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THE RADIUS OF THE CORE–MANTLE BOUNDARY INFERRED FROM TRAVEL TIME AND FREE OSCILLATION DATA; A CRITICAL REVIEW

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A review is given of the present status of the information on the core radius, indicating its most likely value, but with the understanding that it must be made compatible with the remaining features of the Standard Earth Model, which are not known to us at the present time.

1. Introduction

The core–mantle boundary represents the most significant discontinuity within the earth's interior, and precise knowledge of the radius at which this discontinuity occurs is of importance in investigation of the internal structure of the earth. The question of smoothness of this boundary has a bearing on some aspects of the dynamo theory.

Determination of the core radius cannot be considered separately from the physical properties of the mantle; P-velocity distribution if the core radius is determined from PcP reflections, S-velocity distribution if ScS reflections are used, or density in addition to both velocity distributions for the entire earth, if the free oscillation data are used. Therefore, it is difficult to recommend a particular value for the radius of the core if the structure of the mantle is not established.

It is obvious that the internal consistency of the Standard Earth Model must be ascertained at a certain point, and that adjustments may be necessary at all levels within the earth interior with the core radius being only a single, though important, parameter.

2. Determination of the core radius from travel times of body waves

The first close estimate of the core radius is due to Gutenberg (1913) who inferred a value of 3471 km from determination of the distance at which the P-wave becomes diffracted. The precision of his estimate obtained in days of a very scarce seismograph station network, poor time service, inaccurate epicenter location and, above all, inherent insensitivity of the method he used can be compared to the success of Eratosthenes in his measurement of the earth's circumference.

Jeffreys (1939a) proposed a method of determination of the core radius from the travel times of core reflections which, with some modifications, is still being used. This method represents one of the first applications of linear approximation to a solution of a non-linear seismological inverse problem.

To apply the method of Jeffreys it is necessary to know the velocity distribution in the mantle, and assume an approximate value of the core radius, r_t. Having measured travel times T_i^o of, say, ScS phase at a series of epicentral distances, Δ_i, one calculates the theoretical travel time $T_i^c = T(\Delta_i, r_c = r_t)$ and differential kernels $K_i = \partial T(\Delta_i, r_c = r_t)/\partial r$. Then, the correction to the core radius is found from the least-squares condition:

$$\sum_{i=1}^{N} [(T_i^o - T_i^c) - K_i \delta r]^2 = \text{min.}$$

which leads to:

$$\delta r = \sum_{i=1}^{N} (T_i^o - T_i^c) / \sum_{i=1}^{N} K_i$$

Using the data from the earthquakes whose epicentral location and depth were determined as accurately as possible, Jeffreys obtained a value for the core radius of 3473 ± 4.2 km. Soon afterwards, Jeffreys (1939b) added new measurements of ScS and PcP travel times and obtained a new estimate of r_c = 3473.1 ± 2.5 km. This value has withstood many subsequent investigations, and only in the most recent years a need for revision became evident.

Taggart and Engdahl (1968) have used a large number of travel times of PcP phases from nuclear explosions, where location and the origin time were exactly known. They have also applied station corrections determined by Herrin and Taggart (1968) and assumed that these station corrections include the differences between the model (Herrin et al., 1968) and the world-wide average crust and upper mantle. As the earthquakes tend to occur in the regions characterized by an anomalous crustal and upper-mantle structure, this assumption may not be sustained. Taggart and Engdahl gave a value of 3477 ± 2.0 km as their estimate of the core radius.

Hales and Roberts (1970b) have used the differential travel times of $(T_{ScS} - T_S)$ obtained by measuring the elapsed time between the corresponding peaks of the S_H and ScS_H pulses in a particular recording.

