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The Earth's Interior and Geomagnetism

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INTRODUCTION

THIS review represents an effort to summarize the state of our knowledge of the earth's far interior, omitting the very extensive information that pertains to the crust proper, i.e., the topmost 30–50 kilometers of the earth. For the latter problems which are, of course, closely related to dynamical geology the reader is referred to existing summaries (edited by Gutenberg, 1939).

There are, in the main, two *direct* methods of exploring the earth's far interior. These are, first, the study of seismic waves that have penetrated to some depth inside the earth, and second the analysis of the geomagnetic field and its secular variation. We shall give a general survey of the state of quantitative knowledge achieved in these subjects. Passing beyond the organization of the data, one must proceed mainly by indirect methods, and in practice this means that one has to rely to a large extent on inductive reasoning. The power of logical induction is a notoriously controversial matter. One can, of course, point to some of its familiar successes in the history of research. Clearly, one of the main shortcomings of the inductive method lies in the circumstance that the discovery of a hitherto unsuspected fact may at any time overthrow past generalizations. A classical example of this is to be found in geophysics, in the discovery of radioactivity which obliterated the sweeping cosmological consequences that Kelvin and others had drawn from the computed rate of cooling of the earth on the assumption of thermal conduction without internal sources.

With this proviso, it might be said that our knowledge of the earth's interior seems to converge, as time goes on, toward a reasonably well-defined picture. In this connection the chemistry of the earth plays a fundamental role. It would clearly make little sense to explore the mechanical, thermal and electrical conditions prevailing inside the earth unless one obtained at the same time at least some rough idea of its chemical constitution. Astrophysicists find that the mechanics of stellar models depends on the "mixture" of elements assumed, primarily on the ratio of hydrogen to heavier nuclei; similarly it is almost trite to say that the physics of the earth's interior is dependent on its chemistry. At the extreme temperatures and pressures prevailing in the stars the equation of state of the matter involved becomes fairly independent of finer details of the constitution. This condition is not yet reached inside the earth, although a tendency in that direction might exist (Bullen, 1946, 1947).

In view of the situation just sketched a rather extensive chapter of this review is devoted to geochemistry. A surprising amount of meticulous and fundamental work has been done on this subject, and we can only indicate some highlights. The writer is not acquainted with a good, comprehensive modern review of geochemistry, or with any review for that matter. These problems have of late become of considerable interest in connection with the cosmological importance of nuclear abundance ratios and a review written from the viewpoint of the geochemist would be most valuable. The third chapter deals with whatever little knowledge we possess, or such inferences as we can draw, about the mechanical and thermal state of the earth's interior. From the viewpoint of the geologist the most important question by far is what mechanical mass motions could take place or have taken place during the earth's geological history. So far as the solid part of the earth is concerned, we must confess that the question of possible plastic flow under thermal or other influences is still entirely open. We confine ourselves, therefore, to a description of the static situation. Matters are somewhat more favorable with respect to motions in the liquid core where one can perhaps gradually evolve a more specific model. While this is of importance for the phenomena of terrestrial magnetism, it seems at present unlikely that such motions have a significant influence upon geological events of the earth's crust.

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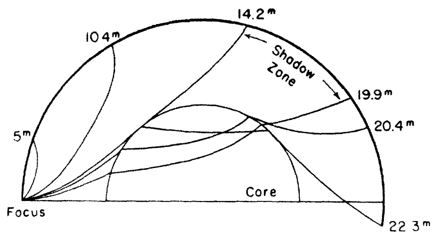


FIG. 1. Seismic trajectories and travel times.

Success in a subject of this kind depends largely on the confrontation of results obtained by various specialists using widely differing tools. This imposes certain limitations upon the author of a review who must needs take a large part of his information second-hand. In some matters of a mildly controversial nature the writer has tried to defer to the opinions of his better instructed colleagues. The last chapter is of a somewhat different character. It gives an account of the mechanics of terrestrial magnetism developed in recent years by this writer and by E. C. Bullard. Not enough time has as yet elapsed to permit exhaustive criticism by others, but we believe that this work is sufficiently an integral part of the model of the earth's interior to warrant inclusion.

SEISMIC DATA

According to our present knowledge the earth consists of three principal layers, the *crust*, the *mantle*, and the *core*. The terminology used here is at present accepted by practically all geophysicists. We shall be concerned primarily with the latter two strata which are separated from each other by an extremely sharp discontinuity at a depth below the surface of 2900 km. This discontinuity forms the boundary of the core (the radius of the core being about 3500 km). The seismological work of the past fifteen years has indicated that there exists near the center of the earth another distinct layer forming a sphere of about 1300 km radius. No generally accepted terminology for this has as yet been established; we shall designate this layer as the "central body." In the older literature, based on less complete seismic data, there are found more or less speculative subdivisions into a larger number of strata, but most of these have been gradually abandoned in favor of the simpler scheme indicated, with such minor subdivisions as will be discussed later on.

The *crust* as revealed by seismology consists of at least two comparatively thin layers composed of materials of low density. For the purpose of investigating the structure of the earth's far interior, it is sufficient to schematize the structure of the crust; it is found that the seismic data pertaining to the deeper parts of the earth are quite insensitive to assumed changes in the characteristics of the crustal layers. Jeffreys (1939) assumes a superficial layer composed of granite, of mean thickness 11 km, density 2.65, and below this a layer, termed intermediate, of thickness 24 km, density 2.87. Its chemical composition is not entirely established; it probably consists in large part of basaltic rocks. The lower limit of this layer is a surface of discontinuity which, in seismology, is usually taken as the lower boundary of the crust (Mohorovičić discontinuity). The term "crust" is sometimes understood in a different sense. Especially in the older literature it is customary to designate as crust that part of the earth's body where permanent elastic stresses are of importance because hydrostatic equilibrium does not obtain. Conceptions of this kind were first developed at a time when it was assumed that the phenomena of vulcanism indicate that the interior of the earth consists of a hot and viscous magma. This interpretation of vulcanism is now abandoned; it is conceded that the appearance of volcanic magma is a secondary and rather superficial phenomenon, the melting being caused by local processes which are not yet fully identified. The earth's mantle, although solid as the seismological evidence shows, yields to stresses by permanent deformation in plastic flow. This is shown by gravimetric measurements which prove that below a certain depth there prevails hydrostatic equilibrium to a high degree of approximation. Again, the hydrostatic equilibrium is not entirely complete as indicated by the phenomena of deep-focus earthquakes, reaching on occasion depths down to 700 km or even more. A large literature exists on deep-focus earthquakes, but so far their appearance has not yet been satisfactorily related to any dynamical concepts. For this reason, and because these quakes can obviously not be dissociated from the mechanics of the crust, deep-focus earthquakes will not be further considered in this review. The transition layer, below which one can for many purposes assume hydrostatic equilibrium, is often schematically replaced by a single level whose depth has been variously located below 30 and above 100 km. There is ample evidence that with respect to the transmission of seismic waves the earth below this level behaves as a body in hydrostatic equilibrium, that is, the mechanical properties are constant on surfaces of constant pressure which in equilibrium are also equipotentials.

We shall now give a brief review of the methods by which seismological information about the interior of the earth is obtained. One can either compare the times at which signals from the same seismic shock arrive at different seismographs, that is, determine the *travel time* of the disturbance as function of the distance, or else, compare the amplitudes produced at different stations. The first method has been developed to a point where very accurate data are obtained; the second often gives corroborative qualitative evidence. It may be assumed for simplicity that the focus of the quake is at the surface, the actual depth of the focus being accounted for by a correction term which we shall disregard in this brief survey. Seismological tables were first constructed in the early years of the century; they contain the travel time T as function of the distance Δ in angular measure along a great circle connecting the focus with the point of observation. Tables constructed in more recent years by various authors show excellent agreement with each other. Perhaps the most complete data, compiled just before the war, are those of Gutenberg and Richter and of Jeffreys and Bullen (Bibliography in Jeffreys, 1945). Jeffreys subjected the data to a thorough statistical analysis in order to achieve maximum consistency of the final travel times with the primary observations. The data of Tables I and III, below, which are simple analytical functions of the travel times are claimed by him to have an intrinsic accuracy of 0.5 percent. With a few exceptions to be considered later the data of other seismologists agree with these results to within about 1 percent or better. With this accuracy it becomes necessary to take into account the ellipticity of the earth; this can be done adequately by applying to the observed travel times a small numerical correction term whose magnitude depends on the location of the terminals of the path with respect to the earth's axis.

In the seismological literature the earthquake waves are classified as follows: The longitudinal waves in the mantle are designated as P ("Primary") waves, the transverse waves as S ("Secondary") waves. Since the P waves have the higher velocity they reach the seismograph at a time when it is almost free from disturbance; the time of arrival of the P wave is the most accurate datum used in the construction of travel time tables. Composite waves in the mantle are designated by a succession of symbols; PP, PS, SS, etc., are waves which have been reflected once from the earth's surface; PPP has been reflected twice; PcP, PcS, ScS, etc., are waves that have been reflected from the boundary of the core. The core itself being liquid, only longitudinal waves are propagated in it; these are designated as K waves. Paths through the core give rise to composite waves of the types PKP, SKS, PKS, etc. (The symbol PKP is usually abbreviated by P'.) There are certain other symbols that refer to wave phenomena at or near the surface of the earth which do not concern us here. In the present review we need not make much use of seismological terminology.

Typical seismic waves have periods of the order of several seconds and wave-lengths of the order of a few to some hundred kilometers. Theory shows (Bullen, 1947; Birch, 1939) that in an elastic medium dispersion arises only through the radial variation in the earth of density and elastic constants. This effect being very small, there is no appreciable dispersion in the approximation of ordinary seismology. The velocity increases with increasing depth in an otherwise homogeneous layer; hence the trajectories of waves are convex towards the center of the earth. Some typical rays computed by methods to be described presently are shown in Fig. 1. Some travel times of longitudinal waves are indicated in the figure at the point of emergence. The rays that pass through the core are shown only very schematically; some finer detail about their behavior will be given presently.

TABLE I. Velocities in km/sec. within the mantle at different depths. $R_0=6338$ km.

r/R_0	Long. (1)	Long. (2)	Trans.
1.00	7.75	7.76	4.35
0.99	7.94	7.94	4.44
0.98	8.13	8.12	4.54
0.97	8.33	8.32	4.64
0.96	8.54	8.54	4.74
0.95	8.75	8.80	4.85
0.94	8.97	9.12	4.96
0.93	9.50	9.47	5.23
0.92	9.91	9.81	5.46
0.91	10.26	10.17	5.67
0.90	10.55	10.50	5.85
0.89	10.77	10.81	6.00
0.88	10.99	10.97	6.12
0.86	11.29	11.28	6.29
0.84	11.50	11.49	6.39
0.82	11.67	11.67	6.48
0.80	11.85	11.86	6.56
0.78	12.03	12.03	6.64
0.76	12.20	12.21	6.71
0.74	12.37	12.38	6.77
0.72	12.54	12.56	6.83
0.70	12.71	12.71	6.89
0.68	12.87	12.87	6.95
0.66	13.02	13.00	7.01
0.64	13.16	13.18	7.07
0.62	13.32	13.31	7.14
0.60	13.46	13.42	7.20
0.58	13.60	13.58	7.26
0.56	13.64	13.65	7.31
0.55	13.64	13.62	7.30

We now describe briefly the procedure whereby the velocity as function of the depth is obtained from the travel time tables; the theory is due to Bateman and Herglotz (Jeffreys, 1929; Bullen, 1947). Let r, ϑ be polar coordinates about the earth's center and let V be the velocity of the seismic waves where V is a function of r only (the ellipticity being neglected). By Fermat's principle the travel time along the actual path is a minimum; this time is

$$T=\int ds/V=\int 1/V[(dr/d\vartheta)^2+r^2]^{\frac{1}{2}}d\vartheta. \qquad (1)$$

If we write $r'=dr/d\vartheta$, we have the Eulerian equation of this variational principle

$$d/d\vartheta(\partial L/\partial r')-\partial L/\partial r=0 \qquad (2)$$

where L stands for the integrand in (1). A first integral of (2), corresponding to the energy integral of ordinary mechanics, is

$$L-r'(\partial L/\partial r')=p$$

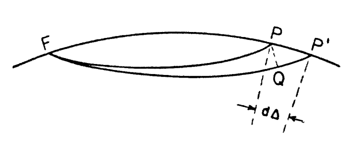


FIG. 2.

where p is a constant. This gives

$$r^2/V = p(r'^2+r^2)^{\frac{1}{2}}. \tag{3}$$

The parameter p has a simple physical meaning; from (1) and (3) we have

$$Vp/r = rd\vartheta/ds. \tag{4}$$

Simple geometry shows that

$$rd\vartheta/ds = \cos\alpha,$$

where α is the angle that the ray makes with the spherical surfaces, $r=$const. Hence $p = r\cos\alpha/V$ is a constant along the ray; this is the generalized Snell's law. For the deepest point of the trajectory we have in particular

$$V(r_{\min}) = r_{\min}/p. \tag{5}$$

Now let P and P' in Fig. 2 be the points of emergence at the surface of two neighboring rays and let Q be the point where the perpendicular from P upon the second ray intersects the latter. Then $\langle PP'\rangle_{\mathrm{Av}} = R_0 d\Delta$ where Δ is the angular distance of the point of emergence from the focus, and $\langle QP'\rangle_{\mathrm{Av}} = VdT$ where T is the travel time. Then $\cos\alpha = VdT/R_0 d\Delta$ and hence

$$p = dT/d\Delta. \tag{6}$$

Eliminating p from (5) and (6) we get

$$V(r_{\min}) = r_{\min}(dT/d\Delta)^{-1}. \tag{7}$$

On introducing (6) and (7) into (2) one can carry out the final integration of the differential equation; the result may be expressed in the form

$$\log\frac{R_0}{r_{\min}} = \frac{1}{\pi}\int_0^{\Delta} \cosh^{-1}\frac{dT/d\Delta}{(dT/d\Delta)_0}d\Delta, \tag{8}$$

where the index 0 indicates that the derivative is to be taken at the upper limit of Δ. The relations (8) and (7) permit one to obtain the velocity as function of depth if $dT/d\Delta$ is known from the travel time tables. If a true surface of discontinuity intervenes, the analytical process becomes more complicated as well as indeterminate to some extent; we shall omit the details of this case.

The method has been used by a number of workers to obtain the variation of seismic velocities within the earth. We reproduce here the values given by Jeffreys (1939), not only because they are very detailed and accurate, but also because K. E. Bullen has used them as basis for additional computations of density, pressure, etc., which will be reviewed later on. Table I (see also Fig. 3) gives the velocities as function of depth in the mantle. $R_0 = 6338$ km is the radius of the discontinuity marking the lower boundary of the crust, taken by Jeffreys to be at a depth of 33 km. The two sets of figures given for the longitudinal (P) waves correspond to different assumptions regarding the existence of a second-order discontinuity near $0.94R_0$ (at a depth of

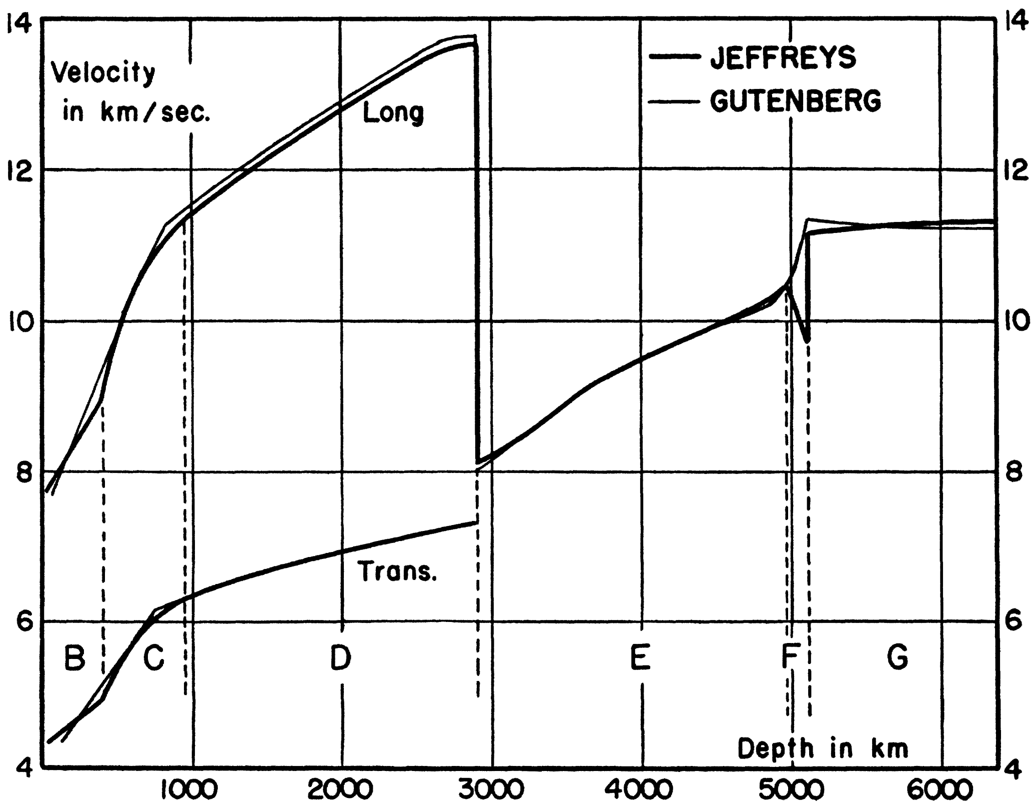


Fig. 3. Seismic velocities.

TABLE II. Velocities in the upper mantle in km/sec. [(G) Gutenberg, 1948; (J) Jeffreys, 1939; (GR) Gutenberg and Richter, 1939; (MD) Macelwane and Dahm, 1939].

Depth, km	G(*a*)	G(*d*)	J	GR	MD	G	J
200	8.1	8.1	8.3	8.1	8.3	4.50	4.60
250	8.3	8.4	8.5	8.4	8.5	4.58	4.68
300	8.6	8.7	8.6	9.0	8.7	4.68	4.76
350	8.8	9.0	8.8	9.3	8.9	4.80	4.85
400	9.1	9.3	9.1	9.6	9.1	4.95	4.94
450	9.3	9.6	9.3	9.8	9.4	5.11	5.12
500	9.6	9.8	9.6	10.0	9.6	5.28	5.32
550	9.8	10.1	9.9	10.2	10.1	5.45	5.55
600	10.1	10.3	10.2	10.4	10.1	5.62	5.65
650	10.4	10.5	10.4	10.6	10.3	5.80	5.80

about 400 km). This so called 20° discontinuity (it occurs near $\Delta=20°$) has been much discussed by seismologists; its weakness makes it difficult to assure its precise character or even its existence. Column (1) of Table I corresponds to the assumption of a second-order discontinuity (discontinuity in slope) at $0.94R_0$, the column (2) to a continuous curve showing a slight "waviness" in this region. The third column, for transverse (S) waves, is computed under the same assumptions as the first longitudinal column. It will be seen that the two sets of values for the P waves are practically identical except near $0.94R_0$, and even there the discrepancy does not exceed 1.5 percent.

Recently Gutenberg (1948) has given an alternative evaluation of the velocities in the upper part of the mantle which he has developed over a number of years. According to these results there is a slight decrease of the seismic velocity in the top layers of the mantle down to about 150 km. From there on down the velocity increases at a steady rate which is nearly linear and does not exhibit the discontinuity or distinct wave of Jeffreys' curves. Gutenberg thinks that the phenomena that led to the assumption of irregularities near 400 km can be explained alternatively by means of the indicated decrease of the velocity above 100 km coupled with certain local effects, and by the difficulties arising in the interpretation of the travel time data.

In Table II, essentially taken from Gutenberg's paper, are reproduced comparative velocity values for the upper part of the mantle. Column (*a*) corresponds to the assumption of a linear decrease of the longitudinal velocity from a value of 8.0 km/sec. at 40 km depth to 7.76 km/sec. at 100 km depth. Column (*d*) corresponds to the assumption, instead, of a slight discontinuous decrease of the velocity by 0.20 km/sec. at 80 km. The data do not as yet permit distinction between assumptions of this type. The next three columns show older values for purposes of comparison. The last two columns give Gutenberg's new values for the transverse velocities, and the corresponding values of Jeffreys.

We next proceed to the velocities in the core. Whereas waves that have only traversed the mantle show a simple monotonical increase of the distance Δ of the point of emergence with increasing travel time, things are much more complicated in the core. Let us illustrate this behavior with reference to Fig. 1 for entirely longitudinal waves (P'), taking as variable the angle which the ray makes at departure with a plane tangent to the earth's surface. Since the refractive index is lower in the core than in the mantle, the general behavior is as shown by the group on the right-hand side of Fig. 1. A ray that approaches close to the core but does not penetrate to it emerges at $\Delta=103°$, but a ray that actually grazes the surface of the core emerges almost diametrally opposite the focus, at $\Delta=187°$. As the angle of departure increases further, Δ decreases until it has reached 143°, then it increases again. These effects, giving rise to the shadow zone shown in Fig. 1, may be described in an optical analogue by saying that the core acts like a spherical lens with high refractive index that concentrates the rays in the forward direction.

TABLE III. Velocity in km/sec., within the core. $R_1=3473$ km.

r/R_1	Vel.	r/R_1	Vel.
1.00	8.10	0.66	9.58
0.98	8.18	0.64	9.65
0.96	8.26	0.62	9.72
0.94	8.35	0.60	9.78
0.92	8.44	0.58	9.84
0.90	8.53	0.56	9.90
0.88	8.63	0.54	9.97
0.86	8.74	0.52	10.03
0.84	8.83	0.50	10.10
0.82	8.93	0.48	10.17
0.80	9.03	0.46	10.23
0.78	9.11	0.44	10.30
0.76	9.20	0.42	10.37
0.74	9.28	0.40	10.44
0.72	9.37	0.36	(9.7)
0.70	9.44	0.36	11.16
0.68	9.52	0.20	11.26
		0.00	11.31

A further complication arises with rays that have large angles of departure, i.e., nearly normal incidence upon the core. One would expect these rays to emerge at points nearly opposite the focus, but a group of them is strongly deflected, showing lower values of Δ with a minimum at $\Delta=110°$. (For the sake of clarity this group has been omitted from Fig. 1.) The phenomenon is interpreted as caused by a sudden steep rise of the velocity not far from the center of the core, at a depth of about 5000 km. (The inside of this region is the "central body.")

