Geochemical Modeling of Mantle Differentiation and Crustal Growth

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A simple two-reservoir model with time-dependent transport coefficients between the reservoirs has been used to model the abundances of K, Ar, Rb, Sr, Sm, Nd, U, Th, and Pb and the isotopic compositions of Ar, Sr, Nd, and Pb in the earth's mantle and continental crust. The transport coefficients, like heat production, are considered to decay exponentially with time. Models which involve the whole mantle in generating the continental crust yield \( \frac{\text{Sr}}{\text{Sr}} \) and \( \frac{\text{Nd}}{\text{Nd}} \) ratios for the residual mantle which are higher than those of midocean ridge basalts, respectively. A model which involves only half of the mantle in the production of continental crust produces a residual mantle with isotope ratios similar to those of midocean ridge basalts. The \( \frac{\text{Ar}}{\text{Ar}} \) ratios in the atmosphere and suboceanic mantle are reproduced by this model without any inequality in the upward transport coefficients of K and Ar but with a smaller downward transport coefficient for Ar than for K. The results imply that the earth's crust may have developed by extraction of material from only half of the mantle and constrain the possible convective regimes that have existed in the mantle.

INTRODUCTION

The degree of chemical heterogeneity in any planetary object at the present day is a function of initial heterogeneity imposed by variations in the composition of the accreting material and of effects resulting from the subsequent thermal evolution of the object. The time interval during which thermally induced mass transport occurred in different planetary objects varies by more than an order of magnitude. For example, in the case of meteorite parent bodies the time interval was generally of the order of 10\(^4\) or 10\(^5\) years. In contrast, it can be demonstrated that mass transport has occurred in the earth from at least 3.8 Gy ago to the present day simply from the occurrence of volcanic rocks in the continental crust. It is to be anticipated that the most significant thermally induced chemical change to accompany the evolution of a planet will be the concentration of heat-producing elements in the outer portion of the planet, from where heat will be efficiently lost. In the case of the earth the continental crust is the prima facie chemical expression of this phenomenon. The timing and rates of supply of the constituents of the continental crust to the outer portion of the earth thus compose fundamental parameters of the earth's evolution. Some authors have proposed that these constituents have resided in the outermost part of the earth since close to the time of its formation, representing either materials supplied at the last stage of heterogeneous accretion or, alternatively, material efficiently differentiated from the remainder of the earth early in its history. Others, however, have suggested that the constituents of the crust have been supplied over a substantial portion of earth history such that the continents have grown continuously through time although not necessarily at a constant rate. Resolution of these opposing viewpoints in favor of the latter one has come about as a result of Rb-Sr, U-Pb, and, recently, Sm-Nd isotopic studies of Archaean (>2.5 Gy) rocks. For example, the oldest terrestrial rocks currently known are the Isua supracrustals in West Greenland, which have been precisely dated by U-Pb, Pb-Pb, and Sm-Nd methods at 3.8 Gy [Moorbath et al., 1973; Michael-Vitrae et al., 1977; Hamilton et al., 1978]. The inferred initial Sr, Nd, and Pb isotope compositions of these and other early Archaean rocks in West Greenland clearly demonstrate that they were not derived from preexisting sialic crust [Moorbath et al., 1973; Moorbath, 1975; Gancars and Wasserburg, 1977; Hamilton et al., 1978]. These investigations considered together with other isotopic studies of Archaean crust [e.g., DePaolo and Wasserburg, 1976a; Hamilton et al., 1979a; McCulloch and Wasserburg, 1978] have lent support to the contention of Hurley et al. [1962] that the continental crust has grown in a quasi-continuous manner and suggest that the maximum rate of development of crust was probably achieved around 3.0 Gy ago [e.g., O'Nions and Pankhurst, 1978; McCulloch and Wasserburg, 1978].

Related to questions of the generation and growth of the continental crust are those of the supply of volatiles to the earth's surface to form the atmosphere and hydrosphere. In this case it has been clear since Brown [1952] demonstrated the paucity of rare gases in the atmosphere that little of the present atmosphere can be residual from the time of earth formation. Furthermore, the abundance patterns of rare gases in mantle-derived materials and the atmosphere [Ozima and Alexander, 1976] and the \( \frac{\text{Ar}}{\text{Ar}} \) inventory of the earth [Turekian, 1959] are consistent with a secondary origin of the atmosphere by degassing. The transport rate of Ar to the atmosphere must have been particularly high in the first 2 or 3 \( \times 10^4 \) years of earth history in order to satisfy the constraint imposed by \( \frac{\text{Ar}}{\text{Ar}} \) ratios in the mantle and the atmosphere [Ozima, 1975]. The rapid and early accumulation of Ar in the atmosphere is in marked contrast to the delayed appearance of significant amounts of sialic crust until about 3.8 Gy ago and the apparent peak in the rate of its generation between about 3.5 and 2.5 Gy. These time-varying rates of transport of mantle-derived material to form the continental crust and atmosphere must reflect the thermal history of the earth and presumably any time-dependent changes in the nature of mantle convection. Comparatively little effort has been expended so far in discovering the constraints imposed by the chemical and isotopic composition of the differentiated components of the earth on the fundamental thermally driven processes which produced them.

Although it is generally agreed that at least the upper
mantle is in an actively convective state, there is no such consensus concerning the state of the lower mantle. McKenzie and Weiss [1975] and Richter [1978] consider that convective communication does not occur between the upper and lower mantle. Others, for example, Peltier [1972] and O'Connell [1977], favor whole mantle convection. These different views of mantle convection have important implications for the generation of the continental crust. The persistence of whole mantle convection throughout a major portion of earth history should be reflected in the comparative compositions of the continental crust and residual mantle. Conversely, if only the upper mantle had been available to supply material to the continental crust and atmosphere, then the bulk composition of the lower mantle would remain unaffected, and the upper mantle would be more depleted in crustal constituents than if the whole mantle had been available. In principle, it is possible to distinguish between these possibilities by modeling the abundances of daughter products of parent isotopes which have half-lives with a similar order of magnitude to the age of the earth. At the present time such models are logically based upon the daughter products of $^{40}$K, $^{87}$Rb, $^{147}$Sm, $^{232}$Th, $^{235}$U, and $^{238}$U, which are $^{40}$Ar, $^{87}$Sr, $^{143}$Nd, $^{206}$Pb, $^{207}$Pb, and $^{208}$Pb, respectively, because considerable amounts of data on the distribution of these isotopes within the earth are now available.