The use of differential travel times has distinct advantages over interpretation of the absolute travel times. In the distance range analyzed by Hales and Roberts (48–70°) the ray paths are at shallow depths very close to the vertical for both S and ScS, thus the effect of the source and station anomalies are eliminated; also, the errors in determination of depth and origin time become relatively insignificant. The method of interpretation is the same as discussed above, except that the calculations of the starting values and differential kernels are performed for a parameter $T_{ScS\text{-}S}$ (Δ). Hales and Roberts (1970b) used two models of S-velocity distribution (SLUTD1 and SLUTD2) and obtained estimates of core radius 3489.92 ± 4.66 km and 3486.10 ± 4.59 km, thus strongly indicating a need for a significant increase in the core radius over the value of Jeffreys. Jordan (1972) has collected a significantly larger set of ScS-S (also PcP-P as well as other phases not relevant to this report) differential travel times. He incorporated these data with free oscillation data and inverted them to obtain the structure of the entire earth. Jordan's results will be discussed in the following section.

Hales and Roberts have observed that the residuals of their observed differential travel times with respect to $T_{ScS\text{-}S}$ (Δ) computed for the model SLUTD1 with a core radius of 3471 km are event-dependent. For example, the average residual for the event KAM 267 is roughly –4 sec, while it is +2 sec for the event RAT 116 in the same range of epicentral distances. The events would have to be mislocated by approximately 1° to account for such a large difference, and this does not appear likely. Hales and Roberts indicated that the alternative explanation may be provided by: (1) lateral heterogeneities in the lower mantle; or (2) lateral variations in the radius of the core–mantle boundary. In future studies it would be appropriate to examine the alternative (1) first, because if the answer is affirmative, there is very little that can be said about alternative (2).

A study by Julian and Sengupta (1973) has provided convincing evidence that there are substantial lateral variations (of the order of at least 1%) of the compressional velocity in the last 500 km of the mantle. They have shown that the rays bottoming within this depth range and within particular regions determined by geographical coordinates show similar travel time anomalies irrespective of the source or receiver.

Julian and Sengupta have also found a significant difference between their P-wave travel time data beyond 90° and those of Herrin et al. (1968). They concluded that this difference results from a bias in sampling. Since the data set of Julian and Sengupta was equally extensive as that of Herrin, their conflicting results indicate that much more data has yet to be collected, in order that the average structure of the earth in this depth range could be established from the body wave studies alone. Even a larger effect has been reported by Jordan and Lynn (1973) for the travel times of the S-waves bottoming near the core–mantle boundary (up to 10 sec).

The presence of the lateral variations of shear velocity in the deep mantle can be deduced indirectly from fig. 6 of Hales and Roberts (1970a) where the scatter of residual travel times with respect to the Jeffreys-Bullen tables increases suddenly by a factor of 3 for distances greater than 82° (bottoming depth of approximately 2300 km).

Recently Engdahl and Johnson (1972) have analyzed the differential travel times PcP-P from three nuclear explosions in the Aleutian Islands. They have concluded that the radius of the core should be increased from 5 to 15 km over the value of Taggart and Engdahl (1968) of 3477 km. Engdahl and Johnson have used a three-dimensional ray tracing technique to account for scattering of the seismic waves by a descending slab. It would appear that uncertainties connected with the model representing this structural complication may contribute significantly to the uncertainty of their results.

We can summarize the results of determination of the core radius from body wave travel times.

(1) Recent results in which differential travel times have been used ($T_{\mathrm{ScS}} - T_{\mathrm{S}}$; $T_{\mathrm{PcP}} - T_{\mathrm{P}}$) favour a core radius between 3482 and 3492 km; a significant increase from the values deduced by Jeffreys (1939a,b) and Taggart and Engdahl (1968).

(2) At the same time, a strong evidence for lateral variations of the velocities in the lowermost mantle has been found. This raises the question whether the sampling of the inhomogeneous mantle has been sufficiently adequate, such that the figures quoted above could represent an unbiased world-wide average.

(3) Any inferences with respect to the large wavelength lateral variations in the core radius (other than ellipticity) are not justified at this time, because of the complications discussed in point (2).