Jeffreys' (1939) values of longitudinal velocities in the core are given in Table III. (See also Fig. 3.) The radius of the core has been determined as

$$R_1=(3473\pm4)\text{ km},\quad R_1/R_0=0.5480.$$

The velocity increases monotonically and without discontinuity from the boundary of the core down to $0.40R_1$. Jeffreys then assumes a decrease of the velocity from there to $0.36R_1$, a strong discontinuous rise at this point followed by an almost constant value of the velocity from there to the center of the earth. Gutenberg and Richter (1938) in their earlier and very complete analysis have simply a steep increase of the velocity between $0.42R_1$ and $0.36R_1$ again followed by an almost constant velocity inside the central body (thin line in Fig. 3). They do not at the present time believe that the data require the "dip" introduced by Jeffreys (personal communication from Dr. Gutenberg).

The passage of transverse waves through the core has not been observed. (A few reported instances of this kind are generally regarded as spurious.) This has early given rise to the assumption that the core is liquid. We shall now consider in some detail the evidence to this effect. First we may well inquire into the nature of the discontinuity at the boundary of the core. All the seismological evidence, derived from travel times as well as from intensities, indicates that this is a genuine first-order discontinuity. The accuracy, as we have noted, is rather high. No contrary evidence has so far become known. The nature of the discontinuity has been studied in some detail by Jeffreys (1945), who advances an additional and very strong argument for the liquid character of the core. He notes that while many cases of refraction of elastic waves at the interface of two solids have been worked out, one thing is common to all of them, so long as the velocities of the longitudinal and transverse waves are in about the same ratio: An incident transverse wave gives a very small transmitted longitudinal wave, and conversely. The wave of the type *SKS* has undergone two such transformations; nevertheless the observations show that its intensity is comparable to that of a transverse wave that has travelled through the mantle only. This observed intensity can easily be interpreted only if the matter below the boundary is a true liquid. Sometimes it has been thought that the core might be a solid, viscous medium of appreciably lower shear stability than the mantle so that in it transverse waves can propagate but are very rapidly attenuated. Jeffreys has pointed out that the existence of strongly damped transverse waves at the same time with virtually undamped longitudinal waves is not to be reconciled with the theory of elasticity except under altogether extravagant assumptions about the elastic constants. The same conclusion has been reached by Eucken (1944).

Additional evidence for the fluidity of the core is obtained from the analysis of the bodily tide of the earth. The lunar and solar tides of the earth's solid body have been measured with great accuracy by Michelson and Gale (1919). These authors determined by an interferometric method the change in water level at the ends of a long pipe. (The maximum tilt of the earth's surface produced bE the lunar tide is about $0.02''$, the maximum amplitude about 30 cm.) The theory is relatively simple only in the case of a homogeneous earth, the deformation is clearly in inverse proportion to the shear modulus. The theory of the non-homogeneous earth being extremely complicated, Jeffreys (1926) has treated a two-layer model. The shear modulus of the mantle can be determined from the seismic velocity data, as will be shown below. Now it is possible to account for the observed amplitude of the lunar bodily tide only if one assumes that the shear modulus of the matter in the core is negligibly small. This shows that the core behaves as a fluid in comparison with the shear stability of the mantle; although it does not prove that the core has a shear stability as near to zero as do actual liquids, it constitutes a valuable piece of independent evidence. On summarizing all the evidence brought forth we may quote Jeffreys (1945) "that any escape from the conclusion that the core is liquid would have to be wildly artificial."

Nothing is known so far about the nature of the central body inside the core or about the character of the transition layer surrounding it. Bullen (1946) has indicated that the rise in velocity can be satisfactorily explained by assuming that the central body is solid, of the same composition as the remainder of the core. So far no other evidence on the subject is known.

We shall now review once more the concrete results of seismological analysis, referring to the curves of Fig. 3. The heavy curves reproduce the data of Jeffreys corresponding to the assumption of a second-order discontinuity at a depth of 400 km. The thin curves give the data of Gutenberg and Gutenberg and Richter at the places where they differ appreciably from Jeffreys' curves. (The most recent values have kindly been supplied by Dr. Gutenberg.) There is also shown a schematical division into "layers" designated by capital letters, as used by Bullen (1947). This is related to Jeffreys' analysis rather than to that of Gutenberg. According to Bullen's interpretation of Jeffreys' data there are three principal layers in the mantle designated as B, C, D (A being the crust). Of these, Bullen considers B as most likely homogeneous physically, its lower boundary being the 20° discontinuity discussed above. Layer C, between about 400 km and about 1000 km, might be considered as a transition layer between the more homogeneous layers B and D. Layer D, extending from about 1000 km to 2900 km, has a fairly constant slope and might possibly be nearly homogeneous in constitution.

In Gutenberg's more recent analysis of the seismic data for the upper part of the mantle there remain only two principal layers in the mantle, the upper one extending down to a depth of about 800 km, the lower one from there on to the boundary of the core. In both layers the slope of the velocity curves is fairly constant with only a slight curvature. In Gutenberg's curve the transition in the neighborhood of 800 km depth is rather rapid; it represents perhaps a second-order discontinuity. This scheme according to Gutenberg might possibly be interpreted as corresponding to a gradual change in the constitution of the mantle in the layer below 800 km.

In the core Bullen distinguishes three consecutive layers, E, F, and G. Layer E is almost certainly a homogeneous fluid (the reasons for this will appear later on). Layer *F* marks the transition to the central body and layer *G* is the central body itself. The velocity values of Gutenberg and Richter (1938) for layer *E* differ very little from those of Jeffreys. Their values near and inside the central body are shown in Fig. 3. As pointed out above, layer *G* might possibly be solid.

CHEMICAL COMPOSITION

Since the earth's interior is not directly accessible to us, we must in many instances rely on indirect evidence about its physical state. This is not nearly as much of an ultimate handicap as one might be inclined to think at first sight. Many inferences can be drawn from the seismic data; but on going beyond the results just presented the conclusions will no longer be formally determinate in the same sense in which the seismic velocities are solutions of a differential equation into which the empirical data enter under the form of initial conditions. The methods to be applied from here on are more in the nature of inductive inferences where narrower and narrower limits are imposed upon the possible models of the earth's interior; variants of a model that do not agree with the whole of the observed facts are progressively eliminated. This method leaves us with a considerable body of quantitative information, as we shall see. From the viewpoint of such a method it is particularly desirable to gain first an insight into the chemical composition of the earth. It will appear that the inferences that can be drawn by reasoning along chemical lines are rather stringent.

There are three sources of information on which we shall draw. These are the chemical composition of the sun, the chemical composition of meteorites, and the chemical composition of the igneous rocks of the earth's crust. It is a fortuitous circumstance that definite inferences can be had from such a heterogeneous collection of data. The main results can also be stated with almost no qualifications attached to them. We shall first summarily indicate these results and then discuss in some detail the findings upon which they are based. The three sources mentioned agree in showing that there is a tremendous variation in the relative abundance of elements. Even elements that are close neighbors in the periodic system may differ by a factor of a hundred or thousand in abundance. In particular the elements heavier than the iron group are so rare on a cosmic scale that they can be neglected altogether for the purposes of this summary. A number of other, lighter elements are rare in the earth for different reasons. There remain only nine elements of appreciable abundance; these are oxygen, silicon, magnesium, iron, nickel, sulfur, aluminum, calcium, and sodium. Of these, again, the first four are much more abundant than the other five. Among all other elements only a few have an abundance in the earth of about one atom per thousand atoms of the four most abundant elements; the vast majority of elements are even more rare. Only a small number of sufficiently stable compounds can be constructed from the abundant elements; specifically, oxides, sulfides, and silicates. It is seen, therefore, that the chemistry of the earth's interior reduces to a rather simple scheme; and it appears likely that the solid mantle contains the silicates, and that the core consists of metallic iron and nickel.

TABLE IV. Atomic abundances per 100 atoms Si.

Z	Element	Sun	Meteorites	Igneous rocks
6	C	1000	0.33	0.27
8	O	2760	347	296
9	F	—	0.02	0.15
11	Na	9.8	4.42	12.4
12	Mg	166	87.24	8.76
13	Al	11.0	8.79	30.5
14	Si	100	100	100
15	P	—	0.58	0.26
16	S	42.6	11.4	0.16
17	Cl	—	0.4–0.6?	0.14
19	K	0.81	0.69	4.42
20	Ca	8.7	5.71	9.17
22	Ti	0.011	0.47	0.92
24	Cr	1.95	1.13	0.04
25	Mn	1.48	0.66	0.18
26	Fe	270	89.1	9.13
27	Co	0.55	0.35	0.007
28	Ni	4.6	4.60	0.017

The sources of the data of Table IV, below, are as follows. The abundances for the sun refer to the solar atmosphere and represent the precision spectroscopic determinations of Unsöld recently published (1947; see also Wildt, 1947a). The question may be raised whether the solar atmosphere is representative of the average composition of the sun. We need not grapple, however, with this problem since we are not concerned with the proportion of hydrogen relative to the other elements, nor with the abundance of the heavy elements. The elements that interest us are comprised between oxygen and iron where the atomic mass changes only by about a factor of three. In view of the vast amount of turbulence in the sun one might think that separation by gravity cannot take place to any appreciable extent within this group. Astronomers generally believe that (apart from hydrogen) the chemical composition of the universe, or at least of the stellar system near the sun, is fairly uniform, so that the data should be representative of the chemical composition of the aboriginal matter out of which the earth and other planets were formed. Unsöld's figures supersede the earlier solar abundance data by Russell, and Unsöld has remarked that wherever his results deviate appreciably from the older data, the new determinations tend to be closer to the abundance ratios found in meteorites.

The chemical composition of meteoritic matter has been a subject of much study. The older analyses (Farrington, 1915) already showed some definite characteristics. The most extensive set of data was given by I. and W. Noddack (1930); the results reproduced in Table IV are the revised data of Goldschmidt (1937) which do not differ appreciably from the Noddacks' values. In forming mean values for the composition of meteorites an assumption must be made about the ratio of stone to iron meteorites. Owing to their different chemical behavior, different resistance to erosion, and the different likelihood of their being identified as meteorites by persons finding them, this ratio is still uncertain within wide limits. (Ni and Co are fixed relative to iron.) Borgström (1937) states that among meteorites that were witnessed to fall about 8 percent by mass are irons, 5 percent stony irons, the remainder stones; Watson (1939) has reason to believe that the ratio of metal to stone masses should lie between 2:1 and 1:4. The figure for the number of iron atoms in Table IV must be considered as rather arbitrary and, in fact, the number given originally by the Noddacks is nearly twice the figure adopted by Goldschmidt and reproduced here.

Note added December, 1949.—In a paper just published, Harrison Brown (1949) uses all available material to compile a table of nuclear abundances in the universe. The deviations from previous data do not seem so large as to require revision of our Table IV for the limited purposes of the present review. The main discrepancy is again in the uncertain ratio of iron to silicon which Brown determines from the mass ratio of the earth's core to mantle (admitting 10 percent metallic iron in the mantle), getting 1.7 as compared with Goldschmidt's assumption of 0.89 for meteorites and Unsöld's figure of 2.7 for the sun.

The question arises naturally as to why the composition of meteorites can give us clues to the chemical constitution of the earth. It must be realized that the common "shooting stars" whose orbits have been photographically studied in recent years have very small masses; the meteorites actually discovered on the ground appear as "fireballs" on traversing the atmosphere. Catalogs have been prepared of such events and the estimates of witnesses point to velocities so large that the orbits are hyperbolic, i.e., these bodies would come from outside the solar system. This is a view held by a school especially of German astronomers. The most recent, mainly American students of the subject, believe that the eye witnesses tend to overestimate the velocities systematically and that the meteorites constitute members of the solar system. The old suggestion that meteorites are fragments of a former planet is based on a number of characteristics of the minerals found in them; particularly on the occurrence of large crystallites that require slow cooling for their formation; more detailed chemical reasons have also been indicated (Brown and Patterson, 1947/48).

The values of Table IV represent the average composition of meteorites. These bodies show a characteristic uniformity of chemical makeup. The main constituents of the stones are the magnesium-iron silicates, olivine and pyroxenes which we shall encounter again, on some different grounds, as the likely constituents of the earth's mantle. Enclosed in these is often a certain amount of iron, either metallic or oxydized. The calcium-rich minerals, so characteristic of the earth's crust, are rarely found in meteorites; they make up only a few percent of the aggregated mass of the stones. A feature typical of meteorites is, not only the occurrence of metallic iron practically never found in terrestrial deposits, but also the joint occurrence of large masses of stone and of metal in almost any proportion, in one and the same meteorite (stony irons, pallasites). It has been pointed out by the Noddacks and by Goldschmidt that this indicates clearly that the formation of meteoritic matter must have taken place in a much smaller gravitational field than that of the earth, since the seismic data make it practically certain (see the preceding and the following section) that in the earth's interior the separation of silicates and metal is about complete. This need not necessarily be interpreted, as Goldschmidt wants, as an indication that the meteorites have condensed in small lumps in interstellar space, but it does mean that if there was a parent planet, this body must have been so small that it solidified before the gravitational separation of the silicate and metal phases was completed.

In the present context we need not take recourse to cosmological hypotheses. It must be admitted, however, that the assumption of a condensation of the earth from originally hot and fluid matter is so natural and consistent with all data that it is hard to circumvene. We need not specify the mode and place of condensation of the meteoritic matter. The chemical composition of these bodies is in a characteristic way intermediate between that of the sun and that of the crustal rocks; moreover the main chemical compounds are the same as those that we believe, on other evidence, to constitute the bulk of the earth. The meteorites differ characteristically from the solar matter by the absence of such "volatile" elements as hydrogen, rare gases, carbon, nitrogen, and a large fraction of the oxygen; in this they agree with the igneous rocks. On the other hand, the study of igneous rocks shows that if one goes from the upper strata to rocks which, by geological evidence, originated in regions near the bottom of the crust, the silicates of calcium, aluminum, and the alkalis, characteristic of the ordinary rocks, are more and more replaced by the silicates of iron and, primarily, of magnesium, so that the composition of these deepest rocks tends to be much closer to that of the meteorites. On the whole it is likely that the meteorites represent a condensation of cosmic matter that is somewhat closer to the aboriginal proportion of elements than the constituents of the earth's crust that have been subjected to an extensive differentiation in the earth's strong gravitational field. In the present context the evidence drawn from the constitution of meteorites is mainly corroborative. Since it fits so well into the over-all picture, it adds a good deal of strength to the conclusions drawn later on about the composition of the earth's interior.

Finally we give, in Table IV, the mean chemical composition of igneous rocks. The results of various workers as to these mean values differ very little, and the earlier determinations of Clarke and Washington (1924) are in substantial agreement with those of later workers (e.g., von Hevesy, 1932). By far the most extensive analytical work in this field in recent times has been done by V. Goldschmidt, who has with especial care investigated the relative abundances of all the rarer elements (not needed here). The values reproduced are from a summary of Goldschmidt's results (1937). In view of the great consistency of the data from different sources these figures may be considered as reliable mean values of the constitution of crustal igneous rocks.

Following Goldschmidt, the abundance data are given as numbers of atoms per 100 atoms of silicon. The elements omitted have abundances of less than 0.1 in both meteorites and igneous rocks. (In the sun the abundances of the heavier elements near and beyond the iron group are roughly parallel to those in the meteorites, although sometimes slightly higher, for instance, Zn, 0.31.) The following elements are absent from Table IV:

Noble gases, He, Ne, A.

Hydrogen and nitrogen.

The light elements, Li, Be, B.

Two metals, Sc, V, just below iron, and all elements heavier than the iron group.

Very little need be said about the absence of the noble gases. The very low abundance of the elements H and N will be discussed below. The comparative scarcity of the three elements Li, Be, B, not only on the earth and in meteorites, but also in stellar atmospheres has an established explanation. It is attributed to the fact that these elements are subject to slow nuclear disintegration at the temperatures prevailing in the interior of the sun and of ordinary stars. We may disregard these elements as constituents of the earth's interior. The remaining elements not contained in Table IV are the heavy elements beginning in the neighborhood of the iron group. According to Noddack, the plot of log abundance *versus* Z shows that the points of the heavier elements in meteorites fall into a rather narrow strip whose mean slope corresponds to a decrease of abundance as Z^{-7} to Z^{-8} in the middle of the periodic table, and flattens out toward the very heavy elements; in the earth's crust the abundance of the heavier elements is, in the mean, even smaller. Hence the heavier elements should not be important for the constitution of the earth.

The last two columns of Table IV show that certain elements are concentrated in the earth's crust as compared to their abundance in the meteorites. Goldschmidt designates such elements as "lithophil," the most common ones being Ca, Al, Na, K. Four others are much more abundant in meteorites than in igneous rocks; these are Mg, Fe, Ni, S. This is what one should expect if the principal components of these elements have been subjected to a process of sedimentation (rather, flotation) in the earth's gravitational field so that the relatively lighter compounds of the "lithophil" elements, especially of the alkalis and earth-alkalis, are concentrated in the crust. This view is strongly corroborated by the recent finding that those igneous rocks that originated in the lowest layers of the crust (and are also rich in magnesium) show a much weaker radioactivity than average crustal rocks (Davis, 1947). This latter condition had long been postulated on the basis of theoretical considerations regarding the heat balance of the earth.

Finally there are a few light elements that are much more abundant in the sun than either in meteorites or in the earth's crust. Unsöld gives the following abundances, again relative to Si=100; hydrogen 1,380,000; carbon 1000; nitrogen 2100; oxygen 2760. Clearly, almost all of the first three elements and the largest part of the oxygen must have escaped during the formation of the terrestrial and meteoritic matter in the form of their more volatile compounds. In this connection, sulfur should also be mentioned as being rather an abundant element in the sun. Even in meteorites the mean ratio of sulfur to silicon is only about one-fourth of the ratio in the sun. For chemical reasons sulfur can be assumed to exist inside the earth almost exclusively in form of several types of iron-sulfides; this is found to be true in meteorites. The older authors on the constitution of the earth often assumed the existence of a sulfide-oxide layer, these compounds being intermediate in density between the mantle and the core, and under laboratory conditions sulfides and silicates are completely immiscible. Jeffrey has made a careful search for such an intermediate layer just above the boundary of the core and reports that it does definitely not exist. The presence of sulfides in macroscopic dispersion in the mantle remains one of the problems of the chemistry of the earth's interior; however, the reduction in sulfur content of the meteorites relative to the sun makes it possible to think that a large part of the sulfur present in the aboriginal matter was able to escape from the earth in form of one of its more volatile compounds. The amount of sulfur in the earth's crust is negligible.

On reviewing the contents of Table IV it appears that there is a definite hierarchy of chemical elements in the earth; the picture is almost completely dominated by a very few abundant elements. Taking once more the proportions in the meteorites as an example, we find that, expressed in mass as percentage of the total mass, the eight most abundant elements are as follows:

O	32.3	S	2.12
Fe	28.8	Ni	1.57
Si	16.3	Al	1.38
Mg	12.3	Ca	1.33.

Owing to the very large uncertainty in the amount of iron present the precise figures have of course very little significance, but the trend is obvious: These eight elements make up 96 percent of the mass of the meteorites, the four most abundant ones make up 90 percent. It might be added that in terms of the conventional atomic and ionic radii the oxygen ions fill up over 90 percent of the volume in the stony component of the meteorites, and a similar statement should no doubt apply to the silicates in the earth's mantle.

We shall now proceed from the elements to their compounds. For this we require no more than some ordinary mineral chemistry as contained in any reference work (Dana-Hurlbut, 1941). It will suffice if we confine ourselves to the more dense and stable compounds of the nine most abundant elements, and among these again the compounds of Fe, Si, Mg, S will be most significant. We may readily disregard nickel altogether in this context, considering it as a chemical homolog of iron. Furthermore, the main constituents of the earth's interior are restricted by the fact that, according to results given later, the density of the material at the top of the mantle cannot be very far from 3.3 and the compressibility must be low enough to produce the observed, very large seismic velocities. There are only three significant types of compounds of the most abundant elements: oxides, sulfides, and silicates. (The sulfates are rare among mineral-forming compounds; they are not sufficiently dense and stable to be of interest as constituents of the earth's interior.)

In the following enumeration the figures in parentheses are densities.

Oxides. Apart from quartz or tridymite, SiO_2, the main representatives are the iron oxides, Fe_2O_3 (5.3) and Fe_3O_4 (5.8). MgO occurs in minerals only in its hydrated form $Mg(OH)_2$. Other compact oxides are Al_2O_3 (4.0) and $MgAl_2O_4$ (3.5–4.1).

Sulfides. Only iron sulfides are of importance here. *Pyrrhotite* is $Fe_{1-x}S$ (4.6) with x between 0 and 0.2. *Troilite*, found in meteorites, is close to FeS. There are two modifications of FeS_2 (4.9–5.0).

Silicates. Of these there are several types. The physicochemistry of the silicates enumerated here has been studied in great detail (see many papers of the Carnegie Geophysical Laboratory). The complex phase equilibria of the magnesium-iron silicates at high temperatures in particular are well investigated (Bowen and Schairer, 1935).

Orthosilicates. The most important mineral in this connection is *olivine*, $(Mg, Fe)_2SiO_4$. In these compounds magnesium and iron are freely substitutable; the whole series from *forsterite* Mg_2SiO_4 (3.19) to *fayalite* Fe_2SiO_4 (4.14) exists and representatives are occasionally found. The common variety of olivine (near 3.3) contains approximately 1 Fe to 9 Mg; in meteorites it is often slightly richer in iron. Crystals are orthorhombic. The rock, *dunite*, consists almost wholly of olivine and is often quoted as representative of the constitution of the earth's mantle.

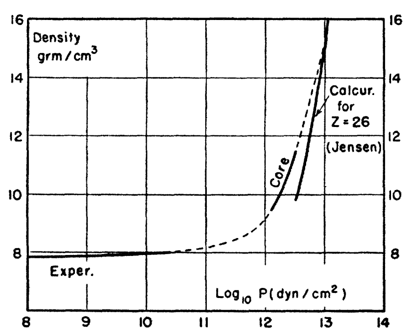


Fig. 4. Dashed line: Probable density of iron.