In this paper a simple model of mantle differentiation is presented and investigated numerically. The models are constrained by the ratios $^{40}$Ar/$^{36}$Ar, $^{87}$Sr/$^{86}$Sr, $^{143}$Nd/$^{144}$Nd, $^{206}$Pb/$^{204}$Pb, $^{207}$Pb/$^{204}$Pb, and $^{208}$Pb/$^{204}$Pb in the crust/atmosphere and the mantle and by the time of formation of continental crust as estimated from geochronological investigations of Archaean rocks. It is envisaged that material transport occurs between a specified volume of mantle and an outer 50-km layer of the solid earth which contains all of the continental and oceanic crust, and, for convenience, the Ar contained in the atmosphere. Models of this general type have been investigated previously to explain the Pb isotope [Russell, 1972; Russell and Birnie, 1974] and the Sr and Pb isotope [Armstrong, 1968; Armstrong and Hein, 1973] evolution of the earth. The present study differs in several important respects from these earlier investigations and reflects the enhanced data base now available for such exercises and our improved understanding of continental growth. (Subsequent to the submission of this paper it was learned that DePaolo [1978] has made some calculations of this type as part of his Ph.D. thesis dissertation at California Institute of Technology.)

THE TWO-RESERVOIR MODEL

It is assumed that the earth formed 4.55 Gy ago with a chemically homogeneous silicate portion. Subsequent to that time the silicate portion is considered to consist of two parts: an outer layer (L) 50 km thick and a mantle (M) layer. The chemical composition of the 50-km layer changes through time in response to the preferential transport of heat-producing and related elements from the mantle into this layer. At the present day the 50-km layer includes all of the continental and oceanic crust, and the largest proportion of the inventory of heat-producing elements is contained in the continental crust. However, the models considered here view the 50-km layer (L) and the mantle (M) as chemically homogeneous entities.

The rate of change in the number of moles of some elemental component $i$ in the 50-km layer ($n_i^L$) depends upon the number of moles of $i$ in the mantle, $n_i^M$, and the coefficients for upward ($\alpha$) and downward ($\beta$) transport according to

$$ \frac{dn_i^L}{dt} = \alpha(t) \cdot n_i^M - \beta(t) \cdot n_i^L $$

(1)

After a time $T$ the total number of moles of $i$, $n_i^L$, will be

$$ n_i^L = \int_{t_0}^{t_0+T} \alpha(t) \cdot n_i^M \cdot dt - \int_{t_0}^{t_0+T} \beta(t) \cdot n_i^L \cdot dt $$

(2)

In the case of isotopic components where radioactive decay must be considered, such as the decay of $^{87}$Rb to $^{87}$Sr, the equation is of the form

$$ \frac{d^{87}\text{Sr}}{dt} = \alpha(t)^{87}\text{Sr}^M - \beta(t)^{87}\text{Sr}^L + \lambda^7\text{Rb}^L $$

The transport coefficients $\alpha$ and $\beta$ are considered to decay exponentially with time (as does heat production in the earth) with the respective time constants $\tau_\alpha$ and $\tau_\beta$, where

$$ \alpha = \alpha_0 e^{-t/\tau_\alpha} \quad \beta = \beta_0 e^{-t/\tau_\beta} $$

(3)

In these expressions the earth commences its evolution 4.55 Gy ago at $t = 0$. The preexponential terms $\alpha_0$ and $\beta_0$ have different values for each element considered but are identical for all the isotopes of an element. In the models investigated here, $\alpha_0$ values for each element are adjusted to produce a 50-km layer of the required bulk composition after 4.55 Gy; however, $\beta_0$ values are equated for all elements considered except Ar.

Expression (2) has been solved numerically for K, Rb, Sr, Sm, Nd, U, Th, Pb, and Ar using increments of 50 m.y. to approximate the integrals. At the end of each increment in time, new isotopic compositions of Ar, Sr, Nd, and Pb are calculated for each reservoir before and after transport is effected. The present-day $^{40}$Ar/$^{36}$Ar, $^{87}$Sr/$^{86}$Sr, $^{143}$Nd/$^{144}$Nd, $^{206}$Pb/$^{204}$Pb, $^{207}$Pb/$^{204}$Pb, and $^{208}$Pb/$^{204}$Pb ratios computed in this way have thus evolved via a 90-stage model.

In comparison with previous attempts to model the isotopic evolution of Pb [Russell, 1972; Russell and Birnie, 1974], Pb and Sr [Armstrong, 1968; Armstrong and Hein, 1973], and Sr and Nd [Richter and Ribe, 1979], the present model differs in some important respects. First, all of the above mentioned models envisage exchange occurring between the mantle and a crustal layer of fixed volume and composition. The implication of this approach is that the crust formed early in earth history by rapid differentiation. This assumption is clearly at variance with geochemical observations. Second, the above models generated an isotopically inhomogeneous upper mantle, whereas the present model generates a single isotopic composition for each of the two reservoirs considered.

BOUNDARY CONDITIONS FOR MODEL

In order to fit the simple model described above to the broad geochemical features of the continental crust and mantle a number of boundary conditions must be specified. Ideally, these should include both the initial and present-day chemical states of the earth. The initial chemical state comprises the abundances of K, Ar, Rb, Sr, Sm, Nd, U, Th, and Pb and the isotopic compositions of Ar, Sr, Nd, and Pb in the undifferentiated mantle or bulk earth 4.55 Gy ago. Specification of the present-day chemical state of the earth requires a knowledge of the appropriate elemental abundances in the outer 50-km layer and in the residual mantle which contributed material to this layer. The initial and final thermal states of the earth do not enter into the model explicitly and need not be specified. However, because the thermal state will be closely tied to the nature of mantle convection, it can be ex-
TABLE 1. Abundances of Trace Elements in the Bulk Earth (Crust and Mantle)

<table>
<thead>
<tr>
<th>Element</th>
<th>Abundance, ppm</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>200</td>
</tr>
<tr>
<td>Rb</td>
<td>0.67</td>
</tr>
<tr>
<td>Sr</td>
<td>21.0</td>
</tr>
<tr>
<td>U</td>
<td>0.020</td>
</tr>
<tr>
<td>Th</td>
<td>0.08</td>
</tr>
<tr>
<td>Pb[^{+}]</td>
<td>0.10</td>
</tr>
<tr>
<td>Sm</td>
<td>0.32</td>
</tr>
<tr>
<td>Nd</td>
<td>0.97</td>
</tr>
<tr>
<td>(^{38})Ar, cm(^{3}) STP g(^{-1})</td>
<td>4.6 \times 10(^{-8})</td>
</tr>
<tr>
<td>Rb/Sr</td>
<td>0.03</td>
</tr>
<tr>
<td>K/U</td>
<td>10(^{2})</td>
</tr>
<tr>
<td>K/Rb</td>
<td>300</td>
</tr>
<tr>
<td>Sm/Nd</td>
<td>0.330</td>
</tr>
<tr>
<td>Th/U</td>
<td>4</td>
</tr>
<tr>
<td>Heat production</td>
<td>4.8 pW kg(^{-1})</td>
</tr>
</tbody>
</table>

*Abundances assuming that earth has the same relative abundances of Ca, U, Th, Sm, and Nd as carbonaceous chondrites; all ratios are by weight [after O'Nions et al., 1979].

\[^{+}\]Pb\[^{+}\] denotes primordial lead.