3. Determination of the core radius from inversion of normal mode data

A new kind of seismic data – periods of free oscillations of the earth, became available for the first time from the analysis of a number of recordings of the great Chilean earthquake of May 22, 1960.

Information that can be obtained through observations of free oscillations of the earth is important not only because of the sensitivity of their eigenperiods to the density distribution, but also because of their property of averaging the lateral inhomogeneities in the earth structure.

Rotation, ellipticity and lateral inhomogeneities in the distribution of elastic parameters within the earth interior will cause splitting of spectral peaks; thus, a multiplet of a mode with an angular order number l is characterized by $2l + 1$ spectral lines. But Gilbert (1971a) has shown that the mean value of all eigenperiods belonging to a multiplet is the multiplet's completely degenerate eigenperiod ("terrestrial monopole") belonging to the spherically averaged earth.

Theoretically, a bias could be introduced because of nonuniform distribution of stations and sources, but Dziewonski and Gilbert (1973a) compared eight different sets of average eigenperiods of fundamental spheroidal modes and have shown that the bias, if it does exist, is not greater than 0.05–0.03%, and that it is probably less for overtone modes with high phase velocities.

Thus, the normal mode method provides absolute information on the properties of the spherically symmetric average earth; no station or source corrections are necessary. This property of the normal modes makes them ideally suited to become a reference data set for derivation of the Standard Earth Model, and they can be used to establish the absolute base-line values for travel times (Gilbert et al., 1973; Jordan, 1972).

3.1. Review of developments until 1971

Numerical calculations of the free oscillation periods for a number of the earth models representing combinations of the velocity distributions of Jeffreys or Gutenberg and density distributions of Bullen (models A′ and B) have shown that the observations of normal modes confirm the basic validity of these models. However, it also became obvious that some adjustments in the radial distribution of the physical parameters were necessary since there were systematic differences of the order of 1 or 2% between the observed and computed eigenfrequencies.

One of the first attempts at inversion of normal mode data was made by MacDonald and Ness (1961) who improved the agreement between the observed and computed periods of low-order toroidal oscillations by perturbing the shear velocity distribution in the lower mantle. However, their solution was obviously unsatisfactory, because the S-wave travel times computed for their model had residuals of over 25 seconds at teleseismic distances.

The observed travel times of body waves represent a serious limitation on the permissible range of perturbations in velocity distributions. Realizing this, Landisman et al. (1965) attempted to satisfy the available free oscillation data by perturbing the density distribution. They obtained agreement with the data for a model in which the density was nearly constant in a depth range from 1600 to 2800 km. Although this solution is considered implausible from the viewpoint of geochemical and thermal considerations, it attracted attention to the non-uniqueness of the free oscillation inverse problem. Soon after publication of the paper by Landisman et al., Dorman et al. (1965) have suggested an alternative solution in which the core radius was increased by 10 km and a "soft" layer was introduced at the base of the mantle. Thus, for the first time the importance of the value of the core radius in inverting the free oscillation data has been recognized.

Bullen and Haddon (1967) were the first authors to derive a model which was reasonably consistent with both free oscillation and travel time data available by 1965 and which at the same time was satisfactory in other essential respects. Their model HB_1 was derived by modifying the original model A′ of Bullen on only those respects which appeared to be demanded by all the evidence brought to bear. Full numerical details of the model HB_1 and some variant models were published by Haddon and Bullen (1969). While these models were of course not intended to be final, it is interesting to note that many of their essential characteristics have survived in currently preferred models.

An important result demonstrated was that travel time and oscillation data could be reconciled with a postulated Adams-Williamson density gradient in the lower mantle by increasing the core radius by 15–20 km from the Jeffreys's (1939b) value of 3473 km.

The large core radii of the HB_1 models were subsequently, for a time, thought to be unacceptable because they were in disagreement with the result of Taggart and Engdahl (1968; 3477 ± 2 km; see discussion in the previous section). Muirhead and Cleary (1969) proposed an alternative solution in which shear velocity in the D″ layer has been reduced according to Cleary (1969) and the core radius was reduced to an "acceptable" 3478 km, with all the other parameters of the HB_1 model being retained. However, the recent discovery of the lateral variations of velocities in the lowermost mantle and uncertainties connected with the applicability of ray theory cast some doubt on the interpretation of Cleary (1969).