The term *garnets* designates silicates of the general composition $R_3''R_2'''(SiO_4)_3$, the most important combinations of positive ions being Mg_3Al_2 (3.5); Fe_3Al_2 (4.2); Ca_3Al_2 (3.5); Ca_3Fe_2 (3.7). Most of these are octahedral.

Metasilicates. The common mineralogical term is *pyroxenes*. The magnesium-iron silicates range from *enstatite*, $MgSiO_3$ (3.1–3.4), to minerals where part of the Mg is replaced by iron. The minerals richer in iron are called *hypersthene* (3.3–3.5); if they contain iron oxides they are called *bronzite*. They cease to be stable if Fe exceeds 90 percent. These minerals are common in meteorites. Orthorhombic.

Diopside. $CaMgSi_2O_6$ (3.2–3.3). Here again Fe may replace Mg in any proportion up to $CaFeSi_2O_6$ (3.6).

Jadeite. $NaAlSi_2O_6$ (3.3–3.5). Triclinic.

The rock, *eclogite*, consists mainly of pyroxenes and garnets.

There are some other silicates of the abundant elements, but they are of an extremely complex structure and their existence in the earth's interior is not compatible with the high seismic velocities observed there.

The simplest types of silicates enumerated are distinguished by their tendency to avoid the vitreous state in favor of straight crystallization. Olivine in particular has only a very small vitreous range and normally crystallizes directly from the liquid state. Also, the volume contraction on devitrification is very large (near 15 percent) for these simple silicates (Birch, 1942). All this refers of course to laboratory pressures.

We can now discuss the approximate chemical constitution of the core, the mantle and the crust. We expect the core to consist of iron with some slight admixture of nickel. A strong corroboration of this view is obtained from an extrapolation of the density values of iron to the range of pressures prevailing in the core. Theoretical calculations about the behavior of metals under extreme compression have been carried out by Slater and Krutter (1935), Jensen (1938), and Feynman, Metropolis, and Teller (1949). These are based on a Thomas-Fermi model for the electrons which become applicable when the compression begins to obliterate the specific structural effects of the valence electrons. It appears from Jensen's results, and from general theoretical arguments, that this is the case at about 10^7 atmospheres. In Fig. 4, following Jensen, the density of iron is plotted against $\log p$, the values for layer E of the core are bracketed between Bridgman's laboratory values and the Thomas-Fermi curve.

If one tries to ascertain the composition of the central body, he finds at once that there are apparently not enough heavy elements to fill a volume of the required size. In Goldschmidt's tables the abundance decreases with extreme rapidity between the iron group and the palladium group. From there on the average abundances remain fairly constant (on a logarithmic scale at least) up to the heaviest elements. The abundance of the heavy elements is about $10^{-6\pm1}$ of that of iron. Counting about 50 such elements the aggregated number of atoms would be 5.10^{-5} of that of iron, not enough to fill the central body whose volume is 5–6 percent of that of the core. The elements in the neighborhood of iron, on the other hand, are mostly metals; since the iron group represents a minimum of the atomic volume it is very unlikely that such elements will form a separate phase of greater density. We need not discard, however, the idea that whatever fraction of heavy elements or compounds there is has become concentrated to some degree in the central body.

Bullen (1946) suggests that the central body consists mainly of solid iron. This will almost account for the rise in wave velocity, assuming a negligible change in density and compressibility. By the formulas of the next chapter a rise in longitudinal velocity from $V=10.44$ to $V=11.16$ (Table III) would correspond to an increase of Poisson's ratio from zero to 0.37, somewhat large since other effects have been neglected. It seems also possible to assume a liquid-liquid phase transition corresponding to a rearrangement of electronic wave functions (Elsasser and Isenberg, 1949); assuming no change in density this would correspond to a decrease in compressibility by 13 percent. It seems not quite impossible that this problem can ultimately be investigated by theory, in view of the remarkable success achieved by the theoretical analysis of the wave functions of solid iron by Manning and Greene (1943).

Turning to the mantle, it is characterized by extraordinarily high values of the seismic velocities. The pressures near the top of the mantle, of the order of some 10,000 atmospheres, are well within the experimentally accessible range. The temperatures are unknown, but since it is possible to make laboratory experiments on rocks under a few thousand atmospheres and at moderately high temperatures, no serious extrapolation is involved with regard to the physical conditions near the top of the mantle. It can be shown that among the known minerals only some few silicates, introduced by name above, yield the observed high velocities. These are olivine and the pyroxenes, some garnets and jadeite. It is characteristic that in the measurements of Bridgman (1945/48) the minerals of this composition are found to have a lower compressibility than any other of the many substances investigated, elements or compounds. As we go down below the earth's crust we may expect the magnesium-rich silicates, olivine and pyroxenes to preponderate. Many geophysicists believe that the mantle consists largely of olivine. This view is held for instance by Jeffreys and by Adams (1947). Birch (1939) points out that the experimental data do not permit one to decide between olivine and the pyroxenes. Both are the most common constituents of meteorites. As Jeffreys (1939) notes, the density of the moon is 3.33 which on extrapolation to zero pressure becomes 3.29, practically the density of pure ordinary olivine. The density at the top of the mantle is assumed by Jeffreys as 3.32, the density of olivine at the pressure prevailing. He indicates that for a variety of geophysical reasons the actual density there should not deviate from this value by more than 0.1 or 0.2 at the most. As Birch remarks, only the members of the olivine series with low or moderate iron content have the requisite mechanical strength; this is also reflected in the fact that forsterite, the magnesium end of the series, has a melting point of 1890° while fayalite, the iron end, melts at 1205°. Enstatite, the magnesium end of the pyroxene series, transforms to a monoclinic variant at about 1100°, and at 1560° it melts incongruently to forsterite and liquid SiO_2. This complex behavior at ordinary pressure and many other traits of those silicates make it difficult to be more specific about the physical condition of the mantle beyond the statement that there is no reason to doubt its composition almost wholly of silicates among which the magnesium silicates should greatly preponderate. The seismic velocities are very large throughout the entire mantle, and this would seem to make it unlikely that more than rather modest proportions of other compounds, such as oxides and sulfides, could exist in the mantle. The writer has been unable to find any specific indications in the literature: a quantitative upper limit for the amount of iron oxides, iron sulfides and iron silicates in the mantle would seem a highly desirable datum.

In connection with the somewhat controversial question of second-order discontinuities in the mantle, as discussed in the preceding chapter, an interesting suggestion of Bernal (1936) should be mentioned. He notes that most of the space in the silicate lattices is taken up by oxygen; the silicon ions, being comparatively small, are placed in the holes between them. Compression of the lattice would have qualitatively the same effect as substitution of a larger ion for silicon. Goldschmidt has studied magnesium germanate, Mg_2GeO_4, and finds that it has two stable modifications, one isomorphic with olivine, and another one which is cubic. The density of the latter is larger by 9 percent than that of the orthorhombic variety. This makes it possible to assume the existence of a cubic modification of olivine at extremely high pressures. Adams (1947) points out that if there exists an upper boundary of cubic olivine, then this surface might have shifted in the course of the earth's life owing to a small change in temperature. It seems not, however, necessary to assume that such polymorphous transitions occur almost instantaneously when the temperature changes. Experience shows that very high pressure often has an effect similar to a lowering of the

TABLE V. Mantle.

Depth	r/R_0	Density	Gravity	Pressure
$\times 10^3$ km		g/cm^3	cm/sec.^{2}	$\times 10^{12}$ c.g.s.
0.033	1	3.32	985	0.009
0.10	0.989	3.38	989	0.031
0.20	0.974	3.47	992	0.065
0.30	0.958	3.55	995	0.100
0.40	0.942	3.63	997	0.136
0.413	0.940	3.64	998	0.141
0.50	0.926	3.89	1000	0.173
0.60	0.911	4.13	1001	0.213
0.70	0.895	4.33	1000	0.256
0.80	0.879	4.49	999	0.300
0.90	0.863	4.60	997	0.346
1.00	0.847	4.68	995	0.392
1.20	0.816	4.80	991	0.49
1.40	0.784	4.91	988	0.58
1.60	0.753	5.03	986	0.68
1.80	0.721	5.13	985	0.78
2.00	0.690	5.24	986	0.88
2.20	0.658	5.34	990	0.99
2.40	0.626	5.44	998	1.09
2.60	0.595	5.54	1009	1.20
2.80	0.563	5.63	1026	1.32
2.90	0.548	5.68	1037	1.37

temperature; it increases viscosity and reduces ion mobilities and reaction rates. Thus the time required for the formation of cubic olivine in the mantle might be long even by geological scales. Hence Bernal's idea might open up the possibility of a gradual contraction of the earth during its life, a phenomenon that is strongly suggested by geological evidence.

Turning now to the earth's crust we find it again largely composed of silicates. Chemical analysis of igneous rocks shows a SiO_2 content of about 60 percent. These silicates contain of course larger amounts of Ca, Al, Na, K than can be expected of the silicates of the mantle. One of the great differences between the mantle and the crust lies in their different degrees of homogeneity. It is of course more difficult to detect inhomogeneities in the mantle than in the crust, but no evidence for inhomogeneity in the mantle other than the systematic vertical variation of the seismic velocities has so far been given. The crust is very strongly inhomogeneous, both physically and chemically, so much so that in contradistinction to the mantle great difficulties are encountered in defining its average seismic character. To understand this behavior we best adopt a dynamical view.

As will be shown below it is almost certain that the solidification of the mantle took place from the bottom up while the liquid overlying the solid stratum at any stage was in convective agitation. This explains why the specifically lighter compounds can have become concentrated in the crust by a process of flotation quite analogous to the one encountered in a smelting furnace. We might expect that the substances that are congregated at any one level in the mantle have densities reasonably close to each other. We might perhaps assume that the original cosmic matter from which the planet was formed was physically homogeneous. The pronounced chemical differentiation that we find in the crust and that leads to concentration of different minerals at different places on the earth, must have originated through the action of strong temperature or pressure gradients. Such gradients have been acting upon the crust since its inception and also during its formation, but it is difficult to see how they can have a counterpart inside the mantle. This shows that it is dangerous to draw inferences about the composition of the mantle on starting from purely empirical observations of the crust.

The question of the earth's internal constitution is closely related to that of the constitution of the planets, a subject reviewed in detail by Wildt (1947). The planets fall clearly into two groups, the inner small ones and the outer large ones. The outer planets have mean densities of the order of unity (e.g., Jupiter 1.30, Saturn 0.69) whereas the inner planets are more like the earth (Mercury 2.86, Venus 4.86, Earth 5.52, Mars 3.86). Clearly only Venus (having about the same mass as the earth) can have an appreciable iron core; Mercury and Mars have smaller mass and they are similar in density to the moon (3.33); they may be suspected to consist of silicates. We do not know how this condition came about.

Kothari (1936) first derived a relation between the mass and the radius of a "cold" celestial body in whose interior the temperature effects are negligible compared to the pressure effects. He could show that the white dwarf stars and the planets follow a common law of this type. His assumptions about the compressibility were very crude and were refined by Sommerfeld (1938), who used a Thomas-Fermi model for the atoms. He derives a radius-mass curve for a body made of iron and another for hydrogen and finds that the points for the inner planets fall very close to the iron curve whereas those for the outer planets fall between the iron and hydrogen curves. Scholte (1947) has, apparently independently, derived the same relation in more detail and has computed such a curve for silicon. The points for the inner

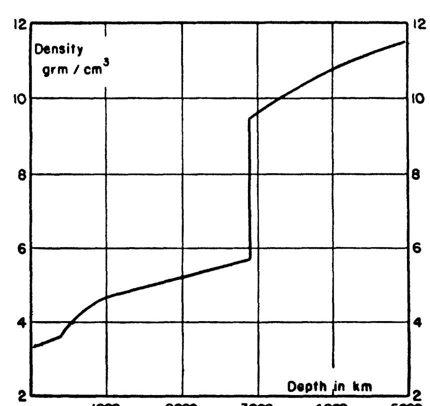


FIG. 5. Density variation after Bullen. Curve stops at boundary of central body.

planets fall rather closely on this curve. From this, and from the fact that the main constituent of these planets should really be oxygen, we may conclude that the relationship is rather insensitive to the choice of the material so long as the latter is appreciably heavier than hydrogen.

In this connection we must mention an effort at reinterpreting the data on the earth's interior by Kuhn and Rittmann (1941; Kuhn, 1942) which has thrown some confusion among those inclined to geophysical speculations. These authors suggest that the earth's core consists mainly of hydrogen, or rather of undifferentiated solar matter of which hydrogen is the main constituent. In their opinion the viscosity in the central parts of the earth is so large that the escape of hydrogen from there can never have taken place. They interpret the boundary of the core as a continuous but rapid (exponential) change in viscosity over a short distance such that transverse waves that penetrate this region are rapidly damped out by viscosity. These views have been severely criticized by Eucken (1944; also by others quoted by Wildt, 1947). Eucken recapitulates and in part redevelops the analysis of Jeffreys and other seismologists given in the preceding chapter which shows that the boundary of the core is a genuine and sharp discontinuity and that the data on the reflection coefficients do not seem to permit any other interpretation than that of a boundary between a solid and a true liquid. Detailed calculations (Kronig, de Boer and Korringa, 1946) show that the density of hydrogen under the pressures prevailing in the core is near unity, whereas the actual density in the core is about 9–13. On the other hand, the large planets, Jupiter, Saturn, etc., have mean densities near unity, indicating that they do consist largely of hydrogen. There seems, then, scant hope of finding much hydrogen inside the earth.

Another suggestion has recently been made by Ramsey (1948) on astronomical grounds. He indicates that he can satisfactorily account for the density of the inner planets on assuming them to consist of silicates throughout. Arguing from the fact that at sufficient compression any substance ultimately becomes a conductor, he interprets the boundary of the core as a phase transition to a denser, liquid, and electrically conducting form of the silicates that constitute the mantle. A further pursuit of this line of thought leads into great difficulties.

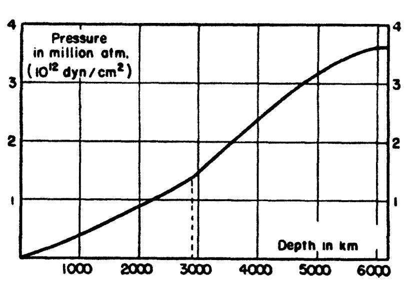


FIG. 6. Pressure variation.

TABLE VI. Core.

Depth	r/R_1	Density	Gravity	Pressure
$\times 10^3$ km		g/cm^3	cm/sec.^{2}	$\times 10^{12}$ c.g.s.
2.90	1	9.43	1037	1.37
3.00	0.971	9.57	1019	1.47
3.20	0.913	9.85	979	1.67
3.40	0.855	10.11	936	1.85
3.60	0.798	10.35	892	2.04
3.80	0.740	10.56	848	2.22
4.00	0.683	10.76	803	2.40
4.20	0.625	10.94	758	2.57
4.40	0.568	11.11	716	2.73
4.60	0.510	11.27	677	2.88
4.80	0.452	11.41	646	3.03
4.98	0.400	11.54	626	3.17
5.12	0.360	(14.2)	585	3.27
5.12		(16.8)		
5.40	0.280		460	3.41
5.70	0.193		320	3.53
6.00	0.107		177	3.60
6.37	0	(17.2)	0	3.64

The compressibility can hardly increase on going to the denser phase, and hence the change of wave velocity at the boundary of the core must be caused by a corresponding change in density. From the data of the next chapter one finds readily that the change in thermodynamic potential, $p\Delta V$, would be 14 electron-volts per molecule of olivine which seems rather excessive for a substance so stable to begin with. (By the same argument one can show that any density change of iron at the boundary of the central body must be very small.)

Note added December, 1949.—There exist measurements of Bridgman (1945/48) on numerous elements and compounds going up to $1\cdot 10^5$ atmos. Using these and using the Thomas-Fermi values from $1\cdot 10^7$ atmos. on up one can successfully interpolate across the remaining gap and can estimate densities with quite good accuracy. In this way one obtains not only isolated points but data for groups which have a higher degree of consistency. The conclusions drawn from this material corroborate the picture of a silicate mantle surrounding an iron core; they do not lend support to Ramsey's assumption.

MECHANICAL AND THERMAL PROPERTIES

We have yet to determine the major mechanical and thermodynamical features of the earth's interior. The first step in this direction consists in a further evaluation of the seismic data. Following the usage of seismologists, we introduce two elastic constants, the bulk modulus and the rigidity. The bulk modulus, k, is the reciprocal of the volume compressibility,

$$1/k = (1/\rho)(\partial\rho/\partial p). \tag{9}$$

The rigidity, μ, is identical with the conventional shear modulus of elasticity. In terms of the conventional elastic constants, λ and μ, we have $k=\lambda+2\mu/3$. The velocities of longitudinal and transverse waves, V_l and

TABLE VII. Limiting densities.

Layer	Depth	Density	
	$\times 10^3$ km	Assumpt. (1)	Assumpt. (2)
—	0.413	3.64	
	0.50	3.88	3.90
C	0.60	4.11	4.14
	0.80	4.46	4.52
—	1.00	4.65	4.71
	1.40	4.88	4.95
	1.80	5.10	5.17
D	2.20	5.31	5.37
	2.60	5.51	5.57
	2.90	5.66	5.72
—	2.90	9.7	9.1
	3.00	9.9	9.2
E	3.50	10.5	9.8
	4.00	11.1	10.3
	4.50	11.6	10.8
—	4.98	11.9	11.1
F			
—	5.12	12.0	
G			
—	6.37	12.3	22.3

V_t, can be expressed as follows:

$$V_l^2 = (1/\rho)(k+4\mu/3), \qquad (10)$$
$$V_t^2 = \mu/\rho,$$

and hence

$$k/\rho = V_l^2 - 4V_t^2/3. \qquad (11)$$

In the core where there are no transverse waves, (10) reduces to the familiar relation $V=(k/\rho)^{\frac{1}{2}}$. From Tables I and II we at once get k/ρ and μ/ρ as function of depth.

These elastic constants refer clearly to changes of the medium that are adiabatic in the sense of thermodynamics. Now assume that we have a *homogeneous* layer of the earth in which the temperature varies along the adiabat as the pressure increases with depth, i.e.,

$$(dT/dr)(dr/dp) = (\partial T/\partial p)_{\text{adiab}}.$$

In this case we have by (9)

$$d\rho/dr = (d\rho/dp)(dp/dr) = -g\rho(d\rho/dp) = -g\rho^2/k. \quad (12)$$

Now let $M(r)$ designate the total mass contained within a sphere of radius r about the center of the earth,

$$M(r) = M(R_0) - 4\pi\int_{R_0}^{r} \rho r^2 dr.$$

Since $g=\gamma M(r)/r^2$ where γ is the gravitational constant, we finally have from (12)

$$(1/\rho)(d\rho/dr) = (\rho/k)[\gamma M(r)/r^2]. \qquad (13)$$

Since k/ρ is known from (11) it is possible to get ρ as function of r by numerical integration. This method, originally due to Williamson and Adams (1923) has been used extensively by Bullen to obtain the data which are reproduced below. The meaning of the thermodynamical limitation will be discussed later.

Any determination of the density distribution inside the earth must comply with two extremely stringent conditions. The integrated density must give the correct total mass of the earth, 597.7×10^{25} g (Olczak, 1938) and also the correct moment of inertia as determined from the precession of the equinoxes. Its value (about the earth's axis) is 81.04×10^{43} gcm^2. Now let θ be the moment of inertia of a sphere of mass m and radius r; we shall write

$$\theta = zmr^2, \qquad (14)$$

where for a homogeneous sphere $z=0.40$. For the earth one finds $z=0.3341$, indicating a strong increase of density towards the center. Now if we determine the density distribution in the earth by a process of integration from the outside inward, we know at any depth r the residual mass $M(r)$ and we also must get a plausible value, slightly below 0.4, for the parameter z at that depth. These conditions are found to put extreme limitations on the admissible density distributions.

In Tables V and VI are given the results of density computations of Bullen (1940/42) based on these principles. These data as well as all the other numerical and analytical work by Bullen and Birch reported in the remainder of this chapter use the velocity values of Jeffreys' analysis, as given above. The more recent values of Gutenberg will of course in all these cases lead to modifications for the upper part of the mantle. It is almost certain, however, that this will have no serious effect upon the results for the deeper layers or the core.

Table V refers to the mantle, Table VI to the core. The tables also contain numerical values for the acceleration of gravity, g, and the hydrostatic pressure which are readily computed when the density distribution is known. The density is shown in Fig. 5, the pressure in Fig. 6. The pressure at the boundary of the core is 1.37 million atmospheres, that at the center of the earth 3.64 million atmospheres.

In order to obtain these values Bullen proceeds as follows: First, on deducting the mass and moment of inertia of the crustal layers he finds a mass of 593.0×10^{25} g and a moment of inertia of 79.80×10^{43} gcm^2 for the remainder. Next he assumes that (13) is applicable to the top layer, B of the mantle. The density at the top of the layer enters as a constant of integration, and this is taken as 3.32, being the density of olivine at that pressure, for reasons detailed above. Birch (1939) has voiced some criticism of this particular value, but Bullen points out that there are many geophysical arguments for such a value, and even if the initial density was in error by as much as 0.1 g/cm^3, this would not affect the densities below layer B by more than about 0.5 percent. If the application of (13) was not justified, considerable errors in the slope of the density curve in layer B might result, but no serious errors in the density values below. There are some reasons to believe that (13) is applicable; one of these has been advanced by Birch and will be indicated later on.

If the density distribution in the entire mantle is determined by the integration of (13), an impossibly large value, $z=0.57$, results for the core. This constitutes independent proof that the mantle is not homogeneous. In order to compute densities in layers C and D, Bullen uses simple polynomials with undetermined coefficients. He then makes some simple physical assumptions: that there is no discontinuity of ρ in the mantle, and that the upper part of layer D is fairly homogeneous so that (13) can be applied to it; finally, he assumes a suitable value of z for the core. Using various consistency tests he finds $z=0.380$ to be a rather likely value, justified by the ulterior density distribution in the core. These conditions determine the density values as given in Table V. The mass of the core under these assumptions is 187.6×10^{25} g.