...expected that the magnitude of \(\tau_a\) and \(\tau_p\) will in some way reflect the difference between the initial and final thermal states. Since the initial thermal state of the earth is a topic surrounded by considerable speculation, no attempt is made to fix the values of \(\tau_a\) and \(\tau_p\) a priori.

**Initial Chemical State**

The first step toward defining the initial chemical state is to specify the ratios K/U, U/Pb, Th/U, Rb/Sr, and Sm/Nd in the bulk earth. The K/U weight ratio was estimated at 10\(^{2}\) by Wasserburg et al. [1964], and a review of more recently published data by O'Nions et al. [1978a] has shown that this value is not in need of revision as far as the crust plus upper mantle is concerned. The U/Pb and Th/U ratios of the bulk Earth (Table 1) can be constrained by the Pb isotope composition of mantle and crustal materials (Table 1), although some uncertainty still surrounds the significance of the different U/Pb ratios required to produce the Pb isotope compositions of formable lead ore deposits, oceanic volcanics, and Archean gneisses [Cumming and Richards, 1975; Stacey and Kramers, 1975; Ganczar and Wasserburg, 1977]. The Sm/Nd ratio of the bulk earth is known precisely from the initial \(^{143}\)Nd/\(^{144}\)Nd ratios of Archean volcanics and metavolcanics [DePaolo and Wasserburg, 1976a; Hamilton et al., 1977, 1978, 1979a, b] and is within error of the cosmic abundance ratio [Evensen et al., 1978]. This latter observation, coupled with the demonstrated inverse correlation between \(^{143}\)Nd/\(^{144}\)Nd and \(^{87}\)Sr/\(^{86}\)Sr in oceanic volcanics, has led to an improved estimate of the bulk earth Rb/Sr ratio of 0.03 [DePaolo and Wasserburg, 1976b; O'Nions et al., 1977].

The second step is to establish the absolute abundances of these elements in the bulk earth, and this requires that some additional assumptions be made. On the one hand the heat-producing element abundances have been obtained from the mean global heat flow by assuming that the heat lost at the surface is equal to the heat generated within the earth [e.g., Tera et al., 1974; Langseth et al., 1976; O'Nions et al., 1978a], with abundances of other elements of interest derived using the assumed values for Sr/U, Rb/Sr, K/U, etc. in the bulk earth. On the other hand, if it is assumed, or can be shown that those elements predicted to have condensed at high temperatures from the solar nebula [Larimer, 1967; Grossman, 1972] are fractionated coherently into the earth, then their abundances can be calculated if a value for one of them, such as Ca or Al, can be fixed. It has been amply demonstrated that this group of elements fractionates coherently between different meteorites [e.g., Ganapathy and Anders, 1974; Grossman et al., 1977], and it is probably a valid assumption for the earth [O'Nions et al., 1979]. This latter model yields abundances for the bulk earth which are about a factor of 2 lower than those derived from a model based on heat flow considerations (Table 1) and, if they are accepted as the correct ones, imply that heat is lost from the earth with a significant time constant. This result was further analyzed by O'Nions et al. [1979], who showed that the time constant must exceed 1 Gy. In this respect it is interesting to compare these conclusions with those of Daly and Richter [1978] and Sharpe and Petter [1978], all of whom have proposed on the basis of geophysical arguments that long time constants are associated with heat loss from the earth. The abundances in Table 1 are the preferred ones and form the initial chemical state for all models examined in this study. It is noteworthy that these abundances are very little different from those derived for the bulk earth by Smith [1977], on the basis of a present-day mantle model derived from garnet peridotite analyses, a meteorite analog model of the core, and a preferred model of the continental crust. Smith [1977] also noted that the heat-producing element abundances he adopted were incompatible with simple thermal models of the earth. The initial isotopic compositions of Sr, Nd, and Pb in the earth are the values derived for meteorites approximately 4.55 Gy ago (Table 2), and the initial isotopic composition of Ar is estimated from nucleosynthetic considerations [Cameron, 1968].

**Present-Day Chemical State**

The final chemical state is specified from the composition of the continental crust, the oceanic crust, the atmosphere, and the residual mantle from which the constituents of the continental crust and atmosphere were derived. The continental crust comprises about 30% by mass of the outer 50 km layer but contains a much larger percentage of the inventories of the elements of interest here. Various models for the chemical composition of the crust have been suggested on the basis of, for example, the average composition of Precambrian shield areas [Poldervaart, 1955; Gast, 1960; Taylor, 1964; Holland and Lambert, 1973; Shaw, 1972; Heier, 1973a, b; Turner, 1976] and also on the results of seismic reflection data combined with analyses of the expected lithologies [Smithson, 1968]. The major uncertainty in the trace element abundances of interest here, and particularly the heat-producing element abundances, arise from an inadequate knowledge of the composition of the lower crust. Both Heier [1963, 1973a] and Gast [1972] have presented crustal models based upon a lower crust composed of granulite, which have overall lower abundances of heat-producing elements than some other models. Further insight into the problem can be gained from heat flow considerations. If the mantle contribution to the surface heat flow in

TABLE 2. Initial Isotope Compositions (4.55 Gy Ago)

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{87})Sr/(^{86})Sr</td>
<td>0.69988</td>
<td>Papanastassiou and Wasserburg [1969]</td>
</tr>
<tr>
<td>(^{143})Nd/(^{144})Nd</td>
<td>0.50682</td>
<td>Lugmair and Morii [1977]</td>
</tr>
<tr>
<td>(^{209})Pb/(^{208})Pb</td>
<td>29.476</td>
<td>Tatsumoto et al. [1973]</td>
</tr>
<tr>
<td>(^{208})Pb/(^{206})Pb</td>
<td>10.294</td>
<td>Tatsumoto et al. [1973]</td>
</tr>
<tr>
<td>(^{40})Ar/(^{36})Ar</td>
<td>9(^{-4})</td>
<td>Cameron [1968]</td>
</tr>
</tbody>
</table>
TABLE 3. Continental Crust and Model 50-km Layer