A different approach has been adopted by Press (1968, 1969, 1970a and b) who, recognizing non-uniqueness of the inverse problem, used the Monte Carlo method to generate trial earth models, tested them against the data and then investigated the common properties of those models that satisfied the data within prescribed error limits. The successful models in his most recent study (1970b) have not shown strong preference for any particular value of the core radius within the bounds from 3463 to 3483 km.

Dziewonski (1970) in his investigation of the singularities of the auto-correlation matrix of differential kernels (partial derivatives) explained the reason for the failure of free oscillation data to narrow the range of permissible solutions based on the fundamental mode data alone. He has pointed out that only acquisition of particular overtone data or relevant body wave information free of usual uncertainties may remedy the situation.

Perhaps the only important conclusion that could have been made on the basis of the results obtained during the period 1961–1970 was that it is possible to satisfy the observed normal mode data with a model in which density distribution in the lower mantle follows closely the Adams-Williamson equation. All the published solutions in which this condition was fulfilled had either the core radius substantially greater than the value of Jeffreys (1939b) or anomalous velocity gradients in the region D″. However, the possibility of a significant super-adiabatic density gradient in the lower mantle could not be excluded on the basis of the limited set of free oscillation data brought to bear.

3.2. Recent developments in identification of normal mode data

Dziewonski and Gilbert (1972) identified and measured eigenperiods of 70 spheroidal and toroidal overtone modes from the spectra of 84 recordings of the Alaskan earthquake of March 28, 1964. It became obvious from the first attempts at inversion (Dziewonski, 1971b) that these overtone data impose new constraints on the distribution of elastic parameters within the earth's interior.

The following unpublished experiment of Dziewonski may serve as a graphical illustration of the increased resolving power of the new data set. The data set of Dziewonski and Gilbert (1972) was inverted using the approach of Dziewonski (1971a,b) for 41 successful models of Press (1970b; oceanic structure). The solid lines in Fig. 1 designate the extreme range of solutions of Press; the hatched area limited by the broken lines envelops the range of solutions obtained from the inversion experiment. It is easily seen that the range of permissible density solutions in the lower mantle has been reduced by a factor of 2 or 3 by the application of the overtone constraints. The range of shear velocity solutions is also somewhat less even though the travel time data were not used as a constraint. The average shear velocity from 41 solutions is very close to that of Hales and Roberts (1970b). The upper mantle has been obviously overparameterized and, as a result, the bounds were not appreciably changed in this region.

Fig. 2 shows the histogram of the core radius values before (broken line) and after inversion (solid line). All solutions after inversion fall within a relatively narrow range between 3486 and 3491 km, while for the starting models they were scattered between 3463 and 3483 km.

The experiment described above was not entirely satisfactory from the formal point of view. The parameterization of the models (pivotal depths) was the same as in the study of Press, and it appears that the models were underparameterized in the lower mantle with respect to the increased resolving power of the augmented data set. Thus, certain bias might have been introduced to the solutions. For example, the models UTD124A′ and B′ of Dziewonski and Gilbert (1972) represent successful solutions with the core radius of only 3482 km.

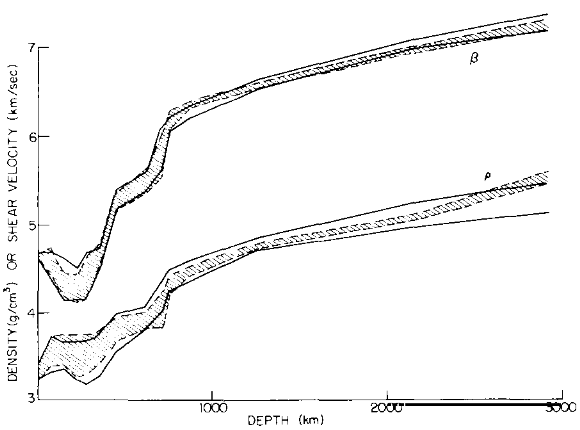

Fig. 1. Comparison of the extreme bounds of 41 models of Press (1970b, oceanic structure) – solid lines, bounds of the models obtained by perturbing the models of Press to satisfy the data of Dziewonski and Gilbert (1972) – stapled area.