In the core things are somewhat simpler. Layer E is almost certainly a physically homogeneous fluid; in this case the temperature gradient cannot exceed the adiabat. As we shall show later, if it is less than the adiabat, the difference is negligible; hence (13) may be applied in this layer. Bullen finds that on grounds of consistency the mean density in the central parts, layers F and G must be no less than 12.3 and no more than 22.3. The values of Table VI are determined by an arbitrary assumption regarding the variation of density between the bottom of layer E and the top of layer G, giving a mean density in F and G about intermediate between the two limits. Later on Bullen (1947) has also computed density curves for the two limiting cases of low or high density in the central body. These are reproduced in Table VII. It appeared to the present writer that there is some advantage in having one internally consistent solution of the density problem, carried through in all numerical detail. This is the reason for giving Tables V and VI, not any superior accuracy. Bullen is emphatic about the provisional character of his solution, and from the viewpoint of our present knowledge the values of Table VII define an interval of equally good solutions of the density problem. On the other hand, the margin of variability is limited; Bullen believes that the density values outside the central layers F and G are now determined to within ±3 percent. It might, moreover, prove difficult to assume enough heavy elements to justify a density as high as 22 or even 17 at the earth's center, and this would further restrict the admissible distributions.

It is found that the pressure is rather insensitive to small changes in the density distribution; its values in Tables V and VI are not likely to undergo much further change. Gravity is found to be constant to within 1 percent at a mean value of 990 throughout almost the entire mantle; this circumstance has been used as an approximation in dealing with the mechanics of the mantle. The values of g in the central part of the earth are sensitive to changes in density; they change rather considerably within the range defined in Table VII.

The elastic constants, k, μ, can now readily be computed. The values corresponding to the density distribution of Tables V and VI are given in Table VIII (Bullen, 1947). For k and μ the unit is 10^{12} dyne/cm^2. The value of Poisson's constant, σ, is independent of ρ and is given by

$$[2(1+\sigma)]/[1-2\sigma]=V_l^2/V_t^2.$$

The elastic constants in the central body are not included; they are subject to large uncertainties.

Another quantity that can be computed when the density is known is the ellipticity as function of the depth, by Clairaut's theory (see Gutenberg, editor, 1939, Chapter 13). Bullen (1947) gives numerical values; he finds that it decreases from 0.00337 at the earth's surface to 0.00257 at the boundary of the core.

Birch (1938/39) has shown that important advances in the study of the earth's interior can arise from the application of Murnaghan's (1937) theory of finite strain. Birch developed in particular the theory of the case where the infinitesimal deformation by a seismic wave is superposed upon the finite deformation due to hydrostatic pressure. On applying this theory to the upper layer, B, of the mantle, he assumes as given the two velocities at the top of the layer and the density at the top. This permits him to determine the two elastic constants of the theory and by means of those he can compute the variation of the velocities and of density through the layer. These agree with the seismic values of Jeffrey and with Bullen's values of ρ, respectively, within the errors of measurement. This result bears out Bullen's assumption that Eq. (13) can be applied to layer B, since it shows that the same elastic constants that determine the seismic parameters also give the compression with finite strain. Birch has carried out similar calculations for the layer D of the mantle, finding a somewhat less complete agreement with the observations. It would seem that Murnaghan's theory is a

TABLE VIII. Elastic constants.

Depth	k	μ	σ
$\times10^3$ km	$\times10^{12}$ c.g.s.	$\times10^{12}$ c.g.s.	
0.033	1.16	0.63	0.269
0.10	1.24	0.67	0.272
0.20	1.38	0.74	0.275
0.30	1.54	0.81	0.277
0.413	1.73	0.90	0.280
0.50	2.15	1.10	0.283
0.60	2.57	1.32	0.282
0.80	3.19	1.69	0.275
1.00	3.59	1.89	0.276
1.40	4.20	2.15	0.281
1.80	4.87	2.39	0.288
2.20	5.57	2.63	0.295
2.60	6.23	2.88	0.300
2.90	6.51	3.03	0.300
2.90	6.2		0.5
3.00	6.5		0.5
3.50	8.1		0.5
4.00	9.7		0.5
4.50	11.1		0.5
4.98	12.6		0.5

valuable tool for a study of the elastic properties inside the earth.

Up to now we have dealt with the mechanical properties only as related either to the equilibrium case or to wave motion. The phenomena of plastic flow are of great importance for the physics of the earth's interior. The theory of such phenomena is generally non-linear; it is notoriously complicated and not yet very well understood. In order to survey the situation with respect to the plastic flow of solids, the rather simple linear relation between stress and strain proposed by Maxwell has often been used:

$$\epsilon = S/\mu + (1/\nu)\int S dt, \tag{15}$$

where ϵ is a shear strain, S the corresponding stress, μ is the shear modulus, ν the viscosity. We may define a time

$$\tau = \nu/\mu, \tag{16}$$

known as the relaxation time. For periods small compared to τ, or for large values of ν, the second term of (15) becomes small and we are left with the stress-strain relation of classical elasticity. For periods large compared to τ the first term of (15) becomes small and the relation between strain and stress reduces to that of the conventional Stokes' theory of the flow of viscous fluids. The thorough study of Birch and Bancroft (1942) has shown that Maxwell's relation is not supported by observational fact; moreover an effort at founding it on a theoretical molecular model leads to contradictions. Particularly the concept of viscosity loses most of its ordinary meaning for values above about 10^{10} poises (roughly the viscosity of pitch).

Notwithstanding the inadequacy, Maxwell's relation has had a certain heuristic value. It leads to the idea that for one and the same body rapid motion can be treated by ordinary elastic theory and very slow motion by viscous hydrodynamics. The former seems well enough justified and is employed consistently in seismology; the latter is quite possibly erroneous. A few results have been obtained by a straightforward application of the Stokes-Navier equations of viscous flow to the mantle. Haskell (1935/36) used the known rate of rise of the Fennoscandian peninsula after the disappearance of the ice load to derive a coefficient of viscosity for the upper part of the mantle: $\nu = 1.10^{22}$. Meinesz (1937) on correcting an error in Haskell's treatment obtains $\nu = 3.10^{22}$. In this case (16) would give a relaxation time of 1000 years, well above the periods of seismic waves and well below the periods of geological changes that require millions of years. Gutenberg (1941) has given a very detailed survey of all such uplift phenomena; he shows that those in North America and in other parts of the world are similar and lead to similar relaxation times. He also gives a review of the attempts at theoretical analysis. Using Haskell's value of $\nu = 10^{22}$; Pekeris (1935) calculated a model of viscous thermal convection in the mantle introducing plausible deviations of the temperature distribution from spherical symmetry, of about 100°. He finds velocities of about 1 cm/year. This seems to be typical of the general order of velocity in quasi-viscous models. Meinesz (1948) has recently reviewed the question of cellular convection in the mantle and has propounded some stimulating ideas.

Viscosity is very strongly dependent on temperature and also on pressure. Most experiments on the pressure effect of viscosity (Bridgman, 1931, 1946) have been made on organic substances such as oils; there is one study, by Dane and Birch (1938) undertaken especially in view of geophysical applications. The viscosity of B_2O_5 glass was found to increase exponentially with increasing pressure between 1 and 1000 atmospheres, more rapidly at lower than at higher temperatures. If the dependence of viscosity on temperature is also exponential, as is likely, there would still be a rather well defined and relatively narrow temperature interval of the "softening" transition between the vitreous solid and liquid states.

It is, however, extremely questionable whether the model of a quasi-liquid viscosity gives even a qualitatively satisfactory description of the phenomena of plastic flow of solids. We cannot enter into a discussion of this subject, as its application to geophysical problems is almost non-existent. We hope, however, that studies of this interesting question might be stimulated by mentioning a number of experiments on the plastic flow of stressed solids which were at the same time subjected to extreme hydrostatic pressures (Bridgman, 1935, 1945). A preliminary discussion of the bearing of the results on geology was given by Bridgman (1936). Under the applied very large hydrostatic pressures (up to 50,000 atmos.) rupture of the solid is delayed so that extremely large strains can occur before rupture takes place. It is known that the stress-strain curve of many solids consists, very roughly speaking, of two nearly linear parts joined together by a transition region. The lower part is the region of Hooke's law, the upper part, very much extended by hydrostatic pressure, is that of plastic flow before rupture. The slope, $dS/d\epsilon$, of the stress-strain curve in the region of plastic flow is known to be very small. Hence one can expect (as has been pointed out to us by Professor Bridgman) that under large pressures and with large stresses the material assumes its strained configuration without appreciable time-lag. The condition of the strained solid is then at any moment determined by the boundary conditions, with only such stresses remaining as cannot be released by the plastic deformation.

These observations apply not only to longitudinal, but also to shearing stresses. Plastic flow is extended to higher strains and rupture is delayed, but, for many substances at least, rupture does occur under correspondingly higher shears. Hence the material does not approach the behavior of a liquid as the stress increases, it seems to retain some of its elasticity of form. The experimental evidence agrees with the geophysical fact that deep-focus earthquakes occur in the mantle to depths of 6–700 km; they generate very strong transverse waves and it is generally concluded from this that they represent in the main release of shearing stresses.

The above mentioned behavior of solids under pressure with respect to stresses, especially shearing stresses, applies to both crystalline substances and glasses. No evidence has as yet appeared that would permit one to tell whether the matter of the mantle is vitreous or crystalline. A number of arguments can be brought forth in favor of a crystalline state. Such are the extremely high wave velocities and high densities derived from them, and the tendency of olivine to crystallize readily from the melt without vitrification. On the other hand, the time interval during which the mantle solidified might have been comparatively short, of the order of a few thousand years (see below). This would be enough to permit crystallization of an olivine mantle at atmospheric pressure, but whether this still holds at the tremendous pressures actually prevailing has not been investigated.

The viscosity of liquid metals behaves quite differently from that of silicates. The viscosity of mercury increases by about 30 percent on compression to 10,000 atmospheres (Bridgman, 1931). On linear extrapolation to the 200 times larger pressure in layer E the viscosity would still be so low as to be characteristic of a liquid. Since at the same time the viscosity decreases very rapidly with increasing temperature, there seems to be reasonable evidence that the matter of layer E of the core is truly liquid in the conventional sense.

The final problem to be treated in this section is that of the earth's internal temperature. There seems to exist only one effort at estimating the temperature that will stand close criticism, and even if it should not be correct it represents at least a model with closely specified conditions. This is Jeffreys' hypothesis (1929) that the temperature in the mantle is everywhere about that of the point of solidification, thus determined by the hydrostatic pressure alone. This is based on the fact that heat conduction in the solid mantle is an extremely slow and inefficient process, even over times as long as the age of the earth (see below). The dependence of the melting point upon depth in the earth is described by the Clausius-Clapeyron equation:

$$dT_m/dr = -g\rho(dT_m/dp) = -(gT_m/L)\Delta\rho/\rho \qquad (17)$$

where T_m is the melting point, L the latent heat of fusion and $\Delta\rho$ the density decrease at fusion. On substituting numerical values for olivine the gradient is found to be 4.7°/km (Bowen and Schairer, 1935). This should then apply to the upper part of the mantle. Jeffreys (1932) considered that a linear extrapolation from the laboratory data for olivine to the pressure at the lower boundary of the mantle should give an upper limit for the temperature there; he finds this to be near 10,000°. One might be safe in expecting that $\Delta\rho$ decreases pronouncedly with increasing pressure whereas L should be less affected by the pressure. Daly (1943) on making what amounts to an instructed guess at the melting point curve arrives at temperatures of the order of 4–5000° for the bottom of the mantle. A number of authors that have given thought to the problem arrive at comparable figures. The present writer is inclined to attribute also some weight to the fact, to be reported later on, that the electrical conductivity of the mantle is rather low throughout, probably less than that of sea water. Since silicates are semiconductors it is hard to see how they could fail to develop good electronic conductivity at sufficiently high temperatures. Again, this argument is in favor of a fairly low temperature.

Jeffreys' model presumes that the mantle has solidified from an originally liquid state. A fluid earth is rather readily accessible to theoretical treatment (Jeffreys, 1929). Heat transport is by convection, heat loss at the surface by radiation. Jeffreys estimates the lifetime of the gaseous and liquid stages of the earth as about 15,000 years. This is very short compared to the total length of life of the earth, known from radioactive determinations to be $2\text{–}3\times10^9$ years (Jeffreys, 1948; Houtermans, 1947).

It was pointed out by Adams (1924) that the solidification of the mantle must have taken place from the bottom up. If this had not been the case the lower part of the mantle could hardly have solidified at all, since the heat conductivity of the solid part is inadequate to carry off the heat of solidification. Adams compares the melting point gradient with the adiabatic temperature gradient that prevails in a convectively agitated liquid. For the latter we have by simple thermodynamics

$$dT/dr = -g\rho(dT/dp) = -g\alpha T/c_p, \qquad (18)$$

where α is the coefficient of thermal expansion, c_p specific heat at constant pressure. With the best values for solid olivine Birch finds this gradient to be 0.4°/km, thus very much smaller than the melting point gradient. For liquid olivine the gradient should be somewhat larger, but since the thermal expansion of liquids decreases rapidly with increasing pressure, it is likely that this gradient also decreases at some depth. So long as the melting point gradient exceeds the adiabatic gradient it is readily seen that solidification proceeds from the bottom up.

We can apply similar arguments to the thermal state of the core. Layer E of the core, being liquid, has a temperature gradient equal to the adiabat if thermal convection is occurring, and this is most probably the case as will be shown later. Hence the application of formula (13) to compute the density variation in layer E is fully justified. This formula, as we have seen, holds exactly for adiabatic conditions.

The temperature gradient in the mantle, provided it is the melting point gradient, is seen to be several times

TABLE IX. Spherical harmonic analysis. Unit 10^{-4} gauss.*

Source	Epoch	g_1^0	g_1^1	h_1^1	g_2^0	g_2^1	h_2^1	g_2^2	h_2^2
Gauss	1835	−3235	−311	+625	+51	+292	+12	−2	+157
Erman-Petersen	1829	−3201	−284	+601	−8	+257	−4	−14	+146
Adams	1845	−3219	−278	+578	+9	+284	−10	+4	+135
Adams	1880	−3168	−243	+603	−49	+297	−75	+6	+149
Fritsche	1885	−3164	−241	+591	−35	+286	−75	+68	+142
Schmidt	1885	−3168	−222	+595	−50	+278	−71	+65	+149
Dyson-Furner	1922	−3095	−226	+592	−89	+299	−124	+144	+84
Afanasieva	1945	−3032	−229	+590	−125	+288	−146	+150	+48
Vestine-Lange	1945	−3057	−211	+581	−127	+296	−166	+164	+54

* For the purposes of this review the number of gauss is equal to the number of oersted. This will also be considered to hold inside the core, assuming that ferromagnetic effects are negligible. Non-rationalized electromagnetic units are used in conformity with the tradition of geomagnetic measurements.

the adiabatic gradient. Hence the stratification is unstable, much energy is available in principle for viscous convection, although actually the stratification might have a high degree of conditional stability owing to the large "viscosity." We shall not enter here into the highly controversial question as to the slow convection motions that might have taken place during the geological history of the earth and their connection with the folding of the large mountain ranges.

The old problem of the thermal history of the earth has recently been resumed by Slichter (1941). He finds that with reasonable assumptions about the heat conductivity of the silicates the cooling effect produced by conduction to the surface makes itself felt, at the present age of the earth, to a depth of about 200 km. In a layer of this depth a stationary state of heat flow has been reached while the temperature lower down would be essentially unaffected by the presence of the surface. Lowan (1933–5) has applied rigorous mathematical methods to this problem.

Observations in deep shafts, etc., show a temperature gradient of the surface layers that is extremely variable depending upon the location, and is between 10°/km and 50°/km. Most of the flow corresponding to this gradient removes the radioactive heat output of the crustal rocks. It is a well-known fact that the matter of the mantle must contain a much lower concentration of radioactive substances than the crust does contain according to direct measurement; otherwise an impossibly large amount of heating of the whole earth would result. The recent work of Davis (1947) shows that eruptive rocks which originated near the bottom of the crust have a radium content so low, about 10^{-14} g/g, that this catastrophe is avoided. Moreover, as Slichter points out, an extreme rarefication of radioactive matter in the interior need not be assumed, provided it be admitted that the earth as a whole heats up very slightly in the course of time, not enough to melt; an effect that could hardly be perceived at the outside.

MAGNETIC PHENOMENA

A magnetic field observed at the surface of the earth could be produced by sources in the interior of the earth, by sources outside the earth's surface, or by electric currents crossing the surface. The first two components of the field have a scalar potential. Here we are only concerned with the first component, the field whose sources are inside the earth; it is known that this is by far the largest component. Some older authors estimated the external field and the non-potential field as about 3 percent each of the total, but these estimates are now considered as extremely doubtful or spurious.

The separation of the field into its components can be achieved by means of spherical harmonic analysis; this method has been used since Gauss. Let U designate the potential of the combined internal and external field at the earth's surface; we develop U into a series of spherical harmonics†

$$U=R_0\sum_{nm}(A_n^m\cos m\varphi+B_n^m\sin m\varphi)\lambda_n^mP_n^m(\cos\vartheta),\quad(19)$$

where ϑ and φ are the geographical co-latitude and longitude and where λ_n^m is a numerical factor,‡

$$(\lambda_n^m)^2=2(n-m)!/(n+m)!\quad(20)$$

Since (19) does not involve r, it is a potential for the horizontal component of the field alone. Now let a similar development of the vertical component, of the force itself, be

$$Z=\sum_{nm}(Z_{(1)n}^m\cos m\varphi+Z_{(2)n}^m\sin m\varphi)\lambda_n^mP_n^m(\cos\vartheta).\quad(21)$$

A three-dimensional potential, $\phi(r,\vartheta,\varphi)$, is related to (19) and (21) by

$$U=(\phi)_{r=R_0},\quad Z=(\partial\phi/\partial r)_{r=R_0}.\quad(22)$$

If we write ϕ in the form

$$\phi=R_0\sum_{nm}[c_n^m(r/R_0)^n+(1-c_n^m)(R_0/r)^{n+1}]\times A_n^m\cos m\varphi\lambda_n^mP_n^m(\cos\vartheta)+R_0\sum_{nm}[d_n^m(r/R_0)^n+(1-d_n^m)(R_0/r)^{n+1}]B_n^m\sin m\varphi\lambda_n^mP_n^m(\cos\vartheta),\quad(23)$$

† R_0 is the radius of the earth, larger by 33 km than the value of R_0 used previously.

‡ It may be noted that $(2n+1)^{\frac{1}{2}}\lambda_n^m$ is the normalization factor of P_n^m. The seminormalized functions $P_{(n)}^{(m)}=\lambda_n^mP_n^m$ which apparently go back to Gauss are generally used by workers in geomagnetism.

the first Eq. (22) is identically fulfilled and the second yields

$$(2n+1)c_n{}^m = Z_{(1)n}{}^m/A_n{}^m + n + 1,$$
$$(2n+1)d_n{}^m = Z_{(2)n}{}^m/B_n{}^m + n + 1,$$

so that the components of the internal and the external field are determined separately if A, B, and $Z_{(1)}$, $Z_{(2)}$ are known. The last two are directly obtained from the observed vertical component by (21). The A's and B's are obtained from corresponding series for the horizontal force components which are the derivatives of the series (19).

The coefficients of the field of internal origin are

$$g_n{}^m = (1-c_n{}^m)A_n{}^m, \quad h_n{}^m = (1-d_n{}^m)B_n{}^m. \tag{24}$$

The symbols $g_n{}^m$ and $h_n{}^m$ are commonly used in the geomagnetic literature. If the external field is negligible (24) becomes

$$g_n{}^m = A_n{}^m, \quad h_n{}^m = B_n{}^m.$$

In this approximation the potential is determined by the horizontal components alone and its coefficients must agree with those found independently from the vertical component of the field. In practice the random errors are very large. For this reason it is hardly worth while to eliminate the non-potential field separately. There has been much discussion about the reality of the external field and of the non-potential field. Chapman and Bartels (1940) indicate that the external field could be simulated by very small systematic errors in the magnetic maps, and there seems at present no definite way of ascertaining the reality of these components. These arguments do not apply to the rapidly variable components of the external field with periods of the order of a day or several days. Such fields are produced by solar and lunar perturbations of the ionosphere and their properties are well known (Fleming, 1939; Chapman and Bartels, 1940). Their spherical harmonic components have been determined, but they have no apparent relation to the stationary components of the field or to the very slow secular variation of the latter. In investigations of stationary components of the non-potential field their magnitude is determined directly by computing the line integral of the magnetic force along closed contours on the surface of the earth.

In order to carry out a spherical harmonic analysis one prepares a table of the field components at a grid of points with regular intervals in latitude and longitude. The values of the magnetic force components at these points are scaled off magnetic maps. The values of the Legendre functions at these points being tabulated, one can then proceed to solve a system of normal equations for the spherical harmonic coefficients. Since some parts of the globe are not too closely surveyed magnetically, considerable errors are introduced into the final numerical data. In view of this and of the great irregularity of the field which results in relatively slow convergence of the spherical harmonic series, students of the subject prefer whenever possible to use the maps directly. We shall give a brief survey of the results of the spherical harmonic analysis, as some of the conclusions drawn from them are significant. The most complete analyses are those for the year 1885 by Schmidt (1895), for 1922 by Dyson and Furner (1923), for 1945 by Vestine (1947) and an additional one for the same year by Afanasieva (1946). Schmidt is the only one who separated the internal and external field, but in view of what has been said above no particular significance can be ascribed to these results.

TABLE X. Dipole components.

Epoch	Mean	Dipole	Mean	ϑ	φ	Mean
1835		1.061		12.2°	−63.5°	
1829	1836	1.047	1.053	11.7	−64.7	−64.2°
1845		1.052		11.3	−64.3	
1880		1.036		11.6	−68.0	
1885	1883	1.033	1.035	11.4	−67.8	−68.4
1885		1.035		11.3	−69.5	
1922		1.011		11.6	−69.1	
1945	1937	0.989	1.000	11.8	−68.8	−69.3
1945		0.998		11.4	−70.0	

The numerical values of the dipole and quadrupole coefficients as collected by Dyson and Furner and supplemented by Vestine are given in Table IX. The most outstanding feature of this table is the secular variation of the individual coefficients, which is clearly apparent in spite of the individual scatter. The data for the dipole terms are given in Table X in another form. The relative magnitude of the total dipole moment¶ is given in arbitrary units, putting the last value in the "mean" column equal to unity. Table X contains in addition the polar angles of the dipole axis. The observations fall conveniently into three groups about 50 years apart, for which means have been taken. The over-all decrease of the dipole moment amounting to about 5 percent in a century can hardly be ascribed to observational errors alone; its absolute magnitude might not be given correctly by Table X. Vestine (1947) carried through a spherical harmonic analysis for the secular variation of the field; his values for the preponderant dipole term, $dg_1{}^0/dt$, are as follows:

Epoch	1912.5	1922.5	1932.5	1942.5
Percent per year	−0.08	−0.09	−0.08	−0.03.