<table>
<thead>
<tr>
<th>Continental</th>
<th>Crust Model*</th>
<th>Estimate of Composition†</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>12,400</td>
<td>3800</td>
</tr>
<tr>
<td>Rb</td>
<td>50</td>
<td>15.3</td>
</tr>
<tr>
<td>Sr</td>
<td>400</td>
<td>122</td>
</tr>
<tr>
<td>Sm</td>
<td>3.7</td>
<td>1.13</td>
</tr>
<tr>
<td>Nd</td>
<td>16.0</td>
<td>4.90</td>
</tr>
<tr>
<td>U</td>
<td>1.0</td>
<td>0.31</td>
</tr>
<tr>
<td>Th</td>
<td>2.5</td>
<td>0.77</td>
</tr>
<tr>
<td>Pb* ‡</td>
<td>5.0</td>
<td>1.5</td>
</tr>
<tr>
<td>*Sr/56Sr</td>
<td>...</td>
<td>0.7120</td>
</tr>
</tbody>
</table>

*Continental crust model of Taylor [1977, 1979]. †Estimate of present composition of 50-km layer which excludes ocean floor sediment. Inclusion is unlikely to increase K by more than 200 ppm. ‡ Pb* denotes primordial lead.

The continents is zero \( (q_m = 0) \), then the crust should contain approximately 1.7% K if it is assumed that crustal K/U and Th/U are the same as the bulk earth values (Table 1). However, if \( q_m = 28 \text{ mW m}^{-2} \) as suggested by Pollack and Chapman [1977], then the K content will be only 0.96%. Taylor’s [1977] recent model for the composition of the crust gives a K content of 1.2%, which is sufficiently close to the latter estimate to encourage us to use Taylor’s trace element abundances in the models that follow (Table 3). The composition of the 50-km layer (Table 3) is dominated by the continental crust component, so uncertainties in the composition of the oceanic crust are comparatively negligible.

The present-day \( \text{Sr}/\text{Sr} \) ratio of the 50-km layer is used to limit the possible values for \( \tau_d \) and \( \tau_p \). For a given Rb/Sr ratio the average \( \text{Sr}/\text{Sr} \) ratio of the continental crust can be estimated if the mean age of the crust is known. For example, using Rb/Sr = 0.13, Taylor’s [1977] value (and also the value recommended by Gast [1972] for a model including a granulite lower crust), and a mean age of 2.5 Gy, the \( \text{Sr}/\text{Sr} \) ratio is 0.7130. Alternatively, an estimate can be made from the average \( \text{Sr}/\text{Sr} \) ratio in continental runoff, although it is not obvious that this should be a representative sample of the crust. One estimate of \( \text{Sr}/\text{Sr} \) in continental runoff comes from the isotopic composition of seawater, since Sr has an oceanic residence time of about 106 years and does not appear to have a large component derived from the ocean floor [Brass and Turekian, 1977]. Seawater has an average \( \text{Sr}/\text{Sr} \) ratio of 0.7091, which for a Rb/Sr ratio of 0.13 gives a mean crustal age of 1.9 Gy. At the present time it is not possible to distinguish rationally between these possibilities: a value of 0.7120 is adopted here for the whole 50-km layer with the understanding that it may well be slightly on the high side. The trace element abundances in the residual mantle are not used as boundary conditions in the models investigated here, and no attempt is made to estimate them at this stage. However, the Nd and Sr isotope compositions of midocean ridge and ocean floor basalts are used as indicators of the isotopic compositions in the suboceanic upper mantle. Basalts generated at midocean ridges are likely to provide a good estimate simply because intraplate volcanics are of trivial volume in comparison. Nd and Sr isotope investigations of ocean floor basalts [Richard et al., 1976; DePaolo and Wasserburg, 1976b; O’Nions et al., 1977] suggest that the average \( ^{143}\text{Nd}/^{144}\text{Nd} \) ratio should be about 0.5130 and the \( \text{Sr}/\text{Sr} \) ratio about 0.7028. The Nd and Sr isotope composition of subcontinental mantle can in principle be estimated from basalts erupted in continental environments and ultramafic nodules transported from the subcontinental upper mantle to the surface in kimberlites and alkali basalts. The major difficulty in using continental volcanics to infer properties of underlying mantle arises from the possibility that they have reacted with the continental crust through which they have been erupted. This possibility is often difficult to assess, but in the case of Tertiary basalts of northwest Scotland, Carter et al. [1978] have demonstrated that some of the basalts were contaminated by Precambrian basement to an extent which obscures their original isotope compositions. Although in other instances [DePaolo and Wasserburg, 1976b; Carter et al., 1978, 1979] continental flood and plateau basalts show no evidence of crustal contamination, the extent to which subcontinental mantle can be considered to have been enriched as proposed by Brooks et al. [1976] and Brooks and Hart [1978] is uncertain. Very few ultramafic nodules have been analyzed for Rb-Sr and Sm-Nd to date, but those that have [O’Nions et al., 1978b; Basu and Tatsumoto, 1979; Allegre and Shimizu, 1978] indicate that depleted mantle akin to suboceanic mantle is present beneath southern Africa at depths of \( \leq 200 \text{ km} \). The kimberlite hosts of these nodules, however, appear to have

![Fig. 1. Influence of the time constant for downward transport \( \tau_d \) on the amount of K in the 50-km layer. The time constant for upward transport \( \tau_u \) is fixed at 2.0 Gy, and the \( a_0 \) parameter for potassium is \( 7.5 \times 10^{-13} \text{ yr}^{-1} \). A value of \( \tau_u \) of 1.2 Gy generates a 50-km layer with the appropriate K content in 4.35 Gy, whereas a smaller time constant (e.g., \( \tau_u = 0.5 \text{ Gy} \)) generates a layer with more than double the required amount.](image1)

![Fig. 2. Model I: whole mantle. Variation of the \( \text{Sr}/\text{Sr} \) ratio of the 50-km layer as a function of \( \tau_d \) and \( a_0(K) \), where \( \tau_d \) is the time constant for upward transport and \( a_0(K) \) is the coefficient for potassium. The curves drawn are the loci of all solutions for given values of \( \tau_d \) (in gigayears), which are indicated in parentheses. It is assumed that the 50-km layer has a present-day \( \text{Sr}/\text{Sr} \) ratio of 0.7120 (see text), and this constraint is used to limit the possible solutions.](image2)
been generated from undepleted areas beneath the continents [Banu and Tatsumoto, 1979]. Thus the isotopic character of subcontinental mantle cannot be clearly defined at present except to say that it certainly contains depleted and undepleted parts but in unknown proportions.

