Gaia's daisy word

great papers, Eli

November 2, 2017

1 Model formulation

Following Lenton and Lovelock (2001), and Watson and Lovelock (1983).

Let area fraction occupied by black/ white daisies be a_b, a_w. Corresponding temperatures are T_b, T_w. The temperature of bare ground is T_g. Let also a_g be the fraction of the area not occupied by daisies (that is, bare **g**round, denoted x in the above papers),

$$a_g = 1 - a_w - a_b.$$

Growth equation for the areas are basically the logistic equation,

$$\begin{aligned}
\frac{da_w}{dt} &= a_w(a_g\beta - \gamma) = a_w((p - a_w - a_b)\beta - \gamma) \\
\frac{da_b}{dt} &= a_b(a_g\beta - \gamma) = a_b((p - a_w - a_b)\beta - \gamma).
\end{aligned}$$

The growth rate is temperature dependent, with a maximum at the optimal temperature,

$$\beta(T) = \max(0, 1 - (22.5 - T)^2/17.5^2).$$

Let the planetary temperature, T_p, be defined as the area-weighted average,

$$T_p = a_gT_g + a_wT_w + a_bT_b.$$

The energy balance of the areas covered by black and white daisies, and that of bare ground, includes the solar input, the albedo of each of the daisy types, and a transport (mixing) term between each of the two areas and the planetary temperature, represented by a transport coefficient q,

$$\begin{aligned}
\frac{\partial T_w}{\partial t} &= 0 = S(1 - \alpha_w) + q(T_p - T_w) - \sigma T_w^4 \\
\frac{\partial T_b}{\partial t} &= 0 = S(1 - \alpha_b) + q(T_p - T_b) - \sigma T_b^4 \\
\frac{\partial T_g}{\partial t} &= 0 = S(1 - \alpha_g) + q(T_p - T_g) - \sigma T_g^4.
\end{aligned}$$

This way if one adds the two equations to get one for the total heat content

$$\frac{d}{dt}(a_b T_b + a_w T_w + a_g T_g) = \dots,$$

the transport term drops out, as it should because it only moves heat within the earth and does not affect to total heat content.

2 Solving the model equations

To solve these equations, assume that the temperature adjustment time is rapid so that we can treat the temperature equations as being in a steady state to solve for the temperatures as function of the areas. But first, need to linearize the long wave radiation around a reference temperature, $T_0 = 273$, e.g.,

$$\sigma T_b^4 = \sigma T_0^4 + 4\sigma T_0^3 (T_b - T_0) = A + B(T_b - T_0).$$

The temperature equations therefore become,

$$\begin{aligned}
0 &= S(1-\alpha_w) + q(T_p - T_w) - (A + B(T_w - T_0)) \\
0 &= S(1-\alpha_b) + q(T_p - T_b) - (A + B(T_b - T_0)) \\
0 &= S(1-\alpha_g) + q(T_p - T_w) - (A + B(T_g - T_0)).
\end{aligned}$$

Collect terms, remembering that T_p is a function of the other temperatures,

$$\begin{aligned}
T_w\Big[q(a_w - 1) - B\Big] + T_b\Big[qa_b\Big] + T_g\Big[qa_g\Big] &= \Big[-S(1-\alpha_w) + (A - BT_0)\Big] \\
T_w\Big[qa_w\Big] + T_b\Big[q(a_b - 1) - B\Big] + T_g\Big[qa_g\Big] &= \Big[-S(1-\alpha_b) + (A - BT_0)\Big] \\
T_w\Big[qa_w\Big] + T_b\Big[qa_b\Big] + T_g\Big[q(a_g - 1) - B\Big] &= \Big[-S(1-\alpha_g) + (A - BT_0)\Big].
\end{aligned}$$

This may be written in matrix form,

$$\mathsf{A}\begin{pmatrix} T_w \\ T_b \\ T_g \end{pmatrix} = \Gamma,$$

and solved to find the temperatures. If the bare ground area, a_g, approaches zero, we do not need the ground temperature for the planetary temperature average and the third temperature equation is not needed (it also becomes singular). Given the temperatures we can advance the equations for the areas in time. For the transport term to be a significant player but not too dominant, we want $q \lesssim B = 4\sigma T_0^3$.

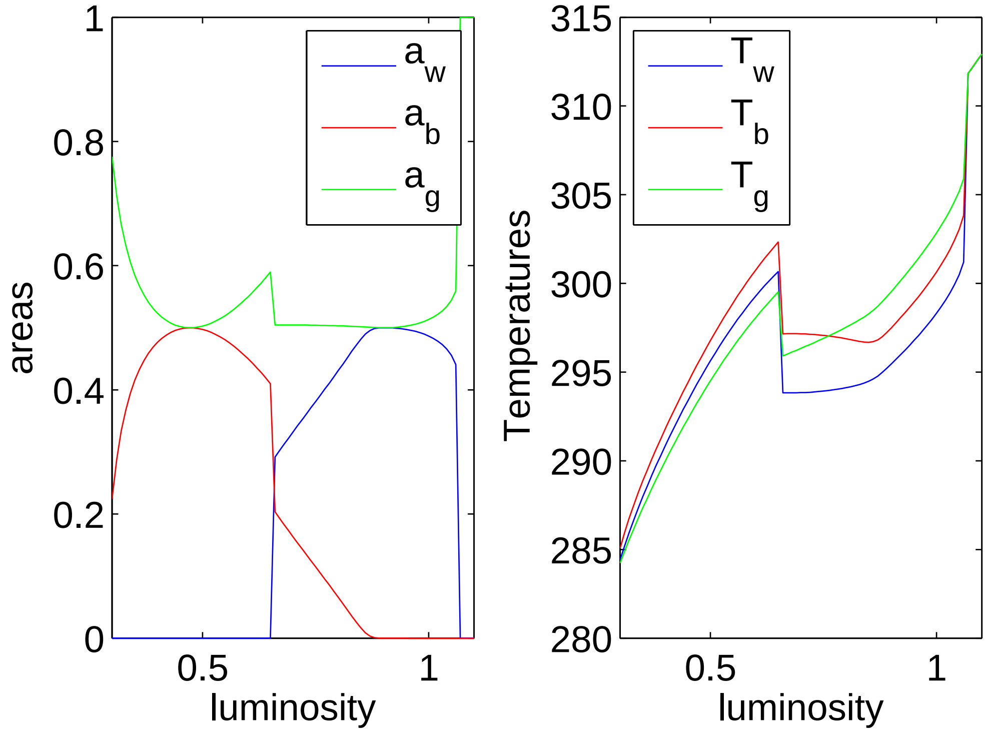

Figure 1: Results of `gaia.m`, solving the equations in these notes for areas and temperatures as function of a nondimensional factor multiplying the solar luminosity. Note the regulation of the temperature for luminosity in the range of about 0.7 to 1.

References

Lenton, T. and Lovelock, J. (2001). Daisyworld revisited: quantifying biological effects on planetary self-regulation. *Tellus Series B-Chemical and Physical Meteorology*, 53(3):288–305.

Watson, A. and Lovelock, J. (1983). Biological homeostasis of the global environment - the parable of daisyworld. *Tellus Series B-Chemical and Physical Meteorology*, 35(4):284–289.