It is hard to say how accurate these values are, but there can again be little doubt as to the over-all decrease of the dipole. It is likely that the reduced rate of change in 1942.5 is real, since Vestine's maps indicate a slight general change of the secular pattern at this epoch. One would of course not expect that the dipole moment has decreased or will continue to decrease over a long period of time at the rate expressed in Table X; this change can be taken as just one component of the general, highly irregular variation of the field, other examples of which

¶ The earth's magnetic moment in 1945 was 8.06×10^{25} e.m.u.

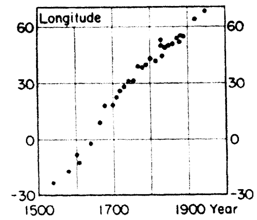


FIG. 7. Movement of the westerly point of zero declination along the equator.

will be encountered later. The inclination of the dipole axis has remained sensibly constant at 11.6° with variations of only a few tenths of a degree. Bauer (1895) has investigated the inclination of the magnetic axis during the period of 1780–1885 by taking averages of the inclination over circles of geographical latitude. He finds that the angle of inclination has remained constant within errors of the same order as those of Table X. On the other hand, the last columns of Table X show a slow if somewhat irregular precession of this axis in the direction from east to west.

We next inquire into the non-dipole part of the field. Some information about the secular variation of the quadrupole terms is contained in Table IX, but on the whole it is better to rely directly on the magnetic maps. Anticipating the main result we may say that the non-dipole components of the field are subject to strong and comparatively rapid secular variations. There are apparently no constant components of the field; there is every reason to believe that the non-dipole field averaged over a sufficiently long interval of time would vanish. Consequently, the non-dipole part of the field cannot be properly treated and understood separate from its secular variation.

In dealing with magnetic maps it might first be said that at many places there are small localized distortions of the field of limited extent caused by iron-bearing minerals in the crust. Such ore deposits are as a rule very close to the surface and permanent magnetic distortions of this nature can rather readily be identified as such. As a matter of experience it is usually possible to reconstruct the field that would be there in their absence. No serious reasons have appeared as yet to indicate that any appreciable part of the remaining non-dipole field originates in the mantle. In the work of the U. S. Coast and Geodetic Survey it has been found that anomalies of a scale intermediate between the distinctly local ones and those of very large extension are absent (Deel, 1945). On considering large areas of the earth's surface and smoothing over the localized crustal distortions one is left with the field that is the subject of this review. A convenient tool for the study of the non-dipole field is

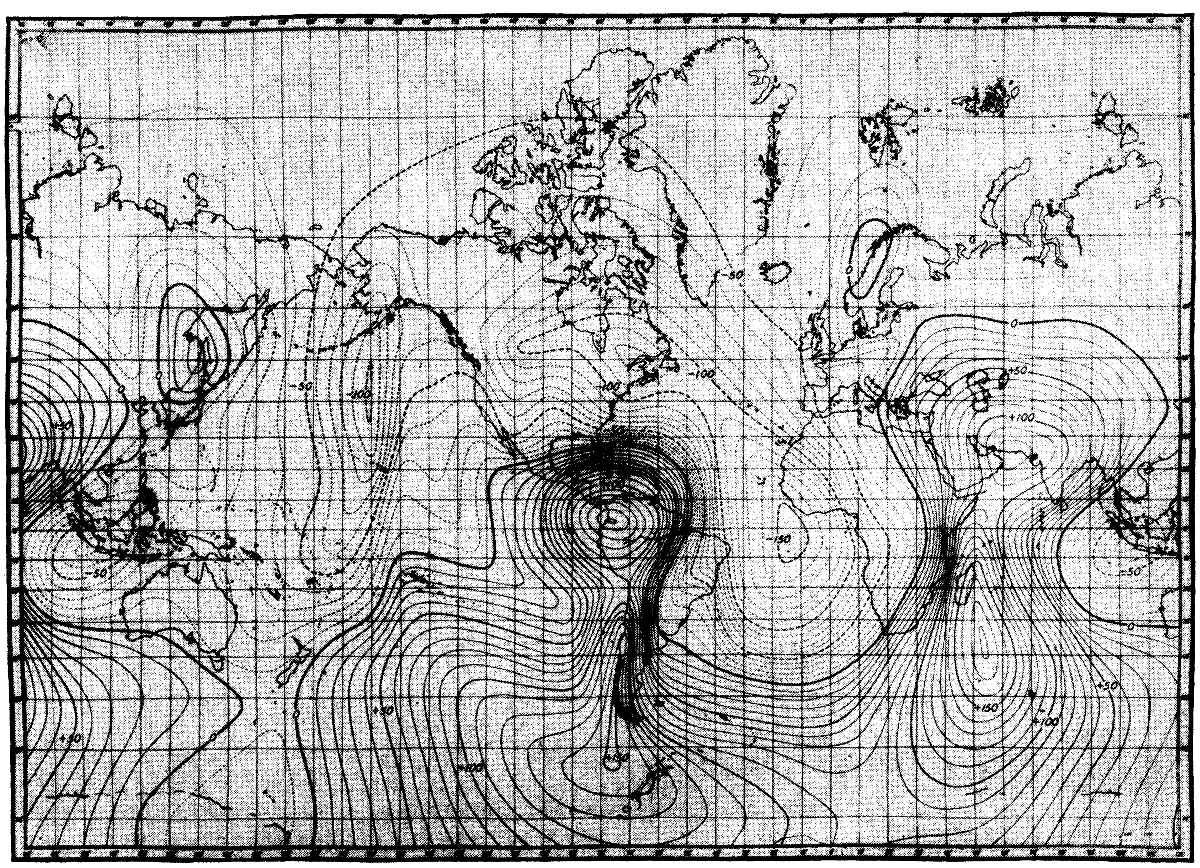


FIG. 8. Geomagnetic secular change in gammas per year, vertical intensity, epoch 1912.5.

maps obtained by subtracting the field of the dipole terms from the total field. Unfortunately, very few such maps have been constructed, and so one has to adopt other, less satisfactory methods for the evaluation of the older maps. The constant components of the field are automatically eliminated in maps of the secular variation. The most important set of these is due to Vestine and his group (1947).

Magnetic maps go back to the second-half of the seventeenth century and some systematic records collected by voyagers cover an additional century. Owing to the irregular character of the field and its secular variation many of the older investigations have virtually no systematic value. Gradually, however, a few more distinct traits became apparent. Thus, Bauer (1895) states quite clearly that there is a trend of the irregular field configuration toward rotation about the earth from east to west, a westerly magnetic "wave" as he calls it. Carlheim-Gyllensköld (1907) also finds a strong preponderance of westerly displacements in his harmonic analysis of the secular variation. Given the overall irregularity of the field the value of such findings lies more in cumulative evidence than in specific quantitative data. As one of the rare examples of the quantitative type we mention the westerly motion of the lines of zero magnetic declination studied by Bauer by means of data going back to the middle of the sixteenth century. The older references are based on a very careful survey of records of navigators from 1540 to 1680 compiled by van Bemmelen (1893). Figure 7 shows the change in longitude of the point at which the westerly line of zero declination intersects the geographical equator. Since the sixteenth century this point has moved from the center of Africa across the Atlantic and is now close to the head-waters of the Amazon. The total displacement is just about 90° in 400 years, or 0.22° per year. The corresponding point for the easterly line of zero declination has also moved westward, but much less regularly, with an occasional standstill, and altogether only by about 30–40°. This seems not due to any inadequacy of the data, as this point has moved through the Malayan archipelago, a region well enough explored for a long time. A glance upon the maps of the magnetic field and the secular variation shows that the westerly line of zero declination is fairly straight whereas the easterly line has some pronounced distortions and passes through regions where the local deviations from the dipole field are large. We are unable at present to explain the large rates of westerly motion as compared to the slow westerly progress of the dipole axis shown in

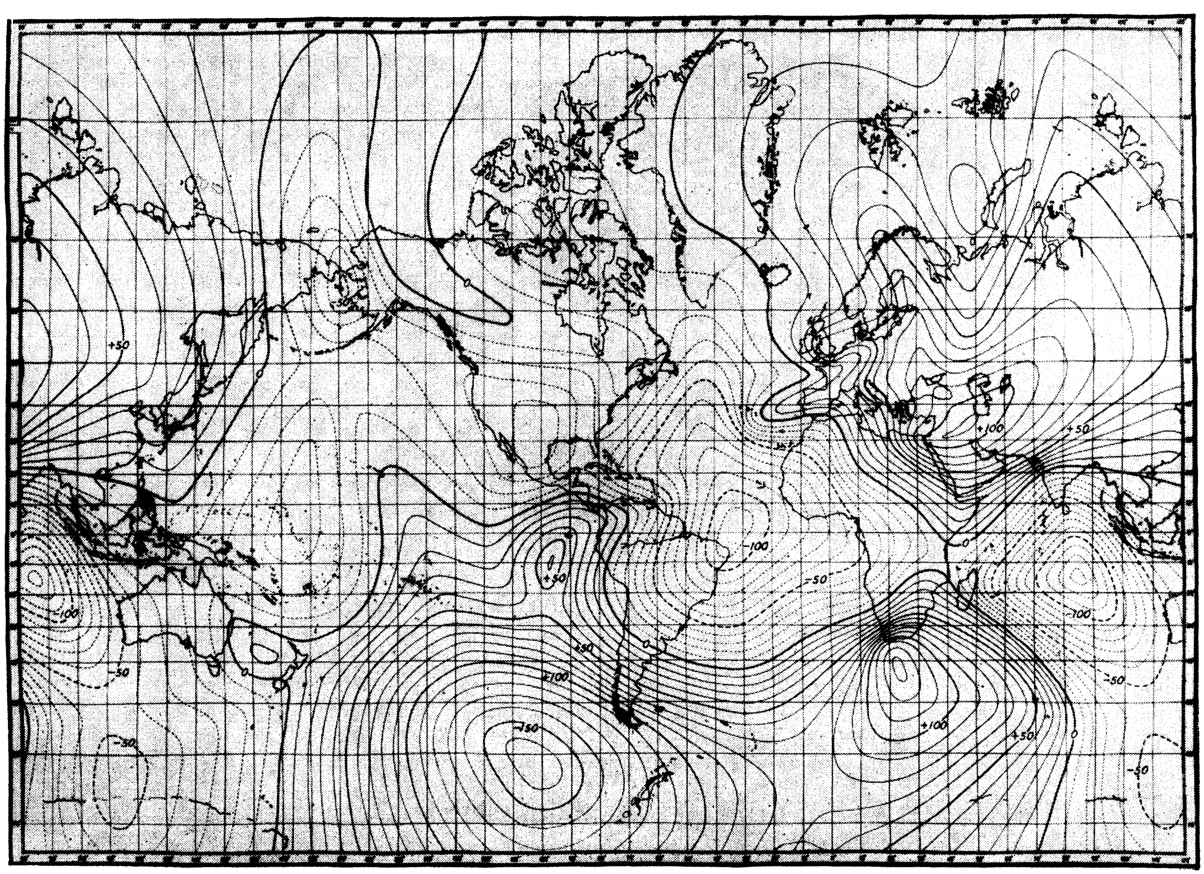


FIG. 9. Geomagnetic secular change in gammas per year, vertical intensity, epoch 1942.5.

Table X. This, together with the difference in speed of the two lines, indicates that the whole phenomenon is highly complex, the dipole component being only responsible for part of the motion; the rest must be due to higher harmonic components which apparently move to the west at a more rapid rate.

An extensive investigation of the secular variation for four epochs at distances of ten years has been carried out by Vestine and collaborators (1947). This work is based on an exhaustive evaluation of magnetic records for the entire globe between about 1905 and 1945. Figures 8 and 9 show two maps for the secular variation of the vertical intensity in units of 10^{-5} gauss per year at epochs 30 years apart. Figures 10 and 11 are two similar maps for the rate of change of the declination in minutes per year. The maps show that there are a number of centers of secular disturbance distributed rather irregularly over the surface of the globe; the rate of change of the field may have either sign. The lifetime of these disturbances is apparently quite short, less than a century in the average to judge from the maps; some of them have disappeared and others have newly appeared during the period of observation. In addition to their increase or decrease in intensity the centers show irregular displacements, but there appears a quite distinct preponderance of drift motion towards the west. This is visible in the motion of individual centers as well as in the displacement of the lines of zero secular change. A few centers show a drift in north-southerly direction, but this seems more in the nature of a random motion. Bullard|| has made a quantitative study of the westerly drift, from Vestine's maps, using a least square method, and finds a value of (0.180 ± 0.013) degree per year. The magnitude of this drift motion is found to be independent of the geographical latitude.

Many efforts have been made to reconstruct the magnetic field of the past, previous to the advent of magnetic measurements. All the methods have in common that they determine the direction or intensity, or both, of the remanent magnetization in a material that has been formed or has solidified at a known epoch of the historical or geological past. Such methods must be used with great caution and must be carefully checked for consistency, and as in particular Haalck (1942) has pointed out, in many cases the results are unreliable.

Chevallier (1925) has studied the magnetization of

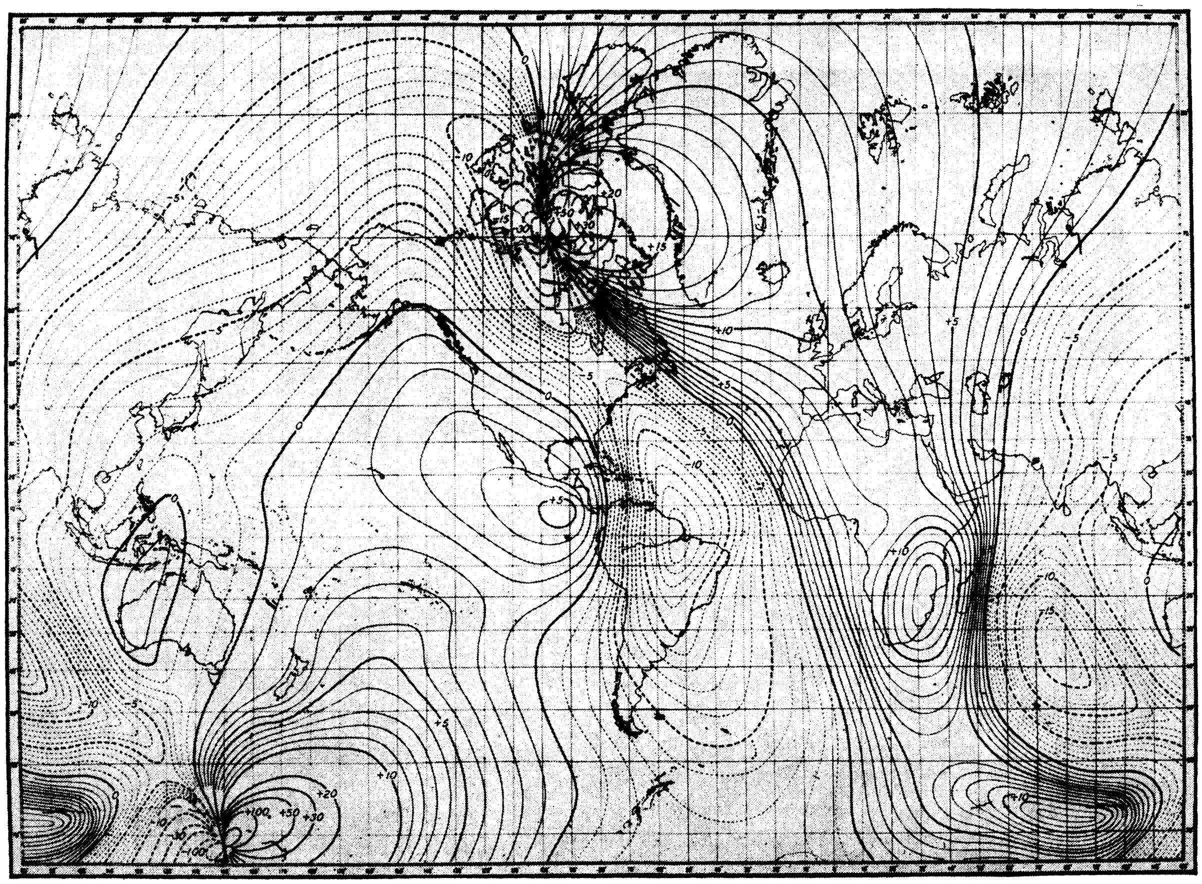


FIG. 10. Geomagnetic secular change in minutes per year, declination, epoch 1912.5.

|| Personal communication. Will be published soon.

lavas of Mt. Etna, from dated eruptions, back to the year 1280. The data for the declination give a curve that, from about 1550 on, coincides with the directly observed values. Thellier (1938) determined the curve of the inclination at Paris back to the year 1400 by means of bricks in historical buildings of known age, it being assumed that a brick is conventionally baked in a horizontal position and that its remanent field has the direction of the earth's field at the time of baking. The results seem sufficiently consistent to warrant the use of the method. An investigation of sediments was made by McNish and Johnson (1938). Their main sample consisted of varved Pleistocene clays from New England. Its time-scale (200 years) is determined, apart from an arbitrary origin, by comparison with an established tree-ring chronology. The curve of the declination obtained is smooth and shows considerable variation. Similar measurements with comparable results were made by Ising (1943) on varved clays in Sweden. A recent investigation by Johnson, Murphy, and Torreson (1948) extends the previous results. The values, for the declination, obtained again from New England varved clays, lie on a reasonably smooth curve over an interval of 5000 years. The relative chronology of this period has been given by E. Antevs; the absolute time is probably from 15,000 to 10,000 B.C. The maxima and minima of the declination are 20° east and 38° west, respectively. Sedimentation tests in the laboratory lead to the conclusion that the declination of the deposits follows quite closely that of the earth's field, while their inclination might differ very appreciably from that of the external field. The authors also indicate evidence that the mean magnitude of the earth's field was of the same order as at present. On the other hand, Thellier and Thellier (1942/6) have tried to determine the intensity of the field by the brick method mentioned above and find that the present observed decline in the field has apparently continued for several centuries in the past. The two results need not be contradictory, as there is no reason to presume that the last-mentioned trend is more than a temporary fluctuation on the secular scale.

Systematic investigations of the remanent magnetization of sedimentary rocks have only been made quite recently. The work of Graham (1949) is concerned with the stability of the magnetization of sediments. If the way in which a bed of sedimentary rock has been folded is known, it is possible by means of a graphical construction to restore the vector of the remanent magnetization to its original undisturbed position, and the reconstructed bed should then appear uniformly mag-

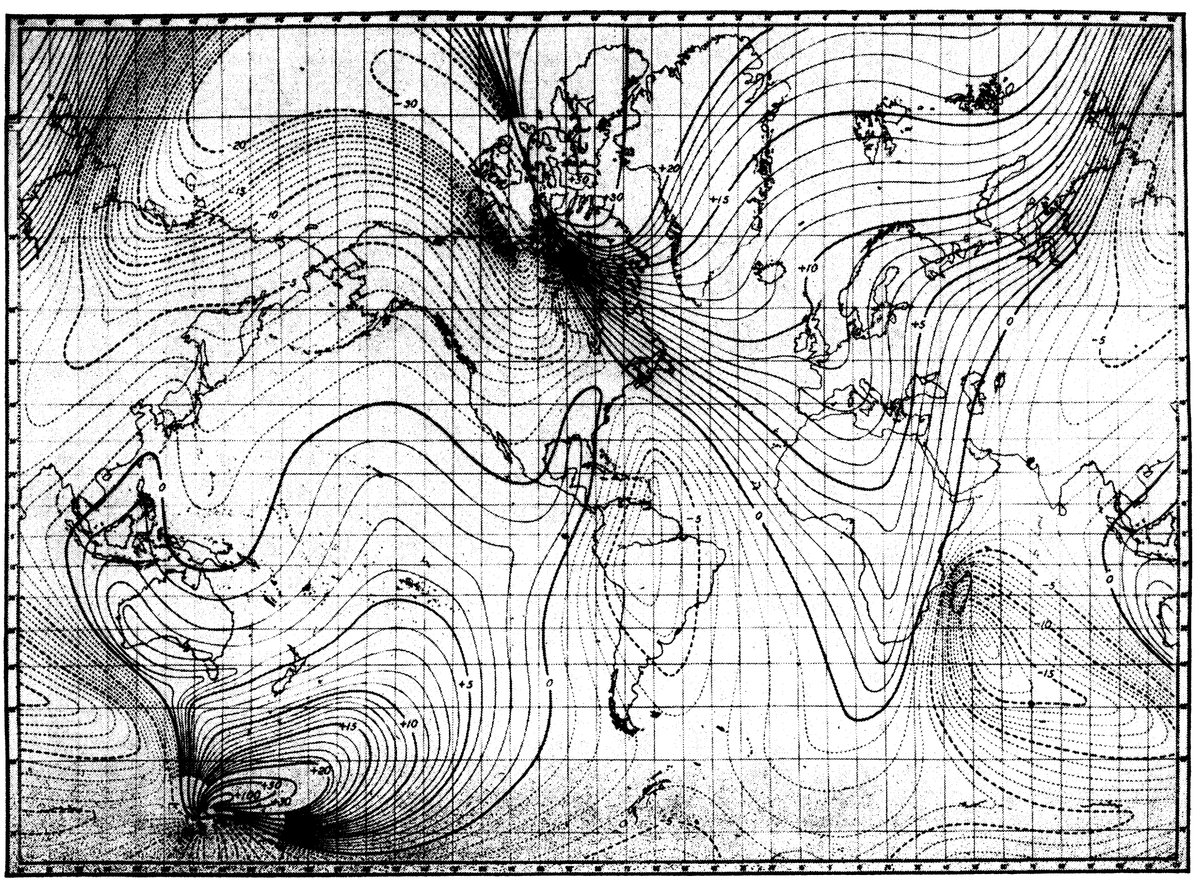


FIG. 11. Geomagnetic secular change in minutes per year, declination, epoch 1942.5.

netized. This was done successfully for some pleistocene varved clays in New England; it was also carried out with great success for a silurian rock bed in Maryland whose age is estimated at 3.5×10^8 years and in which intense folding occurred some 2×10^8 years ago. It may be concluded that the magnetization has remained stable for periods of this order. Another attempt, with an Appalachian limestone, showed, however, a large scatter of the points and further studies must be undertaken to ascertain the cases in which stability can safely be assumed.