The present-day $^{40}\text{Ar}/^{36}\text{Ar}$ of the mantle is a matter of some debate. Mantle-derived materials possess a large range of $^{40}\text{Ar}/^{36}\text{Ar}$ ratios from close to the atmospheric value of about 296 to about 10,000 [e.g., Fisher, 1975; Dymond and Hogan, 1973; Saito et al., 1978; Thompson et al., 1978; Hart et al., 1979]. The highest $^{40}\text{Ar}/^{36}\text{Ar}$ ratios are recorded from deep sea basalt glasses and clearly indicate the existence of Ar with high $^{40}\text{Ar}/^{36}\text{Ar}$ in the suboceanic upper mantle.

**RESULTS**

Values of the transport coefficients $\alpha(t)$ and $\beta(t)$ for any element under consideration may change according to the values assigned to the model parameters $\alpha_0$, $\beta_0$, $\tau_m$, and $\tau_p$. However, in the model evaluated here, variations in these parameters are restricted as follows:

1. The value of $\tau_m$ is identical for each element.
2. The value for $\tau_p$ is similarly identical for each element.
3. The values of $\beta_0$ are identical for each element except Ar and are assigned a value of $2 \times 10^{-4}$ yr$^{-1}$. This identity corresponds to the bulk return of material in the 50-km layer.

<table>
<thead>
<tr>
<th>Table 4. Model II: Parameters and Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>$K$</td>
</tr>
<tr>
<td>$\text{Rb}$</td>
</tr>
<tr>
<td>$\text{Sr}$</td>
</tr>
<tr>
<td>$\text{Sm}$</td>
</tr>
<tr>
<td>$\text{Nd}$</td>
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<td>$\text{U}$</td>
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<td>$\text{Th}$</td>
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<td>$\text{Pb}$</td>
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<td>$\text{He}$</td>
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<td>$\text{Rb/86Sr}$</td>
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<td>$\text{Sm/Nd}$</td>
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<tr>
<td>$\text{87Sr/86Sr}$</td>
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<tr>
<td>$\text{143Nd/144Nd}$</td>
</tr>
<tr>
<td>$\text{40Ar/36Ar}$</td>
</tr>
<tr>
<td>Heat production</td>
</tr>
</tbody>
</table>

*Starting bulk earth composition is given in Table 1, and initial isotope compositions in Table 2.
†The $\alpha_0$ values for each element were adjusted such that the 50-km layer has the present-day composition given in Table 3, third column. For model II, $\tau_m = 2.0$ Gy, and $\tau_p = 0.65$ Gy.
‡See Table 5.
§Weight ratios.
results in a 50-km layer with twice the required K content at the present day.

The proportion of the mantle that has contributed material to the outer layer and chemically interacted with it is not known. Consequently, models involving the whole mantle and 0.5 of the mantle by mass have been investigated.

**Model I: Whole Mantle**

Possible combinations of the parameters $\tau_m$, $\alpha_0$, and $\tau_p$ have been selected to satisfy the adopted present-day elemental abundances for the 50-km layer (Table 3) and a $^{87}$Sr/$^{86}$Sr ratio for the layer of 0.7120. The approach used is illustrated in Figure 2, where $\alpha_0(K)$ is compared with the $^{87}$Sr/$^{86}$Sr ratio of the 50-km layer. It should be noted that the ratios of $\alpha_0(K)$ to $\alpha_0$ values for the other elements are similar to the corresponding ratios of bulk distribution coefficients between garnet peridotite and an equilibrium partial melt for the elements concerned (see, for example, Gast [1968]). For example, solutions for which $\tau_m = 2.0$ Gy generate $^{87}$Sr/$^{86}$Sr ratios for the 50-km layer which are all below the adopted model value (Table 3) if $\alpha_0(K)$ is greater than $10^3$ yr$^{-1}$. Similarly, at values of $\alpha_0(K)$ less than $10^3$ yr$^{-1}$, $^{87}$Sr/$^{86}$Sr exceeds the model value. The $^{87}$Sr/$^{86}$Sr ratio of the 50-km layer is used to limit the possible solutions; thus when $\tau_m = 2.0$ Gy, $\tau_p$ must equal 0.7 Gy in order to generate a satisfactory solution. Computations performed for $\tau_m = 1.5$ and 1.0 Gy are also shown in Figure 2 and demonstrate that generally, larger values of $\alpha_0(K)$ are required for smaller values of $\tau_m$ in order to satisfy the model $^{87}$Sr/$^{86}$Sr ratio.

From Figure 2 it is apparent that solutions are possible corresponding to different various values of $\tau_m$; however, they do involve different rates of accumulation of K (and the other elements) in the outer layer. It can be expected that the rate of increase of K in the 50-km layer will be related in a general way to the rate of increase of continental crust and thus crustal growth. Consequently, a comparison of these results with the available constraints on crustal growth rates as reviewed earlier in this paper should serve to limit the range of possible solutions. In Figure 3 the growth of the K content of the 50-km layer for the solutions obtained in Figure 2 is illustrated as a function of time. If it is accepted as a first approximation that the proportion of K in the 50-km layer at a given time parallels the proportion of continental crust formed by that time, then not all of the solutions in Figure 3 are compatible with our current knowledge of crustal growth. The solution using $\tau_m = 1.0$ Gy would predict that 60% of the continental crust was produced by 4.0 Gy ago and that 90% was produced by 3.0 Gy ago. This result is incompatible with the established radiometric age patterns in the continents. The remaining three solutions in Figure 3 are more compatible with this estimate. The derivatives of the cumulative curves plotted in Figure 3 are shown in Figure 4, where $dN_K/dt$ (the rate of change of the potassium content of the 50-km layer) is plotted versus time. The models computed for $\tau_m = 1.0$, 1.5, and 2.0 Gy each have their maximum values for $dN_K/dt$ prior to 2.5 Gy ago, and in the case of $\tau_m = 1.0$ Gy the maximum occurs prior to 4.0 Gy. In the model where $\tau_m = 3.0$ Gy, $dN_K/dt$ does not reach a maximum value until about 1.5 Gy ago, and the present-day value is 90% of this maximum value. Whereas the model using $\tau_m = 1.0$ Gy is clearly untenable and that employing $\tau_m = 3.0$ Gy unlikely, the models employing $\tau_m = 2.0$ and 1.5 Gy are both compatible with the constraints that can be imposed from our understanding of crustal growth rates. A choice of $\tau_m = 2.0$ Gy has been made as the best solution; however, it should be appreciated that had a slightly smaller or larger value been selected, the following results would be essentially unchanged.