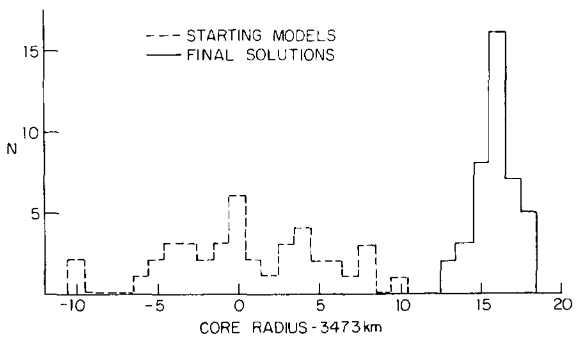

Fig. 2. Comparison of the histogram of the core radii in the set of 41 models of Press (1970b, oceanic structure) – dashed line, with the histogram of the core radii in the models obtained by perturbing the models of Press to satisfy the data of Dziewonski and Gilbert (1972) – solid line. The values on the abscissa represent excess of core radius in a model over a value of 3473 km.

Regardless of these reservations with respect to the experiment described above, it is clear that this new normal mode information can be instrumental in improving our knowlegde of the structure of the earth's interior.

Dziewonski and Gilbert (1973a) have continued their interpretation of the Alaskan data and were able to identify 87 additional spheroidal overtones with periods from 285 to 100 seconds. More than one half of these new overtone data have an effective control over the structure of the outer or inner core.

Mendiguren (1972, 1973) proposed a method which has revolutionized the problem of identification of normal modes. If the earthquake mechanism is known, the theory of excitation of normal modes (Saito, 1967; Gilbert, 1971b) makes it possible to calculate for a particular mode the sign and amplitude of the initial displacement at any point on the surface of the earth. If the data from many stations are available, it is possible to "stack" their spectra for a particular angular order number l. This is achieved by multiplying each individual spectrum by the theoretical amplitude (including its sign) predicted for the particular location and component and summing the results. Should the coverage be continuous, the effect of all modes with the angular order number other than l would be eliminated, because of the orthogonality of spherical harmonics. In practice, the coverage is limited, but the experiments have shown that even using as few as 10 or 15 stations the enhancement is sufficient to isolate the modes which sometimes cannot be seen in the spectra of individual recordings.

Dziewonski and Gilbert (1973b, full report now in preparation) have analyzed spectra of 165 recordings of the Colombian earthquake of July 31, 1970 and using the technique described above as well as a more refined method which they call "stripping", have identified over 400 new spheroidal and toroidal overtone modes. The shortest period of an identified mode is at the present time 83 seconds, but it appears likely that the process can be extended to periods as short as 40 seconds and perhaps even further.

Many of the identified modes have body-wave equivalences (Dziewonski and Gilbert, 1972, 1973a) of the type ScS and PScS, in addition to the mantle modes of S and PS type. It may be expected, therefore, that the entire data set will be sensitive to the value of the core radius.

It is difficult at the present time to predict the progress in identification of normal modes that may take place during the next few years, particularly now that the stacking and stripping methods can be used in processing simultaneously records from different earthquakes. It appears, however, that the quality rather than quantity of the data will be of paramount importance.