Torreson, Murphy, and Graham (1949) undertook a statistical sampling of the magnetization of sedimentary rocks of various geological ages in the Northwestern United States, the great majority being of tertiary origin. It may be assumed that the appearance of definite regularities in such material indicates that at least part of it is magnetically stable. A frequency-distribution curve of the declination showed a very sharp maximum centered about true north (the present local declination is 17–21° east). Over half of the points contribute to this narrow peak, the remainder are distributed at random. This would indicate that in the mean the magnetic and the geographical poles coincide, but the authors indicate that the material is as yet too limited to permit of final conclusions. The curve for the inclination shows an entirely similar maximum near an angle that is slightly smaller than the present inclination.

The literature on the magnetization of volcanic lavas is extensive, and mostly of little interest in the present context, as hardly any definite conclusions have been drawn from them. The question of the stability of the magnetization acquired on cooling below the Curie point of the ferromagnetic component is still largely open. There seems to have been little systematic study of the relation of the magnetic behavior to the structure of the minerals involved.

An exception to this generally unruly behavior has been found in studies of the magnetization of dikes carried out in recent years. ("A dike is a tabular body of igneous rock that has been injected while molten into a fissure"—Webster's dictionary.) Two of these cases have been very fully investigated. There is a system of such dikes in Transvaal extending over an area about 300 km long and up to 150 km wide. The entire volcanic formation is magnetized in direction opposite to the present magnetic field at the locality. The age of these dikes is about 2×10^8 years. Other geological bodies formed in the vicinity before and after this system are magnetized in the same direction as the present field (Gelletitch, 1937). This might conceivably be explained by a shift of the magnetic equator sufficiently far (say 30°) to the south, but such an explanation fails for another large system of dikes that extends across the northern part of England. The detailed investigation by Bruckshaw and Robertson (1949) shows that nearly the entire system of dikes is magnetized in a direction opposite to the present field. Deviations from uniform magnetization can apparently be explained by slight flow in the later stages of solidification, below the Curie point of the material (about 550°). Such flow does not occur in ordinary rocks at these temperatures, but it has been observed to take place in material cooled down from higher temperatures and containing volatile solutes. The regions near the walls of the dikes which cool most rapidly show the most distinct reverse polarization. The authors themselves do not believe that there is another explanation to their findings than the existence of a reversed geomagnetic field at the time of the formation of the dikes. If, however, smaller deviations can be explained by viscous flow below the Curie point, then it might be possible to explain the reversal as being caused by cellular convection that turns the mass upside down before final solidification. Such "half-cycle" convection, though on a vastly larger scale, has been suggested by Meinesz (1948) for the explanation of certain orogenic processes.

Other instances of reverse magnetization of volcanic intrusions have been reported; for instance two from Germany (Schulze, 1930; Reich, 1935) and one from Brazil (Malamphy, 1940). Not all igneous intrusions, however, have reverse magnetizations; in a number of them the magnetization is in the general direction of the present field. These phenomena are quite obscure at the present time, especially if confronted with the results obtained from sedimentary rocks. The assumption of numerous reversals of the earth's field during geological ages does not seem particularly attractive. Much work remains to be done, especially on the petrographic aspects of the problem.

Having now completed our survey of the direct magnetic data, we shall next try to relate them more closely to the other physical facts concerning the earth's interior. It is generally believed that ferromagnetism is not important in the strata below the crust. It is extremely unlikely that the mantle contains large amounts of ferromagnetic material. Moreover, the temperature at greater depth should soon exceed the Curie point. The effect of pressure upon the Curie point can be studied in the laboratory only to a very limited extent, and such data as exist indicate that the Curie point does not rise with rising pressure. It is possible to draw some inferences from the theory of ferromagnetism. In the latter, the permanent magnetization is attributed to exchange effects between electrons in the *d* shells of neighboring atoms. Calculations show that these exchange terms decrease in absolute magnitude and eventually reverse their sign (prohibiting ferromagnetism) as the distance of neighboring atoms decreases. In the iron core where the linear distances of atoms are reduced by some 20 percent, this effect should be rather marked, and ferromagnetism should not exist, quite apart from the temperature effect.

At various times it has been proposed that large celestial bodies become magnetized by rotation owing to fundamental properties of matter not adequately described by conventional theory. The latest suggestion of this type was put forth (Blackett, 1947–9) in consequence of Babcock's discovery (1947–8) of large magnetic fields in rotating stars. We cannot analyze Blackett's suggestion in the present review, but we should remark that Runcorn and Chapman (1948) have proposed a method whereby one might be able to decide experimentally between a theory of this type and those other theories that seek the origin of the field in the earth's core. In the latter case, clearly, the field increases with depth in the earth, as r^{-3}. In a hypothesis such as Blackett's on the other hand, one may assume that any part of the rotating body contributes to the magnetization according to some law, e.g., proportional to its density. The calculations then show that the horizontal component of the field decreases in magnitude as one goes down in the earth. This conclusion can be checked by measurements in deep mines, and a few preliminary measurements (quoted by Runcorn) indicate that an effect of this type might exist. The deviations in question are, however, small and can be masked by local variations of crustal origin. Lengthy series of measurements must be made before the question can be settled by this means.

Within the framework of ordinary physics there remains the possibility that electric currents flow in the interior of the earth. In this connection one would expect the core to have a high electric conductivity and the mantle to have a low one, and this is borne out by such limited evidence as is available at present. We must then expect that the largest part, if not all, of such currents flow inside the core. We shall disregard for the present the permanent dipole part of the field and shall tentatively consider variable currents in the core as sources of the variable part of the field. The question arises whether sources in the core alone are sufficient to produce the field observed at the surface. An approximate mathematical criterion for this can be found as follows: Let the potential at the surface be written

$$U = R_0 \sum_{nm} \alpha_n^m \gamma_n^m(\vartheta, \varphi), \tag{25}$$

where the γ_n^m are a set of spherical surface harmonics. The α_n^m might differ from the coefficients (24) by numerical factors depending on the normalization of the γ_n^m. If it be assumed that there are no sources of the field in the mantle, the potential at the boundary of the core is

$$U_c = R_0 \sum_{nm} (R_0/R_1)^{n+1} \alpha_n^m \gamma_n^m(\vartheta, \varphi). \tag{26}$$

This potential can be constructed if the spherical harmonic analysis at the surface has been carried out. Since the observational data yield only a finite number of spherical harmonic coefficients, it is not possible to establish a rigorous mathematical criterion for the validity of the extrapolation (26) as applied to a real field. Instead, one can use criteria of an approximate nature with a physical basis, and these will be found to be sufficiently stringent.

TABLE XI. Mean (r.m.s.) harmonic components of order n.

$n=$	1		2		3		4		5		6
$m \leq 2$	1433		370		196		92		43		8.5
ratio		0.26		0.53		0.47		0.47		0.20	
$m \leq 4$	1433		370		177		73		32		11
ratio		0.26		0.48		0.41		0.44		0.34	
Model		0.46		0.51		0.53		0.54		0.54	

McNish (1940), on surveying the non-dipole part of the field, found that it lent itself to an approximate reproduction by means of 14 small dipoles of radial direction situated at a depth of half the earth's radius, i.e., somewhat below the boundary of the core. They may have either sign and have an average magnitude of about 1 percent of the earth's moment. Elsasser (1941) compared the observed field with a model of a statistical nature. He assumed that a number of small magnetic dipoles are thrown at random into the volume of a spherical shell centered in the earth, and computed the mean (r.m.s.) values of the spherical harmonic coefficients of the potential produced at the surface of the earth. If, for instance, all the dipoles have radial direction and are located at the surface of a sphere of radius R_1 the result obtained is as follows: Let the γ_n^m in (25) and (26) be normalized. For the r.m.s. average of the corresponding coefficients α_n^m with fixed n and variable m one finds

$$[\alpha_n^m]_{\mathrm{Av}} = [(4\pi)^{\frac{1}{2}}/R_0^2 R_1](R_0/R_1)^n (n/2n+1)[M]_{\mathrm{Av}}, \tag{27}$$

where M is the moment of an individual dipole. The ratio of successive terms of this sequence for $R_1/R_0 = 0.55$ is given in the last line of Table XI. It is interesting to note that the expression (27) is independent of the number of small dipoles involved. This is found to be true so long as the number of dipoles remains moderately large (excluding, e.g., the case of a continuous, regular distribution). As the figures at the bottom of Table XI show, the ratios of successive terms approach rather rapidly to the value $0.55 = R_1/R_0$ as n increases. A similar behavior is characteristic of a number of statistical models investigated, provided the outer boundary of the volume available to the dipoles is the sphere of radius R_1. This shows that for a random distribution of dipoles the series at the boundary of the core converges extremely slowly. Now if one starts from the observed field at the surface and determines the potential at the boundary of the core by means of (26), he would expect on physical grounds that this latter series should not diverge. The statistical models with their extremely slow convergence thus appear as the limiting case of surface potential distributions which permit location of all their sources in the core.

In order to compare these results with the observations the rather complete analysis by Schmidt (1895) was used. All coefficients were converted to normalized harmonics and relative values were used, putting the coefficient of the main dipole field parallel to the earth's axis equal to 10,000. The values of Table XI are obtained by averaging for fixed n over a limited range of m (this is done because coefficients with large m are likely to be less accurate). The first and last columns of the Table have little significance; the former contains only the dipole terms perpendicular to the earth's axis, the latter is rather unreliable on account of observational errors. The table shows that the ratio of successive terms is somewhat below 0.55, thus securing convergence of the series at the boundary of the core. The distribution of sources is slightly more regular than would correspond to the statistical model, or else the sources are appreciably below the boundary of the core. The last alternative can be ruled out on physical grounds, as we shall see presently.

A somewhat different approach was chosen by Bullard (1948). He investigated the secular change in the neighborhood of South Africa, as shown in Vestine's maps. He then tries to represent the field of the secular variation for 1922.5 by a time-dependent dipole located slightly below the surface of the core. The dipole is horizontal and has roughly the direction from SE to NW. This dipole represents well the secular field at the surface in the region vertically above it. At larger distances the field of this dipole decreases somewhat more rapidly than the actual field; this indicates that the agreement would not be improved by putting the dipole at a level above the core. By means of a linear array of dipoles at the boundary of the core, in the direction from SE to NW, one can represent both the horizontal and the vertical component of the surface field over a rather large area. Bullard concludes that the geometrical construction exhibits no contradiction to the concept of sources concentrated in the core.

Using his spherical harmonic analysis Vestine (1947) has computed diagrams of spherical current sheets that would produce the observed field. He places these at various depths, the lowest at a depth of 3,000 km. For greater depth the extrapolated series would soon become divergent as we have seen. In Fig. 12 are shown the currents at 3,000 km depth that would produce the non-dipole part of the field, as of 1945, there being 10^6 amp. between adjacent lines. It must be assumed that in reality the pattern is vastly more complex, as the harmonic analysis stops at $n=6$ and the higher harmonics omitted will be relatively large at the boundary of the core. Such patterns therefore cannot be expected to have much fidelity in the details.

Next, we require some knowledge of the electrical conductivity in the earth's interior, especially in the

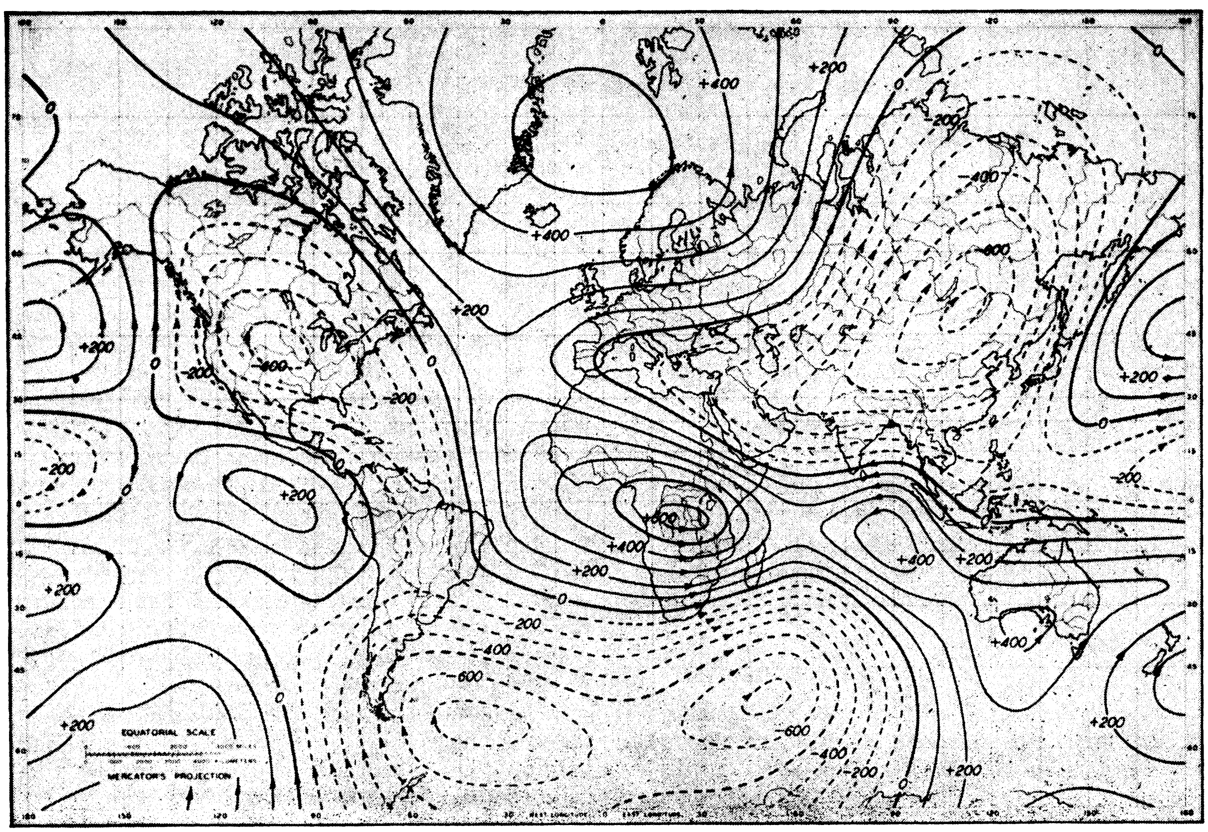


Fig. 12. Current-function in 10^6 amperes for thin spherical shell at depth 3000 km within earth to reproduce residual (non-dipole part) of main field, epoch 1945.

core. The conductivity of ordinary iron is 1.10^5 ohm^{-1} cm^{-1} ($=1.10^{-4}$ e.m.u.) There will be pressure and temperature effects which may be summarized roughly by writing $\sigma = C\cdot\theta^2/T$ where θ is the Debye temperature, T the absolute temperature. The factor C incorporates the changes in conductivity owing to the changed shape of the electronic wave functions whereas the second factor expresses the changes in the statistical behavior of the electron gas. The Debye temperature is proportional to the velocity of sound which in the core is about twice that of ordinary iron. Taking $T=9000°$ and assuming in the absence of other knowledge that C remains constant we find $\sigma = 1.3\times10^4$. Bullard (1948) has made an alternate estimate on using empirically determined pressure and temperature variations and on making an appreciable allowance for alloying and melting. He arrives at $\sigma = 1.10^3$. This seems too small to the present writer; in his latest paper (1949) Bullard uses an intermediate figure, $\sigma = 3.10^3$. This value will also be adopted here.

We can at once draw an important conclusion from these estimates of the conductivity, namely, that the currents which produce the time-dependent part of the field must flow in a thin layer just below the boundary of the core. A layer of metal acts as a shield for variable currents. Given a Fourier component of angular frequency Ω, the depth of penetration, d, into a plane sheet is expressed by the formula, used in the theory of the skin effect,

$$1/d = (2\pi\mu\sigma\Omega)^{\frac{1}{2}}. \tag{28}$$

The situation here is opposite from that of the skin effect, the field here originates inside the conductor and penetrates from there to the outside. The detailed analysis (1946) shows that variable currents flowing at a depth much larger than the critical depth (28) do not produce a field outside the core. Putting $2\pi/\Omega = 100$ years and $\sigma = 3.10^3$ we find $d = 50$ km. This thickness is so small that the result in turn justifies the use of the plane-sheet approximation (28). Furthermore, owing to the square root, the thickness changes only slowly when σ and Ω change. We may say with much generality that the currents producing the time-dependent part of the field originate in a layer that is certainly very thin compared to the radius of the core.

It is possible to estimate the conductivity of the mantle at least within certain broad limits. A glance at the records of individual stations collected by Vestine shows that Fourier components of 50 years in the secular variation are by no means uncommon. Hence these frequencies are not damped out by the mantle. Using this value in (28) together with $d = 2900$ km we obtain $\sigma = 0.5$. This, of course, is a crude estimate and gives no more than an order of magnitude. The average conductivity of the mantle should then be lower, say 0.1 or less. The conductivity of surface rocks on the other hand is $10^{-5}-10^{-6}$ (all in ohm^{-1} cm^{-1}).

There is another method of approaching the problem of conductivity in the mantle, developed by Chapman and several of his pupils. We follow here the latest paper, by Lahiri and Price (1938). The method is based on the fact that there are currents in the ionosphere of short period (e.g., diurnal). These in turn induce currents in the solid earth whose magnitude and phase depend on the conductivity. The theory was developed for a conductivity that is a function of depth. The spherical harmonic analysis of the observed field has been carried out for a number of leading terms and the external and internal fields separated. Lahiri and Price find that the observations can be fitted by the theory only if the conductivity remains very low for some depth and then increases rather rapidly to not much less than 10^{-2} at a depth of about 6–700 km. Not too much emphasis should perhaps be placed on the quantitative side of the results since they are based on a rather limited observational material.

Coster (1948) has investigated the conductivity of a number of rocks, olivine among them, and has found that they exhibit the typical temperature dependence of semiconductors.

DYNAMICS OF THE CORE

We may assume that in the core there exist simultaneously fluid motions and electric currents. The two interact with each other, producing a number of rather complex dynamical phenomena which will be outlined in this chapter. We shall first inquire into the causes of the motions so as to have as far as possible a concrete physical picture. Since the evidence for such motions as gathered from the geomagnetic variation is quite strong, the correct approach would be to postulate the existence of motions in the core irrespective of their presumptive origin, leaving the discussion of their causes for later. The reader might, however, prefer to have a somewhat more concrete picture in his mind; for this reason we have inverted the logical order and shall first deal with assumptions about the causes of the motions. It is clear that the theory of the interaction between the field and the moving fluid, to be outlined thereafter, is essentially a chapter of general dynamics and does not depend on any specific assumptions as to the nature of the primary power supply.

Two causes of motions in the core suggest themselves readily; a rather diligent search has so far failed to unearth others. One of these is found in the variation of the earth's angular velocity of rotation as astronomically observed, and attributed to tidal friction. The other is based on the assumption of a sufficient temperature gradient in the core so as to produce thermal convection. These two mechanisms have been studied in detail by Bullard (1949) whom we follow with some modifications. As will be indicated below, the magnetic field and the fluid motions in the core are very closely coupled so that a transfer of energy from the fluid to the field and vice versa does constantly occur. It so becomes necessary to consider two dissipative agencies, namely, mechanical friction in the fluid and Joule's heat in the electric current system. The postulated sources of power must maintain the motion against this dissipation. Numerical estimates given later make it likely that mechanical friction is negligible compared to electromagnetic dissipation. The latter can be estimated from the formulas given below to be about 5×10^{-18} cal./1 cm^3 sec. Our hypothetical model must generate this amount of power.

Bullard's study leads to the result that of the two potential sources of motive power only thermal convection can produce the observed motions. The deceleration of the earth's rotation which is attributable to friction of the lunar tide (Jeffreys, 1929) is found to have a negligible effect upon the dynamics of the core. The writer had believed for some time past that lunar perturbations play a significant role in the core's motions, but it was more recently found that the conclusions rested on an error. Recent studies of a variety of lunar effects (1950) have verified that no lunar perturbation, tidal or precessional, has an effect on the core large enough to maintain the motions whose magnitude is indicated by the geomagnetic secular variation. The astronomical data regarding the non-uniformity of the earth's rotation have been reviewed and re-evaluated by Spencer Jones (1939). The effects upon the liquid core have been studied from the purely hydrodynamical viewpoint (Bondi and Lyttleton, 1948) and turn out to be exceedingly small. This writer has now come to believe that the theory of geomagnetism can be developed without reference to these astronomical phenomena which seem to be of separate causation.

We therefore consider thermal convection in the core as a means of generating fluid motions. An earlier attempt (Frenkel, 1945) led to satisfactory qualitative results; later Bullard gave a very clearcut thermodynamical analysis that we shall follow here with some additions. The convective stratum may be presumed to consist of all of layer E (see Fig. 3) about 2000 km deep. Since a uniform generation of heat in the layer would not produce convective conditions, there must either be an inflow of heat through the lower boundary or an outflow through the upper boundary. The outflow, again, produces convection only if it exceeds the purely conductive flow of heat inside the core itself. We might, perhaps, expect that the conductive heat flow in the mantle is less than that in the core, a condition that we may designate as thermal insulation of the core, expressed by

$$\kappa_m\tau_m < \kappa_c\tau_c, \tag{29}$$

where κ is the heat conductivity, τ temperature gradient (m and c designating mantle and core, respectively). For τ_c we must use the adiabatic gradient, by (18). Bullard estimates this as 1.1°/km in the core. Again, if Jeffrey's model of the earth holds, τ_m is the melting point gradient which will be several times larger than τ_c. It is comparatively safe and easy to estimate κ_c by the Wiedemann-Franz law, $\kappa/\sigma T=$const., which gives a temperature-independent κ. The value $\kappa=0.18$ cal./cm sec. degree corresponds to the adopted value of σ by virtue of the universal Wiedemann-Franz relation (it also is the laboratory value for iron). It is well nigh impossible to obtain a good estimate of κ_m as the subject of thermal conduction in insulators is very little explored. Omitting pressure effects that are simply unknown, theory indicates that $\kappa\sim T^{-1}$ (Peierls, 1928). Under laboratory conditions the conductivity of olivine is about 1/14 of that of iron. Combining all these data, there results a presumption to the effect that (29) is fulfilled, and we shall therefore assume for the time being that the core is thermally insulated.