For the solution with $\tau_m = 2.0$ Gy the time-dependent changes in $^{87}$Sr/$^{86}$Sr of the residual mantle, the 50-km layer, and the bulk earth are compared in Figure 5. At the present day the bulk earth $^{87}$Sr/$^{86}$Sr ratio is ~0.705, whereas the residual mantle has a $^{87}$Sr/$^{86}$Sr ratio of 0.7043. The residual mantle and bulk earth possessed virtually indistinguishable $^{87}$Sr/$^{86}$Sr ratios until about 2.5 Gy ago. The model parameters were adjusted such that the $^{87}$Sr/$^{86}$Sr ratio of the 50-km layer equals 0.7120 at the present day, but in contrast to the residual mantle the $^{87}$Sr/$^{86}$Sr ratio of this layer departs significantly from that of the bulk earth 4.0 Gy ago. The $^{87}$Sr/$^{86}$Sr ratio of the residual mantle is compared with the measured range of $^{87}$Sr/$^{86}$Sr ratios in midocean ridge basalt (MORB) in Figure 5, from which it is evident that the source regions of midocean ridge basalts have much lower $^{87}$Sr/$^{86}$Sr ratios than the model residual mantle. If MORB are characteristic of the convecting
upper mantle, then the results of this model imply that the remainder of the mantle must be very much less depleted.

Model II: 0.5 Mantle

In addition to model I above, a model involving only 0.5 of the mantle by mass has been investigated (model II). As far as the geochemical modeling is concerned, the precise location of the half of the mantle which contributed to crustal growth is irrelevant. Only the results obtained using \( t_{A} = 2.0 \text{ Gy} \) are discussed here, and the particular values of \( \tau_{e} \) and \( a_{K}(K) \) were selected by using the constraint that the outer 50-km layer has \( ^{87}\text{Sr}/^ {86}\text{Sr} = 0.7120 \) at the present day in a manner similar to that detailed above for model I (Table 4).

The derivative \( dN_{K}/dt \) for this model is shown in Figure 6 and reveals that the maximum rate of increase of K in the 50-km layer occurs between 3.5 and 2.5 Gy and that the present value of \( dN_{K}/dt \) is only about 20% of its maximum value. Given a simple relationship between \( dN_{K}/dt \) and crustal growth, it can be expected from this model that the maximum crustal growth rate would occur between 3.5 and 2.5 Gy with a fairly rapid decline subsequent to about 2.5 Gy ago.

The evolution of \( ^{87}\text{Sr}/^ {86}\text{Sr} \) in the residual mantle is compared with that in the bulk earth and the 50-km layer in Figure 7. The \( ^{87}\text{Sr}/^ {86}\text{Sr} \) ratio of the residual mantle is 0.7029, which is within the observed range for MORB and considerably lower than the ratio obtained from the whole mantle model. It is also significant that the residual mantle and bulk earth in this model have distinguishable \( ^{87}\text{Sr}/^ {86}\text{Sr} \) ratios by 3.5 Gy ago.

Argon Isotope Evolution of Earth

It is well established that \( ^{36}\text{Ar} \) must have been very efficiently released into the atmosphere early in earth history [French, 1971; Ozima, 1975]. In marked contrast, the release of nongaseous constituents to the continental crust, such as K, U, and Th, appears to have been much slower, and very little continental crust accumulated prior to 3.8 Gy ago.

The results of models investigated here demonstrate that these apparently conflicting observations can be readily reconciled. In both model I and model II, \( a(K) = a(\text{Ar}) \), which means that the upward transport coefficients of K and Ar from the mantle to the outer 50-km layer are identical at all times. However, by setting \( \beta_{e}(\text{Ar}) = 0.03\beta_{e}(K) \) the downward transport coefficient of Ar is always 3% of that for K (and the other elements), which implies the bulk recycling of material in the outer layer back into the mantle with the exception of Ar, which is assumed to be readily outgassed as material is added to the layer.

For model II the accumulations of \( ^{36}\text{Ar} \) and K in the 50-km layer are compared in Figure 8. The model yields a present-day \( ^{40}\text{Ar}/^{36}\text{Ar} \) ratio in the outer layer of 300, which is close to the generally accepted value of 296 for the earth’s atmosphere. Whereas less than 20% of the present K content of the 50-km layer has accumulated by 4.0 Gy ago in this model, some 65% of the \( ^{36}\text{Ar} \) was released at this time (Figure 8). The effect of the early rapid accumulation of \( ^{36}\text{Ar} \) at the earth’s surface is to produce a rapid change in the \( ^{40}\text{Ar}/^{36}\text{Ar} \) ratio of the residual mantle (Figure 9). The earth is assumed to have been formed with an \( ^{40}\text{Ar}/^{36}\text{Ar} \) ratio of \( 10^{-4} \) (Table 1), which rapidly increases in the residual mantle, reaching a value of 1400 at the present day not dissimilar to that characterizing the sub-oceanic upper mantle. It is noteworthy that the Ar isotope characteristics of the 50-km layer and residual mantle are generated without ever introducing an inequality between the upward transport coefficients for K and Ar from the mantle, although for the particular parameters chosen, no more than 80% of the total \( ^{40}\text{Ar} \) inventory of the 50-km layer is satisfied. No further adjustment of parameters is made at this stage to redress this deficit. Thus the release of Ar from the earth’s mantle may have been no more catastrophic than the release of K, and the difference in their rates of accumulation in the outer parts of the earth may reflect the less efficient return of Ar to the mantle.

Comparative Isotopic Evolution of Sr, Nd, and Pb

The results obtained for model II demonstrate that if half the earth’s mantle had an \( ^{87}\text{Sr}/^ {86}\text{Sr} \) ratio equal to that of MORB, then the remainder of the mantle could remain essentially unmodified (Figure 7). The \( ^{87}\text{Sr}/^ {86}\text{Sr} \) ratio of the residual mantle is predicted to depart significantly from the bulk earth value from about 3.5 Gy ago, such that by 2.7 Gy ago the model residual mantle value is 0.7012. This ratio is very similar to the average value obtained by Hart and Brooks [1977] for Archaean mantle from a consideration of 2.7-Gy-old volcanics. However, the demonstration of a \( ^{87}\text{Sr}/^ {86}\text{Sr} \) ratio of 0.7012 for the mantle 2.7 Gy ago was used as the pivotal point in Hart and Brook’s [1977] argument for an initial period of chondritic Rb/Sr in the earth, subsequent to which Rb was removed from the mantle either by volatilization or by
Fig. 10. Model II: 0.5 mantle. Comparison of the parameters $\Delta$Nd and $\Delta$Sr for the residual mantle and 50-km layer as a function of time. Note that the $\Delta$Sr scale is 10 times the $\Delta$Nd scale. $\Delta$Nd is defined as $[(^{143}Nd/^{144}Nd)_{Eo} - (^{143}Sm/^{144}Nd)_{Ea}]/(^{147}Sm/^{144}Nd)_{Ea}$ and is expressed here in per mil. The subscript $Eo$ refers to the bulk earth value at the appropriate time, and $Ea$ refers to a single-stage value computed from the model II $^{143}$Nd/$^{144}$Nd ratios and the initial $^{143}$Nd/$^{144}$Nd ratio for the bulk earth 4.55 Gy ago (Table 2). Numbers in parentheses are times in gigayears.

partition into the core. The results obtained for model II suggest that the Rb/Sr ratio of the mantle has indeed been reduced but that the Rb removed may now reside in the continental crust.

