Having passed these two critical radii, they grew rapidly toward planetary dimensions only to have the process frozen at some stage by (possibly) insufficient or declining mass in the nebular cloud, with only the few largest bodies approaching even lunar size.

The importance of the mass loss can be seen from the following calculation. By reasoning similar to that used to derive equation (1), we have the rate of shrinking due to erosion.

$$\frac{dR}{dt} = \frac{v_I}{4} \frac{\rho_n'}{\rho} f , \tag{7}$$

where f is the factor: net mass lost/mass of projectile. The factor f can be estimated from the data of Gault *et al.* (1963, p. 37). For three present-day asteroids, $R = 100$ km, $R = 10$ km, and $R = 1$ km, f is estimated to be 4, 26, and 10^3, respectively, and dR/dt is found to be of the order 10^3 cm/10^9 yr in each case. In this calculation ρ_n' was taken to be the density of eroding debris (mass $< M/125$) now in the asteroid belt, estimated in another paper now in course of publication to be about 2×10^{-18}, 2×10^{-19}, and 2×10^{-20} g/cm^3 in the three respective cases.

This calculation suggests that the meteorite parent bodies and fragments may have been considerably eroded since their formation, if they escaped collisional fragmentation, and replenished by fragmentation of larger bodies. Such replenishing would explain the

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clustered low ages of certain meteories, such as the bronzite chondrites, recently confirmed by Tanenbaum (1967). It will be noted that the estimates of erosion rate made above are comparable to the "space erosion" rate for meteorites derived by Fisher (1966), but the agreement is fortuitous for the rates are not physically the same. Equation (7) refers to erosion of large bodies with significant gravity inside the present asteroid belt; hence the factor f, estimated from Gault's data, can be low and the flux of projectiles is high. Fisher's result refers to erosion of small meteorites after they have reached interplanetary space; Fisher's equivalent of f, estimated from Öpik's (1958) cratering theory, is high and the flux is low. Our estimate of the erosion rate within the asteroid belt is ten times an observed upper limit for the erosion rate experienced by the mesosiderite Patwar (Price, Rajan, and Tamhane 1967); however, this limit may refer to the interplanetary, not the interasteroidal, rate. The point of the present discussion is that much space erosion of an asteroidal meteorite may occur already within the asteroid belt or during a meteorite's orbital passages through the belt, in fact exceeding that for a similar particle in interplanetary space.

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