The simplest model under these conditions is one in which radioactive sources of heat are concentrated in the central body. This seems a very likely assumption since the simple oxides, UO_2 and ThO_2, have (under laboratory pressure) densities around ten. To estimate the amount of heat required, we note that the limiting efficiency, ϵ, of the convective process considered as a thermodynamical engine is, by the second law,

$$\epsilon=\Delta T/T=D\tau_c/T,$$

where D is the mean depth over which convective transport is active. We take $D=10^8$ cm, half the depth of layer E. Even with the extreme value, $T=10{,}000°$, this gives an efficiency, $\epsilon=11$ percent. The assumed value of τ_c might be too large, but in any event ϵ is of the order of a few percent which will turn out to be ample for the purposes required. As Bullard points out, there is good reason to believe that the convective machinery is rather efficient so that the actual efficiency is not lower by orders of magnitude than the theoretical limit.

In order that convection may actually occur, the heat available at the lower boundary of layer E must exceed $\kappa\tau$, the heat carried away by conduction alone; we might assume the total heat to be a small multiple of $\kappa\tau$, say $2\kappa\tau$. As (18) shows, τ is likely to be smaller at the bottom of the layer than at the top; furthermore the temperature of 10,000° on which the previous estimate of τ_c was based is rather extreme; we therefore propose to adopt a somewhat lower estimate, $\tau=0.3°$/km, valid at the bottom of layer E. The convective heat supplied to 1 cm^3 of the layer is then 7×10^{-16} cal./sec. which is 140 times the heat output due to electromagnetic dissipation as quoted above. The total rise in temperature of layer E during the lifetime of the earth is of the order of 100°, not enough to cause any appreciable effects. Again, the heat supplied to the surface of the central body is $1.1\cdot10^{-6}$ cal./cm^2 sec., and this might be compared to the mean outflow of heat from the earth's surface estimated (edited by Birch, 1942) at $1.3\cdot10^{-6}$ cal./cm^2 sec. Since the latter flow is principally due to the radioactive heat output, we might say that the convective mechanism must be operative if the radioactive material of the earth is preferentially concentrated not only in the crust, but also in the central body, and provided the radioactive content of the central body is to that of the crust at least in the ratio of their surface areas, that is 1:20.

In a forthcoming paper Bullard (1950) specifies his previous thermodynamical analysis by proposing a model of convection in which it is assumed that the core is not thermally insulated. This condition can arise in two ways. Either (29) is not fulfilled which, in view of the large errors attendant especially on the extrapolation of the properties of the silicates to the lower mantle, is admittedly possible; somewhat more likely is the presence of slow viscous convection of the type briefly discussed in the third chapter. If such convection occurs it can be shown to provide an adequate means of removal of the heat. In this model the radioactive material may be assumed to be uniformly distributed through the core; also the amount of radioactivity required is somewhat less than in the previous model, of the order of that found in iron meteorites. There are advantages and disadvantages to either of these models and the decision between them will have to await future progress.

We now proceed to the main subject of this chapter, the electromagnetic phenomena of the core and their relation to fluid motions. First, consider the core as a large metallic sphere without internal motions. We are interested in relatively slow changes of the magnetic field and hence we can neglect the displacement current in Maxwell's equations. The latter then read

$$\nabla\times\mathbf{E}+\partial\mathbf{B}/\partial t=0, \quad \nabla\cdot\mathbf{B}=0, \tag{30}$$

$$\nabla\times\mathbf{B}=4\pi\mu\mathbf{J}, \tag{31}$$

where the susceptibility μ will be assumed constant throughout, neglecting ferromagnetic effects. In the absence of mechanical motions we have $\mathbf{J}=\sigma\mathbf{E}$, and σ could be assumed a function of r, but will for simplicity be taken as constant throughout the core, having the value $\sigma=3\times10^3$ ohm^{-1} cm^{-1} given previously. By a familiar procedure we find

$$\nabla^2\mathbf{B}=4\pi\mu\sigma\partial\mathbf{B}/\partial t. \tag{32}$$

One obtains exponentially decaying solutions, "free modes," by putting

$$\mathbf{B}=\mathbf{B}'(r,\vartheta,\varphi)e^{-t/t_0}$$

Fig. 13.

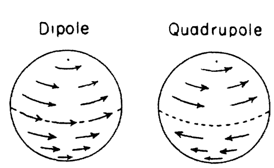
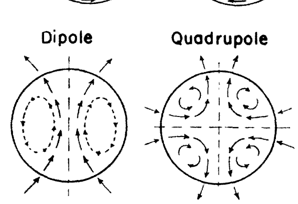


Fig. 14.



where the vector fields $\mathbf{B}'$ are expressible in terms of Bessel functions and spherical harmonics. They form an orthogonal system of vector functions over the sphere.[1] It is important to note that there are *two* types of free modes, schematically illustrated by Figs. 13 and 14. We shall explain their difference for the special case of rotationally symmetrical modes (zonal harmonics). In the first type of free modes, the "magnetic modes," the arrows of Fig. 13 designate electric currents (shown projected upon the surface of the sphere). The corresponding magnetic fields will in general have the form shown by the solid arrows of Fig. 14 (in meridional cross section). The field will extend to the outside of the metallic sphere, but current configurations are possible where it is confined to the inside, as indicated by the dashed arrows. The second type of free modes, the "electric modes," have currents given by the dashed arrows of Fig. 14. The corresponding magnetic fields are those shown in Fig. 13, that is, the magnetic lines of force are now circles about the axis. It can be shown that the magnetic field of these modes vanishes rigorously outside the metallic sphere, so that if such a field exists inside the core it cannot be detected by measurements at the earth's surface. There are, however, reliable indirect methods of inferring its existence as we shall see presently. This type of internal magnetic field will be designated as "toroidal" field, as distinct from the more conventional field of the magnetic modes (which might be called the "poloidal" field).

The mean life of the free, exponentially decaying modes is

$$t_0=a^2R^2\cdot4\pi\mu\sigma, \tag{33}$$

where R is the radius of the core and a is a numerical constant which becomes rapidly small for the higher order modes. For the lowest magnetic dipole mode $a=\pi$, giving $t_0\sim15{,}000$ years. The lifetime of the lowest electric mode is of comparable magnitude. For the sun, by the way, Cowling (1945) has computed the variation of σ with depth, arriving at lifetimes of the lowest free modes of the order of 10^{10} years.

We now consider the electromagnetic effects of motions in the core. If a metallic conductor moves in a magnetic field a current is induced in the conductor, the total current being,

$$\mathbf{J}=\sigma\mathbf{E}+\sigma\mathbf{v}\times\mathbf{B}, \tag{34}$$

where the second term represents the "motional" induction. $\mathbf{v}$ is the local velocity of the fluid. It can be shown that (34) accounts fully for the electromagnetic effects of motions in a fluid conductor, apart from entirely negligible relativistic terms. If (34) is substituted into (31) we have in (30) and (31) the full description of the electromagnetic field in the moving conductor. It should be remarked that these equations do not change if we go from a system at rest to a uniformly rotating system of

[1] J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill Book Company, Inc., New York, 1941), Chapter 7.

reference; hence it is for most purposes adequate to use coordinates that are fixed relative to the rotating mantle.

The phenomena described by these equations were first noticed in connection with solar and sunspot magnetism (Cowling, 1934; Alfvén, 1942). They can be reproduced in the laboratory only under very favorable conditions, the reason being that in the cosmic and geophysical phenomena the free decay periods of the electric currents are large compared to the periods of the mechanical motions, whereas the converse would be the case in bodies of conventional size. Only quite recently the coupling between a magnetic field and sound waves in mercury has been observed in the laboratory (Lundquist, 1949).

In a special case these equations can at once be integrated by conventional methods. If the fluid moves in a large constant field, we may insert the corresponding value of **B** into (34) and interpret the current in (31) as due to a given impressed e.m.f. Along these lines it is possible to account for the secular variation of the earth's magnetic field as a perturbation of the main field caused by the motion of the fluid in the top layers of the core (Elsasser, 1946; Bullard, 1948). As the fluid moves, secondary electric currents are induced in it; these in turn give rise to the secondary non-dipole field and its secular variation. Without going into details it might be said that such an analysis verifies the interpretation of the centers of the secular variation as hydrodynamic entities, specifically, as regions of convergence and divergence of the fluid in the top layers. It may well be assumed that they are at the same time vortices, but the circulating motion of a vortex parallel to the surface gives rise to a field of toroidal character on the inside that does not have a counterpart outside of the core. What we perceive are the effects of convergence and divergence which are connected with such vortices on general dynamical grounds.

One can estimate the magnitude of the fluid velocities required to produce the observed secular variation. This can be done in two independent ways. First we can use direct inspection. The westerly drift motion of 0.18° per year is typical of the rate of change of the geometry of the secular pattern; on the equator at the boundary of the core this represents a linear velocity of 0.034 cm/sec. Secondly, one obtains a velocity from the induction mechanism. Figure 12 gives values for the current densities pertaining to the secular variation in a two-dimensional sheet near the boundary of the core. As a typical average we may take $j=2$ amp./cm. By (34) we may write for the order of magnitude of the velocities, $v\sim 4\pi j/d\sigma B$, where d is the skin-depth; from (28) we have $d=50$ km. Taking $B=4$ gauss at the boundary of the core we find $v=0.04$ cm/sec.

We shall now obtain some more general results regarding the interaction of the moving fluid with the magnetic field. In place of (32) we have in the moving fluid

$$\partial\mathbf{B}/\partial t=\nabla\times(\mathbf{v}\times\mathbf{B})+(4\pi\mu\sigma)^{-1}\nabla^2\mathbf{B}. \tag{35}$$

The first term on the right-hand side describes the motional induction, the second the effects of free decay. In order to estimate their relative proportion in bodies of different size we might, following Bullard (1949), use the principles of dimensional similarity. We may define a "magnetic" Reynolds number as

$$R_m=4\pi\mu\sigma Lv, \tag{36}$$

where as usual L is a representative length, and v is a representative velocity. Setting in the customary way $L=2R$ and taking $v=0.04$, we find $R_m=1060$. If v is actually larger inside the core, as we shall find reason to believe later on, R_m will be quite large. Since R_m measures the ratio of the first to the second term on the right-hand side of (35), the dissipative term is small and may be omitted in a first approximation. A known vector identity then gives

$$\partial\mathbf{B}/\partial t+(\mathbf{v}\cdot\nabla)\mathbf{B}=(\mathbf{B}\cdot\nabla)\mathbf{v}-\mathbf{B}(\nabla\cdot\mathbf{v}).$$

We may safely drop the last term, considering the fluid as incompressible. The left-hand side is nothing but the "substantial" derivative in the usual hydrodynamical sense, thus

$$d\mathbf{B}/dt=(\mathbf{B}\cdot\nabla)\mathbf{v}. \tag{37}$$

These fundamental equations can be given a simple physical meaning. They are analogous to the well-known Helmholtz equations for the conservation of vorticity in an incompressible fluid; they become indeed identical with the Helmholtz formulas if **B** designates the vorticity. The proof of Helmholtz's main statements is independent of the functional relation between vorticity and velocity (the former being the *curl* of the latter); hence these theorems are true for an arbitrary vector fulfilling (37). We can enunciate them by replacing the usual vortex lines and vortex tubes of hydrodynamics by the lines or tubes of the magnetic flux. The theorems then say in the first instance that lines of force can neither be created nor destroyed in the fluid; as the fluid moves, the lines of force move so as to remain attached to the individual particles of the fluid. (Alfvén used these results but did not provide a formal proof.) In the second place one can show (Cowling, 1934) that

$$\int \mathbf{B}_n dS=\text{const.},$$

where this is true in the sense that the substantial derivative, d/dt, of this integral vanishes, and also in the sense that the magnetic flux remains constant along a tube whose boundary is formed by the same lines of force. All this is true, of course, only in the approximation in which free decay is neglected. If free decay is

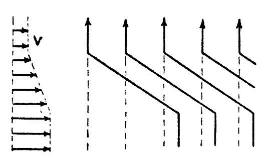


FIG. 15.

taken into account, its effect upon the magnetic lines of force is equivalent to the effect of viscous friction upon the vortex lines of an incompressible fluid (with $1/4\pi\mu\sigma$ as "magnetic" viscosity).

Before proceeding to applications of (37) we may note that our equations imply an exchange of energy between the field and the fluid motion. From (35) one finds after some calculations, on omitting the free decay terms,

$$\tfrac{1}{2}(\partial\mathbf{B}^2/\partial t) = \mathbf{v}\cdot(\mathbf{B}\times\nabla\times\mathbf{B}), \tag{38}$$

where the term on the left is clearly $4\pi\mu$ times the rate of change of the magnetic energy density; this quantity can assume either sign, because the sign of the right-hand side depends on that of $\mathbf{v}$. Since the fluid motion acts on the field, there must clearly be also an action of the field upon the motion; this is the ponderomotive force,

$$\begin{aligned}\mathbf{F} &= \mathbf{J}\times\mathbf{B}\\ &= -(4\pi\mu)^{-1}\mathbf{B}\times\nabla\times\mathbf{B}.\end{aligned}$$

The work done on the fluid by this force per unit time, $\mathbf{v}\cdot\mathbf{F}$, is the negative of the change in field energy (38) as it must be.

When energy is transferred from the moving fluid to the field we may use the term "amplification." The simplest such occurrence is shown schematically in Fig. 15. Assume that there is originally a magnetic field in the y direction (dashed lines) and that at time $t=0$ a fluid motion is started that is in the x direction but has a gradient in the y direction as indicated by the velocity profile at the left of the figure. In the absence of free decay the lines of force become slanted as shown; an x component of $\mathbf{B}$ is generated whose magnitude increases proportional to t. Eventually the fluid motion will be slowed down by the ponderomotive forces exerted by this field. Variations of this simple amplifier mechanism must play a fundamental role in the dynamics of the core, as may be seen as follows. The existence of the westerly drift motion of the secular variation indicates in all probability that the core does not rotate uniformly, as a rigid body, but that the angular velocity inside the core is variable with the depth. Such a motion will deform the magnetic lines of force of the dipole field in a manner schematically shown in Fig. 16. Assume that we first have a dipole line of force in a meridional plane (dashed line in Fig. 16). Assume then for simplicity that the inner part of the core rotates relative to the outer, the latter remaining fixed. The lines of force now get dragged around the circles of latitude as shown, and if this non-uniform rotation continues long enough, the lines of force are "wrapped around" the axis of rotation in nearly a system of circles. A more detailed analysis (1947) shows that if the initial field has the symmetry of a magnetic dipole mode, the toroidal field generated is an electric quadrupole mode whose lines of force are circles centered on the axis of rotation, the directions of the toroidal field vector being opposite in the two hemispheres. The most significant feature of this particular mechanism is its lack of reciprocity: there is no similar interaction whereby, starting from a toroidal field, one can produce a poloidal field of the type represented by the original dipole. To prove this we note that the lines of force of the toroidal field are circles centered on the axis; it is clear that any rotationally symmetrical motion transforms a family of circles into another such family; hence it transforms any toroidal field again into a toroidal field. Since the amplifier mechanism described is very powerful, one can surmise the existence of a toroidal field that is larger than the dipole field. Bullard has calculated the equilibrium between induction and decay of a toroidal field produced by the mechanism of Fig. 16, the primary dipole field being kept fixed. For simplicity he assumes a discontinuity rather than a gradual change of angular velocity. He finds that the rate of amplification is remarkably insensitive to the detailed structure of the dipole field as well as to the size of the smaller volume that rotates relative to the outer part of the core.

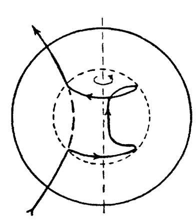


FIG. 16.

The preceding results suggest the idea that the whole magnetic field of the earth is ultimately produced and maintained by a mechanism of induction, the magnetic energy being drawn from the kinetic energy of the fluid motion. This could be achieved if one could construct a "feed-back" mechanism whereby a part of the magnetic energy of the toroidal field is returned to the original dipole field in such a way that it reinforces the latter. In view of the theorem stated in the preceding paragraph such a mechanism, if it exists, must lack rotational symmetry about the earth's axis. Hence one might expect that the direct process of increasing the toroidal field as shown in Fig. 16 is the main step of amplification where most of the magnetic field energy is generated. From this viewpoint it becomes probable that the toroidal field is at least several times larger in the average than the observable dipole field. The figure for the thermal dissipation of the magnetic field given in the beginning of this chapter was estimated under the assumption of an average field in the core of 35 gauss.

If the non-symmetrical interactions are investigated (1946–47) they turn out to be very numerous but also in the main very complicated. Bullard has found a simple model that involves only the superposition of two types of motion. One of the two components of motion is the non-uniform rotation about the axis, the other consists of two ascending streams centered on diametrically opposite points on the equator together with two descending streams centered on points on the equator midway between the former. One finds by a geometrical analysis that the combined motion will deform the toroidal field in such a way that part of it is returned to the dipole mode with the correct sign so as to re-inforce it; this effect takes place whatever the sign of the subsidiary component of the motion.

If such a self-amplificatory model of the earth's field is to be truly satisfactory it must exhibit a high degree of stability so as to be able to account for the maintenance of the field over long periods. This requires consideration not only of the induction process but also of the ponderomotive forces. It would be very difficult indeed to prove stability for a specific model. Stability can, however, be of a different type: it can consist in the average recurrence of certain patterns, that is, it can be statistical. We shall very briefly outline some statistical aspects of the problem of amplification. The magnetic charts of the preceding chapter give evidence of a great deal of irregular motion in the core which, on emphasizing its random character, we may designate as large-scale turbulence, a term commonly used in this sense in dynamic meteorology to designate the ensemble of major atmospheric perturbations.

First, let us ask whether there exists a correlation between the local velocity and the local magnetic field. By (34) the magnetic induction vanishes when $\mathbf{v}$ and $\mathbf{B}$ are either parallel or antiparallel. By (39) the same is the case for that part of the ponderomotive forces that is generated by the induction process. Considering the same problem for sunspots, Gurevitch and Lebedinsky (1946) have concluded that there must exist a very strong correlation between field and velocity whenever the velocity is so large that free decay effects become negligible. The streamlines and the lines of magnetic force will tend to be parallel (or antiparallel) to each other. This simple principle might be said to give us a zero-order approximation to the relationship of motion and field. If the two vectors are not parallel or antiparallel, energy transfer will in general take place and this, as we have seen, can readily occur in either direction.

It might be expected that the "spectrum" of the turbulent motion must have a fairly well-defined upper "cut-off." By virtue of (36) we can assign to each velocity a length, $(4\pi\mu\sigma v)^{-1}$, such that for eddies smaller than this critical size the magnetic quasi-viscosity predominates and tends to damp out the smaller eddies. This is in complete analogy to the action of mechanical viscosity in ordinary turbulence. For the velocity given above, of 0.04 cm/sec., the corresponding size is about 10 km, much smaller than the observed elements of the secular variation. Much of the fine detail of the secular variation, however, if not almost all of it, is no doubt wiped out by the shielding action of the weakly conducting mantle.

The details of the turbulent pattern will depend on the particular model of convective transport. If the heat is supplied by the central body, there must be a pronounced increase of convective activity with depth in layer E. The velocities should be larger at greater depths and the eddies smaller. It might be noted that such a model would also permit us to account for the pronounced preponderance of the earth's dipole terms (see Table IX). The screening action of the outer layers of the core tends to suppress all but the lowest harmonic of the field originating at great depth. The observed higher harmonics, from the quadrupoles on, can no doubt be attributed to the motions in the top layers of the core.

We have seen that thermal dissipation of the currents is small owing to the large numerical value of (36). Frictional dissipation in the fluid may be compared to the electromagnetic dissipation by forming the non-dimensional quantity

$$R_m/R_h = 4\pi\mu\sigma(\nu/\rho), \tag{40}$$

where R_h is the usual "hydrodynamical" Reynolds number, and ν/ρ the specific viscosity. If ν is comparable to unity as it is in the liquids of simple molecular constitution at laboratories pressures, then (40) is numerically small, indicating that the heat of mechanical friction is small compared to Joule's heat. As all dissipative effects are small compared to the mutual interaction of fluid and field, it follows that the generation of motion by thermal inhomogeneities of the fluid, which in the stationary state equals the dissipation, must also be small. Hence the core forms a nearly closed electro-mechanical system. The turbulent motion may then be expected to distribute the available energy in some definite statistical fashion over the accessible degrees of freedom, mechanical and electromagnetic. In order to obtain an estimate of this distribution we consider the hydrodynamic equations of motion in which the ponderomotive forces (39) appear on the right-hand side,

$$\rho\partial\mathbf{v}/\partial t + \rho(\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla p - (4\pi\mu)^{-1}\mathbf{B}\times(\nabla\times\mathbf{B}). \tag{41}$$

On the left-hand side we use the vector identity

$$(\mathbf{v}\cdot\nabla)\mathbf{v} = \tfrac{1}{2}\nabla(\mathbf{v}^2) - \mathbf{v}\times(\nabla\times\mathbf{v}).$$

We have neglected the frictional forces, and in this approximation we may put $\rho=$ const. We now consider stationary isotropic turbulence and form a suitable average over time and space. The term $\partial\mathbf{v}/\partial t$ and all the gradient terms may be assumed to average to zero and (41) becomes

$$\rho[\mathbf{v}\times(\nabla\times\mathbf{v})]_{\mathrm{Av}} = (4\pi\mu)^{-1}[\mathbf{B}\times(\nabla\times\mathbf{B})]_{\mathrm{Av}}. \tag{42}$$

The simplest way of fulfilling (42) is clearly to assume "equipartition,"

$$[\rho\mathbf{v}^2/2]_{\mathrm{Av}} = [\mathbf{B}^2/8\pi\mu]_{\mathrm{Av}}. \tag{43}$$

While this does not of course constitute a formal proof, it makes the existence of equal amounts of energy in the fluid and field rather plausible. It should be noted that from the viewpoint of the present theory the fraction of magnetic field energy outside the core itself is very small.