The Nd and Sr isotope evolution of the 50-km and residual mantle are compared in Figure 10. The major point of contrast is that the $^{143}$Nd/$^{144}$Nd ratio of the residual mantle as reflected in the value of the parameter $\Delta$Nd does not deviate significantly from the bulk earth until the Proterozoic (<2.5 Gy). This result is consistent with the observation that the initial $^{143}$Nd/$^{144}$Nd ratios of Archaean volcanics and orthogneisses are consistent with a source region which evolved with a chondritic Sm/Nd ratio. The present-day $^{143}$Nd/$^{144}$Nd ratio of the residual mantle is 0.5129, close to the average value of ~0.5130 for midocean ridge basalt, and that of the 50-km layer is 0.5120.

The Pb isotope ratios resulting from the fractionation of U/Pb between the two reservoirs are given in Table 5. For the model parameters used, the 50-km layer has a more radiogenic lead isotope composition than the residual mantle. A line joining these two lead isotope compositions in a conventional $^{207}$Pb/$^{204}$Pb versus $^{206}$Pb/$^{204}$Pb diagram has a slope similar to the trend of recent oceanic lead isotope data [e.g., Tatsumoto, 1978], although in detail the model does not satisfy the data. The main deficiency of the model is its failure to generate lead isotope compositions in the residual mantle which are more radiogenic than the assumed bulk earth (Table 5). Clearly, an additional reservoir must be invoked with less radiogenic lead than that of the bulk earth. A detailed discussion of possible additional reservoirs which would significantly affect the Pb isotopic evolution of the earth yet have less influence on the Sr and Nd isotopic evolution is beyond the scope of the present discussion; however, there are two prime candidates. One of these is the earth’s core, and the other would require an outer layer that contains two reservoirs with distinct Pb isotope evolution. The latter possibility appears to be the more likely in view of the evidence for the depletion of the lower crust in heat-producing and chemically related elements [e.g., Heier, 1978]. Specifically, the depletion of U in the lower crust relative to lead and its concentration into the upper crust are well exemplified by the highly unradiogenic lead isotope compositions reported from high-grade Archaean metamorphic terranes [e.g., Moorbath et al., 1969; Black et al., 1971; Chapman and Moorbath, 1977]. The result of such internal differentiation of the continental crust has been to leave the lower crust with lower U/Pb ratios and ultimately less radiogenic lead than the upper crust. Crustal erosion can be expected to preferentially erode and recycle the upper part of the continental crust and thus recycle radiogenic lead into the residual mantle. The addition of radiogenic lead from the upper crust imparts a generally more radiogenic character to upper mantle lead in relation to the bulk earth. More detailed geochemical models of this process are currently under investigation [Evensen et al., 1979].

**Implications for Mantle Convection**

The Scale of Mantle Convection

There is now no doubt that a substantial portion of the upper mantle is involved in some form of convective motion. The extent to which subcontinental upper mantle is currently convecting is not well understood, but recent assessments of the situation [Jordan, 1975, 1978; Oxburgh and Parmentier, 1978] suggest that nonconvecting mantle may extend down to 300 km or more beneath some continental areas. Whereas it can be reasonably assumed that the upper mantle has convected for a major portion of earth history, it is far less certain whether or not the lower mantle is convecting now or indeed whether it has done so in the past. Although geophysical arguments can be mustered in support of both points of view, the geochemical consequences of these different possibilities are such that some distinction should be possible.

Model I investigated above suggests that if whole mantle convection has occurred throughout earth history and if the residuum is reasonably well mixed, it should have an $^{87}$Sr/$^{86}$Sr of about 0.7047. The mean $^{87}$Sr/$^{86}$Sr ratio of midocean ridge basalt is about 0.7028–0.7030, and this cannot be far removed from the mean $^{87}$Sr/$^{86}$Sr ratio of the convecting upper mantle. This mismatch suggests that for the bulk earth model used, the lower mantle must have a higher $^{87}$Sr/$^{86}$Sr ratio than the upper mantle and be significantly less depleted in large ionic radius trace elements. Furthermore, it implies that convective coupling between the upper and lower mantle must have been very limited.

In model II only half of the mantle (by mass) was permitted to contribute to the generation of the continental crust. In this case the Nd and Sr isotope compositions predicted for the residue match the predicted upper mantle composition quite well, suggesting that to a first approximation only half of the mantle may have been in communication with the outer 50-km layer and the remaining half may be chemically undepleted. These geochemical considerations do not in them-

**TABLE 5. Model II: Present-Day Pb Isotope Compositions**

<table>
<thead>
<tr>
<th></th>
<th>$^{206}$Pb/$^{204}$Pb</th>
<th>$^{207}$Pb/$^{204}$Pb</th>
<th>$^{208}$Pb/$^{204}$Pb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk earth</td>
<td>38.08</td>
<td>15.52</td>
<td>17.76</td>
</tr>
<tr>
<td>Residual mantle*</td>
<td>37.03</td>
<td>15.36</td>
<td>16.86</td>
</tr>
<tr>
<td>50-km layer*</td>
<td>38.65</td>
<td>15.57</td>
<td>18.24</td>
</tr>
<tr>
<td>Residual mantle†</td>
<td>37.35</td>
<td>15.41</td>
<td>17.13</td>
</tr>
<tr>
<td>50-km layer†</td>
<td>38.41</td>
<td>15.55</td>
<td>18.04</td>
</tr>
</tbody>
</table>

* $\alpha(U)/\alpha(Pb) = 1.46$  
† $\alpha(U)/\alpha(Pb) = 1.28$
selves directly constrain the geochemical distribution of the depleted and undepleted portions of the mantle. These could, for example, represent the upper and lower mantle (in either permutation) or the exterior and interior portions of whole mantle convection cells that have been stable over the lifetime of the earth. However, any specific physical models of crust-mantle evolution consistent with the above geochemical model are constrained to allow transfer of material from the now depleted mantle to the continental and oceanic crust while disallowing significant transfer from undepleted mantle to depleted mantle or to the crust. Thus for example, a depleted upper mantle and undepleted lower mantle are plausible provided that any convection is confined within the upper and lower mantle segments individually and that any transfer of heat across their interface is not accompanied by transfer of mass [cf. O'Nions et al., 1978a]. In the case of whole mantle convection, on the other hand, there would seem to be a requirement that the convection cells maintain a stable form with little mixing between a depleted exterior and an undepleted interior over the whole of earth history. Such a situation appears feasible, if at all, only in the case of convection driven from below, i.e., by heat sources in the core.