3.3. Results of recent inversion studies that include the augmented set of free oscillation data

Publication of the extensive set of overtone data by Dziewonski and Gilbert (1972) has renewed interest of geophysicists in inversion studies, as it was clear that the additional information contained in these data could be instrumental in resolving questions with regard to the density gradient in the lower mantle or the value of the core radius. The overtone data set of Dziewonski and Gilbert has been considered in the following studies: Johnson (1972), Jordan (1972), Wang (1972), and Worthington (1973); Gilbert et al. (1973) included in their study also additional data of Dziewonski and Gilbert (1973a). Although the detailed composition of the data sets and the inversion methods used were different in each of these studies, the results, with the exception of the upper mantle, are very similar and basically conform to the classical concepts of the structure of the earth's interior.

Of the studies quoted above, the work by Jordan and Anderson (1974) and Gilbert et al. (1973) is perhaps the most relevant to the problem of the Standard Earth Model, as the attempt was made in these reports to integrate the entire available seismic information with regard to the deep structure of the earth's interior.

Jordan and Anderson (1974) have used the normal mode data (primarily those of Dziewonski and Gilbert, 1972) and differential travel times (PcP-P, ScS-S, P_{AB}-P_{DF} and P_{BC}-P_{DF}) to derive their model B1. Having used the inversion method of Jordan and Franklin (1971), they were able to assure smoothness of their final model in the lower mantle and the outer core, while their upper mantle contains two discontinuities at 420 and 670 km. The two main controversial features of their model are the negative gradient of shear velocity between 420 and 670 km and the systematic difference in slowness of the S-waves computed for the model B1 and derived from observations of Hales and Roberts (1970a). Jordan and Anderson estimated the value of the core radius as 3485 km.

Gilbert et al. (1973) have used the normal mode data of Dziewonski and Gilbert (1972, 1973a) and Brune and Gilbert (1974), P-wave travel times of Carder et al. (1966), travel times of S, SKS and SKKS phases of Hales and Roberts (1970a and 1971) and PKIKP travel times of Cleary and Hales (1971). The total number of data was 497 (including the earth's mass and moment of inertia), and hence the name of the final model B497 (for the listing see Appendix I in Dziewonski and Gilbert, 1973a).

However a set of 196 carefully selected normal mode data with a very conservative estimate of standard errors was inverted before the final inversion run, yielding a model C198 (ibid.) This model was used to calculate the absolute base-line values for the travel time data sets specified above. The computed base-line correction for the P-travel times is +2 ± 0.4 sec, for S-travel times +4 ± 0.9 sec, and for SKS and SKKS +3.4 ± 0.9 sec. The computed PKIKP travel time at 180° is 1213 ± 0.2 sec. The P-wave travel time corrections of Jordan and Anderson are approximately 1 second less from those of Gilbert et al., while the S-wave base-line corrections are in good agreement. The reason for this discrepancy is not clear at this time.

Gilbert et al. have computed a table of resolving lengths for calculation of local averages of V_P V_S and ρ at a 0.5% error level. The core radius for model B497 is 3482.6 km, and recently Gilbert and Dziewonski (1973) estimated the error limits to be ± 2.8 km.

The other recent determinations of the core radius are: Wang (1972) – 3481 km; Johnson (1972) – 3481.4 km; Dziewonski and Gilbert (1972) – 3482 km; Gilbert et al. (1973) – 3484.9 km for the model C198.

It is important to state that the inversion and error analysis based on the linear inverse theory of Backus and Gilbert is valid only if the starting model is linearly close to the real earth. If we are willing to assume that this is the case, then a value of the core radius of 3485 ± 3 km appears to be an estimate of the radius of the core–mantle boundary that is consistent with all the recent observations of normal modes and travel times.

4. Conclusions

A value of 3485 ± 3 km is suggested as the best available estimate of the outer core radius. This value is concordant with what we consider to be the best current evidence.

However, it must be realized that this estimate is based on the assumption that the earth models that were used in derivation of the core radius are linearly close to the real earth. Should some new observations, or a more refined analysis of the existing data set, prove that this assumption is incorrect (for example, if it were shown that a major discontinuity is associated with the D″ region), it will be necessary to revaluate all the results obtained so far for the lower mantle and the core, including the estimate of the core radius.

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