Taking $\mathbf{B}=35$ gauss in (43) we obtain an average velocity of 3 cm/sec. Since this is about 100 times the result of the preceding estimates, the discrepancy is serious. It is unlikely that the mean value of 35 gauss is overestimated by more than a factor of two; hence the previous estimates must be too small. In the case of the westerly drift motion it is indeed plausible that this velocity component is much smaller than the average turbulent velocity. The drift motion must needs be of a transient and temporary nature. This follows from the fact that a differential rotation of core and mantle engenders a frictional and an electromagnetic torque which tend to equalize the angular velocities. Such a differential rotation could only be maintained in the long run by some perturbation effect of the moon, but it can be established that all lunar effects are too small to produce differential rotation of this magnitude. Hence the westerly drift must be in the nature of a transient (perhaps comparable in its general character to the present-day decrease of the earth's magnetic dipole moment). Hence we might well expect other velocity components to be larger than those given by the drift.

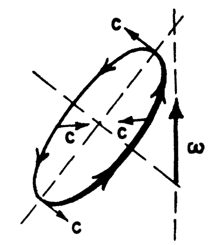
As far as the second method of estimate, from the currents of Fig. 12, is concerned there are several reasons why the result might be considerably too small. In the first place this method gives us only the velocity component perpendicular to the magnetic lines of force; it says nothing about the parallel component which should be much larger. Secondly it is quite possible that velocities in the layer immediately adjacent to the boundary, which are the ones to be seen in the secular variation, are smaller than the velocities in the free liquid at greater depth, especially if a convective model of the core heated from the inside holds. Finally, the projection from the surface of the earth to the boundary of the core as carried out by spherical harmonic analysis might give currents that are too small since much detail is lost in the process.

Note added December, 1949.—Recent quantitative studies carried out by C. Swift in this laboratory (to be published later) lead to the result that a series of spherical harmonics is indeed inadequate for such projection owing to its slow convergence. A satisfactory method of projecting the relatively localized features, as shown in Vestine's maps, has been developed. One finds that the centers of the secular variation are stronger at the boundary of the core than at the earth's surface by a factor of the order of ten. This should remove so much of the above-mentioned discrepancy that the remainder is no longer very serious.

A model of isotropic turbulence would leave out of account the earth's rotation. One might expect a realistic dynamical model of the core to be somewhere intermediate between isotropic turbulence and simple amplification of the toroidal field by non-uniform rotation. Hence one should make allowance for the effect of the earth's rotation upon the turbulence motion. Let s be the distance from the earth's axis. If s changes for a given fluid particle, the latter will tend to conserve its angular momentum, $\rho s^2\omega$. For infinitely rapid turbulent mixing the fluid would approach a distribution of angular velocity, $\rho s^2\omega=\text{const.}$, in which the outer parts rotate much slower than the inner parts of the core. Owing to the large electromagnetic and turbulent stresses the actual angular velocity gradient must be very much smaller than this limiting value. It is seen that any convective type of motion engenders of necessity a non-uniform rotation of the kind required to amplify the toroidal field. The non-uniform rotation is of the same sign as the westerly drift but, for the reasons indicated, the latter is likely to be a transient fluctuation, appearing near the surface, of a larger relative increase of angular velocity at greater depth.

Figure 17 shows the direction of the Coriolis forces acting on a typical eddy which is here drawn inclined relative to the earth's axis. The solid line is a streamline (and also, in a crude approximation, a magnetic line of force). The Coriolis forces reverse their sign when the direction of the motion is reversed. It is seen that there

FIG. 17.



are now forces in the meridional plane which will give rise to motions and magnetic lines of force in these planes. In a theory of the maintenance of the field it will have to be shown that the meridional components of the eddy field do not average out, but have an excess in such a direction as to reinforce the original dipole field. From the very simple notions given here to such a statistic-dynamical theory there is no doubt a long route which has not yet been traveled. The general view which we try to express is that the true state of the core is probably intermediate between the large-scale models containing the toroidal field and small-scale convective turbulence. Bullard's success with a large-scale model of feedback qualitatively worked out, is encouraging. If the general theory of the state of the core as given here is accepted on the basis of its agreement with a large number of geophysical data, then there must necessarily be magnetic fields in the moving metallic fluid, but these could *a priori* be in random directions. The existence of the main dipole field might be taken as evidence for a line-up of eddy fields along the earth's axis, and there is hardly any other way this can be achieved than by the effect of the Coriolis force. It might of course be that the magnetic field itself acts upon the fluid in such a way as to be self-stabilizing once it has come into existence, but this does not seem very likely. In a model with turbulent eddies there is evidently much leeway for changes in magnitude and direction of the earth's field during its past history. It seems therefore to be in good over-all agreement with the observational evidence as summarized previously.

The theory of magnetic fields in moving, electrically conducting media forms a broad chapter of dynamics; it assumes hardly more than that the effects of free decay be not large compared to the motional induction. Cowling's studies of the sun indicate that a celestial body of appreciably larger size than the earth will practically never lose a magnetic field by the action of decay. There is, on the other hand, no limit to the rate of inductive amplification other than the velocity of the fluid. Thus magnetic fields should be a common phenomenon in the universe, and this is in agreement with recent astrophysical findings (Babcock 1947–8). Large-scale motion of fluids is always turbulent, and practically all fluid matter in the universe is hot enough to be a good electrical conductor. Hence turbulent motions should be unstable until magnetic fields have been generated of the general order of magnitude given by (43). It would appear, therefore, that the earth's magnetic field is a sample, close at hand, of a widespread phenomenon of great cosmological interest.

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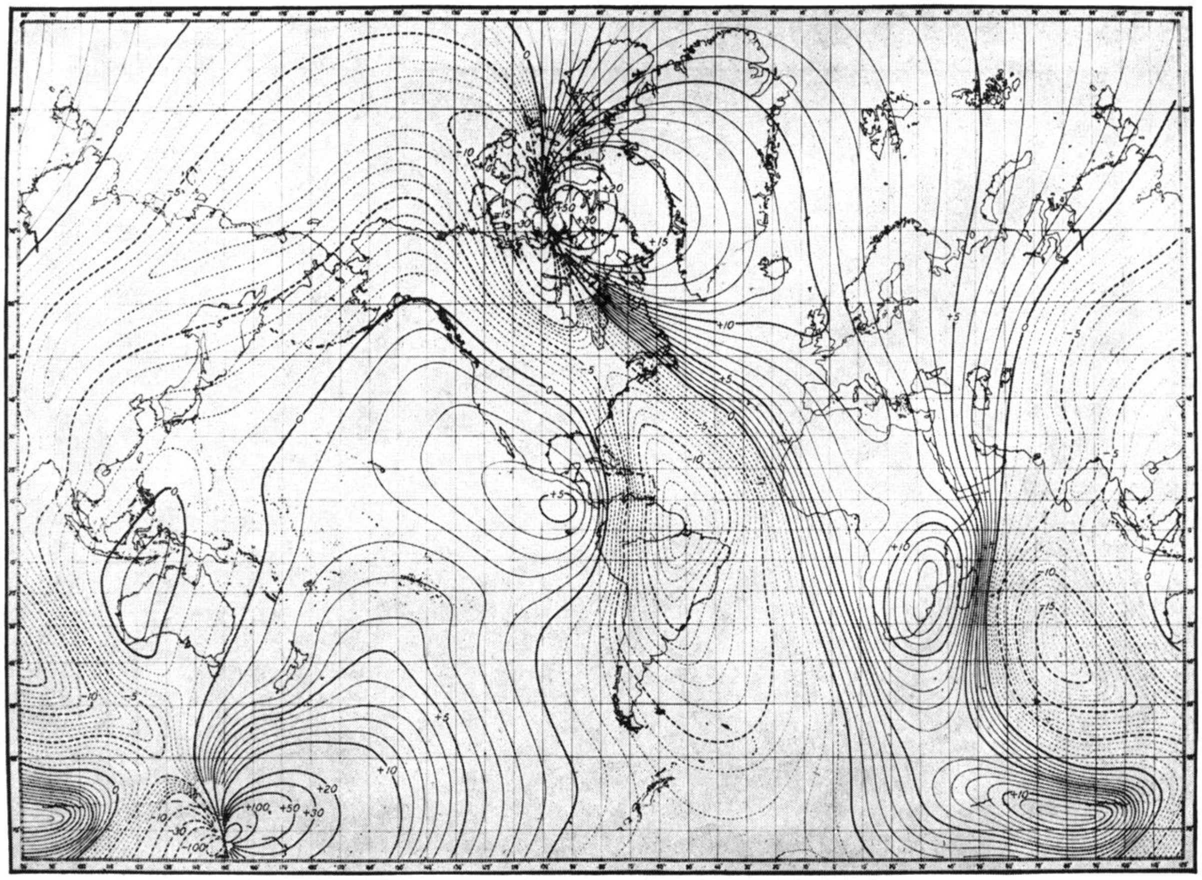

FIG. 10. Geomagnetic secular change in minutes per year, declination, epoch 1912.5.

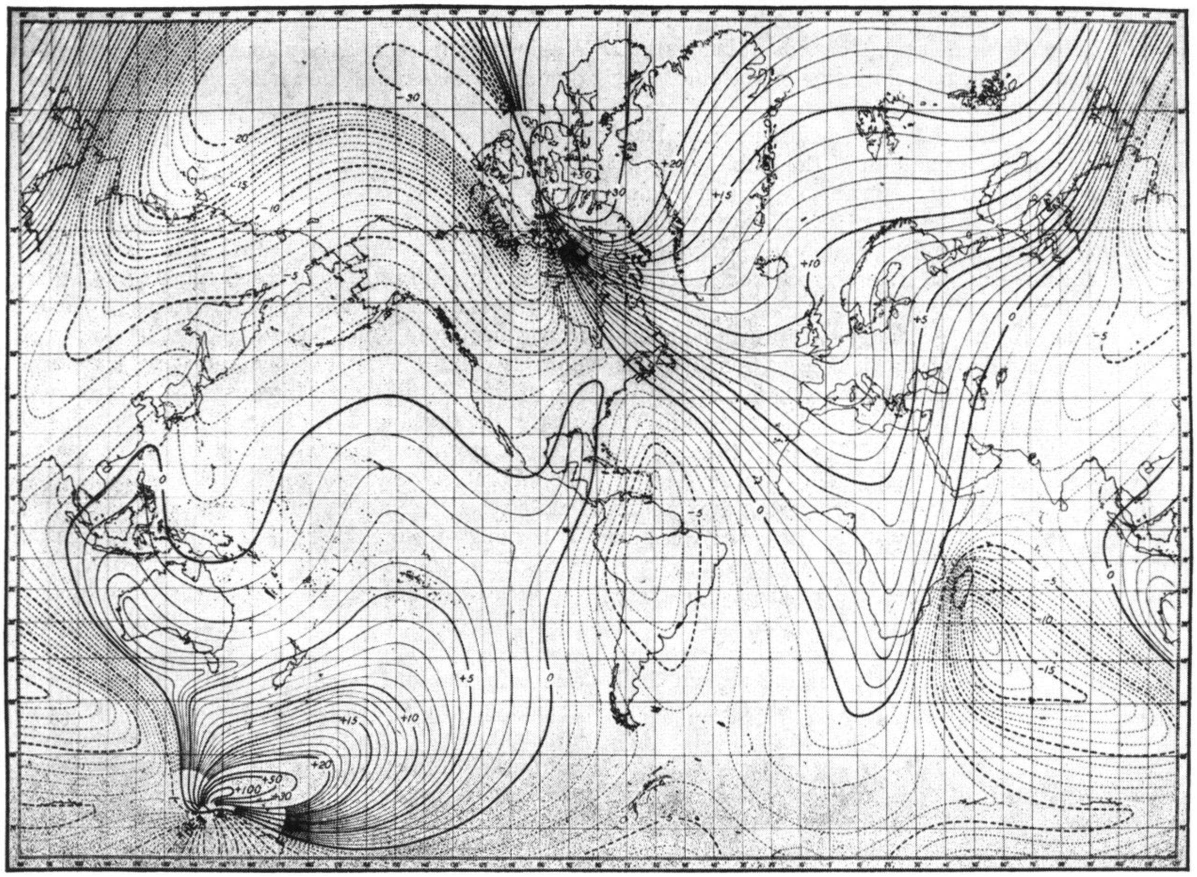

FIG. 11. Geomagnetic secular change in minutes per year, declination, epoch 1942.5.

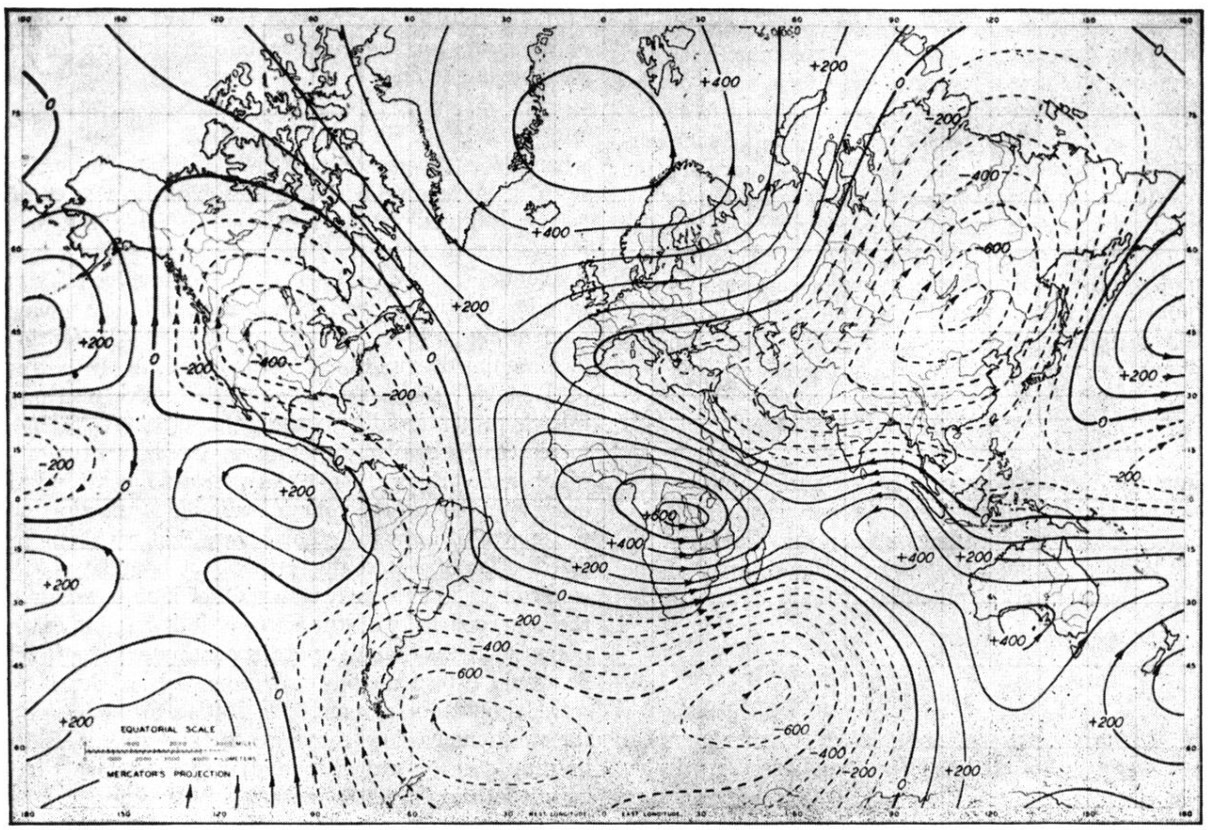

FIG. 12. Current-function in 10^6 amperes for thin spherical shell at depth 3000 km within earth to reproduce residual (non-dipole part) of main field, epoch 1945.

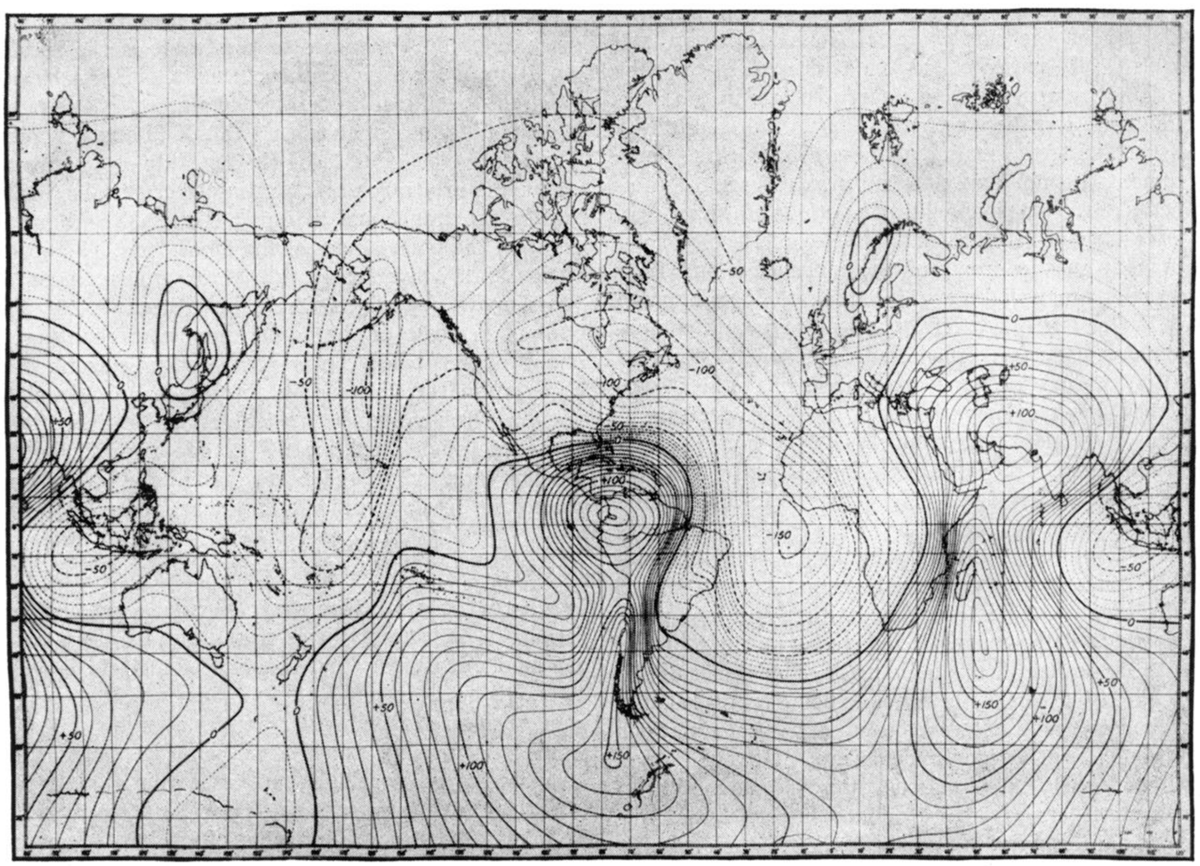

FIG. 8. Geomagnetic secular change in gammas per year, vertical intensity, epoch 1912.5.

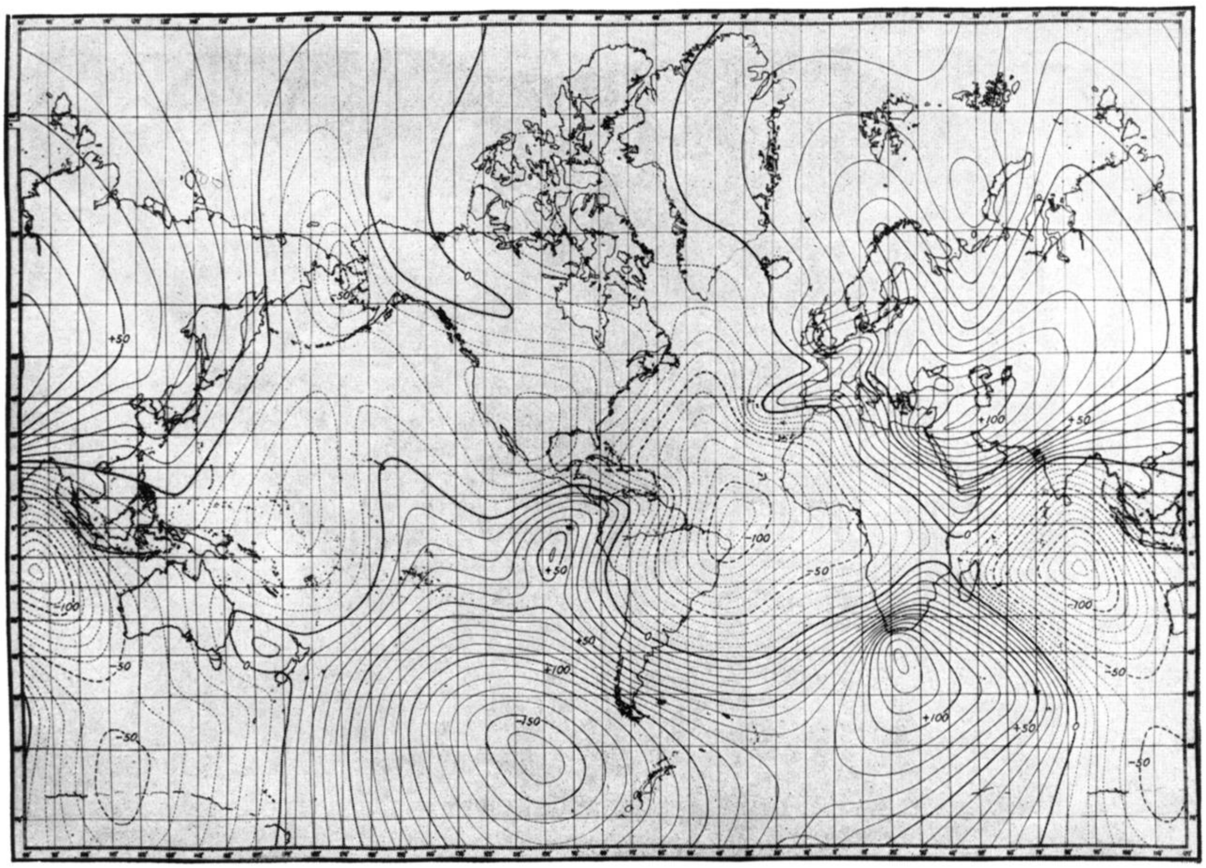

FIG. 9. Geomagnetic secular change in gammas per year, vertical intensity, epoch 1942.5.