**Thermal Considerations**

The results obtained from model I provide an estimate of the present-day heat production in the residual mantle (Table 3) which may be compared with the earth's present-day heat loss. As previously indicated, the abundances of heat-producing elements adopted for the bulk earth can generate only about half of the mean global heat flow if the time constant for heat loss from the earth is zero.

The present-day abundances of K, U, and Th in the model residual mantle are only 25% of the bulk earth abundances. This implies that the earth's residual mantle has lost 75% of its heat-producing elements to the continental crust and, furthermore, that its intrinsic present-day heat production accounts for only a small proportion of the heat flow from the mantle. The present-day heat production of the bulk earth is about 5.7 pW kg⁻¹, and that of the model II residual mantle is 1.4 pW kg⁻¹. These values are in the same relative proportion as those estimated by O'Nions et al. [1978a] for the bulk earth and the upper mantle but are about a factor of 2 lower in absolute terms, reflecting the fact that at that time it was simply assumed that heat loss occurred from the earth with a zero time constant. O'Nions et al. [1979] have estimated that the earth's mantle is losing an average of about 60 mW m⁻² through the continents and ocean basins; the balance of the mean global heat flow is made up by heat generated in the continental crust. If the model II residual mantle is assumed to lose its heat with a zero time constant, it could only contribute 5 mW m⁻² to the 60 mW m⁻² dissipated by the mantle. Even if the time constant for heat loss is increased to 2.5 Gy, the contribution is only doubled. It thus seems that certainly more than half and perhaps as much as 80% of the heat dissipated from the upper mantle comes from undepleted mantle and/or the core. This implies that upper mantle convection would have to be mainly driven by heating from below [cf. O'Nions et al., 1978a].

Little can be said about the location of heat sources in undepleted mantle or the core, but for the models investigated here, heat must be evolved from these regions with time constants of 2.0 Gy or more [O'Nions et al., 1979]. In view of the large magnitude of these time constants it is quite possible that a component of the heat leaving the mantle has been stored from the time of Earth accretion and is associated with the cooling of the whole planet.

**Mantle Convection Through Earth History**

Do the isotopic compositions of mantle-derived materials provide any insight into possible changes in the volume of convecting mantle or time-dependent changes in the degree of convective coupling between different parts of the mantle? It has been argued that the upper mantle is well sampled by midocean ridge volcanism, which, in the light of the results from model II, could be taken to indicate that the remainder of the mantle has not been efficiently coupled to the upper mantle. However, a period of coupling some time in earth history cannot be excluded, nor can a small degree of coupling be excluded at the present day. The constraints imposed on the volume of convecting mantle are still too crude to permit more definitive statements than these or place precise limits on the degree of mass transport between the upper and lower mantle.

For model II the present transport coefficient for K from the mantle is computed to be only about 20% of its maximum value in the Archaean, and since transport of material to the outer layer is inextricably linked with convection, it is clear that mantle convection has become markedly less efficient as a transporting mechanism since the Archaean. This phenomenon may be a reflection of the overall decrease of heat to be dissipated by mantle convection, both from radioactivity and stored primordial heat, exemplified by the 75% decrease in internal heat production in the depleted mantle since the earliest Archaean. The decreasing heat budget can be expected to result in a decrease in convective velocities [McKenzie and Weiss, 1975] such that less material is transported into the outer 50-km layer per unit time. For the case of upper mantle convection the marked reduction of heat-producing elements in the upper mantle may have changed the ratio of internal and external heating of the upper mantle, which is predicted to change the geometrical patterns of convection in the upper mantle [McKenzie et al., 1974]. How this latter change may itself affect the efficiency of material transport into the outer layer is not clear but is probably less important than the effects of a decreasing heat budget.

**Conclusions**

The major geochemical features of the earth's crust and mantle have been reproduced for a given set of bulk earth element abundances using a simple two-reservoir model with time-dependent coefficients for transport between the two reservoirs. The mean $^{87}$Sr/$^{86}$Sr ratio of the outermost reservoir (the 50-km layer) was used to limit possible solutions obtained by varying the model parameters, and considerations of crustal growth narrow down the choices still further.

Reasonable solutions to a model involving material transport between the whole mantle and the outer layer (model I) require the time constant for upward transport ($\tau_u$) to be approximately 2.0 Gy and that for downward transport ($\tau_d$) to be less than 1.0 Gy. The present-day mean $^{87}$Sr/$^{86}$Sr of the residual mantle in model I is 0.7047, compared with 0.7050 for the bulk earth. This ratio is substantially higher than the estimated $^{87}$Sr/$^{86}$Sr ratio of the convecting upper mantle, which is about 0.7028, indicating that parts of the earth's lower mantle must be considerably less depleted than the upper mantle.

A second model which involves exchange between the outer
layer and only 0.5 of the mantle by mass (model II) also requires that \( \tau_a \) is approximately 2.0 Gy and that \( \tau_p \) is less than 1.0 Gy. However, in this case the mean \( ^{87}\text{Sr}/^{86}\text{Sr} \) and \( ^{143}\text{Nd}/\text{Nd} \) of the residual model mantle is closer to the average value of MORB. This result indicates that the generation of the continental crust may have involved no more than half of the earth's mantle. This result is clearly dependent upon the bulk earth element abundances selected. If these are in fact significantly higher than the values selected, for example, then the amount of mantle involved in generation of continental crust would be proportionately lower. The maximum rate of accumulation of K in the outer layer in model II occurs between 3.5 and 2.5 Gy ago, and at the present day the rate of accumulation is only about 20% of the maximum value. These results are quite consistent with the present state of our knowledge of crustal growth.

The residual mantle in model II has only 25% of the bulk earth heat production at the present day, and it is shown that its intrinsic heat production cannot account for more than about 10% of the present heat flux from the mantle. Even if the heat is lost from this depleted mantle with a time constant of 2.5 Gy, it will only contribute about 20%.

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