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Thermal Histories of Convective Earth Models and Constraints on Radiogenic Heat Production in the Earth

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Thermal histories have been calculated for simple models of the earth which assume that heat is transported by convection throughout the interior. The application of independent constraints to these solutions limits the acceptable range of the ratio of present radiogenic heat production in the earth to the present surface heat flux. The models use an empirical relation between the rate of convective heat transport and the temperature difference across a convecting fluid. This is combined with an approximate proportionality between effective mantle viscosity and T^{-n}, where T is temperature and it is argued that n is about 30 throughout the mantle. The large value of n causes T to be strongly buffered against changes in the earth's energy budget and shortens by an order of magnitude the response time of surface heat flux to changes in energy budget as compared to less temperature-dependent heat transport mechanisms. Nevertheless, response times with $n = 30$ are still as long as 1 or 2 b.y. Assuming that the present heat flux is entirely primordial (i.e., nonradiogenic) in a convective model leads back to unrealistically high temperatures about 1.7 b.y. ago. Inclusion of exponentially decaying (i.e., radiogenic) heat sources moves the high temperatures further into the past and leads to a transition from 'hot' to 'cool' calculated thermal histories for the case when the present rate of heat production is near 50% of the present rate of heat loss. Requiring the calculated histories to satisfy minimal geological constraints limits the present heat production/heat loss ratio to between about 0.3 and 0.85. Plausible stronger constraints narrow this range to between 0.45 and 0.65. These results are compatible with estimated radiogenic heat production rates in some meteorites and terrestrial rocks, with a whole-earth K/U ratio of $1\text{–}2 \times 10^4$ giving optimal agreement.

INTRODUCTION

The thermal history of the earth has been a subject of major controversy in geology for over a century. The first major issue was settled with the discovery of radioactivity, which provided a source for the presently observed surface heat flux which could maintain itself through the long ages demanded by geologists to explain their observations [*Kelvin*, 1899; *Strutt*, 1906; *Stacey*, 1977]. The second major issue, still not settled, concerns the means by which heat generated deep in the earth's interior is transported to the surface at the observed rate, and in the last two decades, conduction, convection, and radiation have all been vigorously advocated and disparaged. Since the prevailing view, until recently, has been that the earth's mantle is immobile, conduction and radiation have received more attention. A difficulty which has been encountered with both conduction and radiation is that much larger conductivities (or effective conductivities, in the case of radiative transport) are required at depth in the earth than are observed in rocks at the surface or could be demonstrated to occur at high pressures and temperatures, and these difficulties persist [e.g., *Schatz and Simmons*, 1972; *Shankland et al.*, 1979]. The theory of plate tectonics, by now widely accepted, postulates that the earth's surface is very mobile and thus implies that at least part of the underlying mantle has a complementary mobility. More recently, mobility of the entire mantle [*Holmes*, 1931] has been revived and advocated by some [e.g., *Tozer*, 1972; *Runcorn*, 1972; *Cathles*, 1975; *McKenzie and Weiss*, 1975; *O'Connell*, 1977; *Davies*, 1977; *Elsasser et al.*, 1979], and the inferred velocities of mantle material are such that material and heat could be transported through the mantle in times much shorter than the age of the earth. Tozer, especially [e.g., *Tozer*, 1972], has repeatedly emphasized that this transforms the question of the thermal history of the earth, since it implies that the surface heat flux will fairly closely reflect the rate of internal heat generation. The consequences of this have been most explicitly explored to date by *McKenzie and Weiss* [1975]. While predominantly conductive or radiative transport of the earth's internal heat cannot be considered by any means to have been laid to rest, the intent of this paper is to present results of a further exploration of convective heat transport through the entire mantle. The approach used was demonstrated, but not fully exploited, by *McKenzie and Weiss* [1975]. A closely analogous method, called 'parameterized convection,' has been used by *Sharpe and Peltier* [1978, 1979] and *Schubert et al.* [1979]. Their conclusions are compatible with those of this study, but the present approach to constraining radiogenic heat production is more direct and comprehensive.

If mantle convection is as efficient as suggested above and as assumed by *McKenzie and Weiss* [1975] and *Davies* [1979], then after the initial warming and core segregation the surface heat flux of the earth would have been controlled by the steadily declining rate of radiogenic heat production; this is just the kind of thermal history calculated by *McKenzie and Weiss* [1975]. Nevertheless, one may ask how efficient mantle convection really is: how much of the present surface heat flux might be escaping primordial (i.e., nonradiogenic) heat, and is it even possible that the earth is still warming? These are the main questions which were addressed in the study reported here. These questions relate directly to a third major issue in geothermal history, that of the total abundance of radioactive heat sources in the earth and the fact that the present rate of heat loss per unit mass of the earth is within a factor of 2 of the present rate of heat production per unit mass in chondritic meteorites: the so-called 'chondritic coincidence' [*Urey*, 1956; *Hurley*, 1957; *Birch*, 1958; *Clark and Ringwood*, 1964; *Wasserburg et al.*, 1964].

It might be thought at first that if the problem was too difficult to have been solved by assuming conductive or radiative heat transport, then the problem involving convection is quite

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hopelessly complex. It turns out, however, that certain critical properties of convection and of the earth seem to be well enough known that surprisingly well constrained calculations can be made. In the following sections the relevant properties of convective heat transfer and of the earth's rheology are described, and a simple, approximate differential equation is derived which describes the thermal evolution of an internally heated, convecting body. Some simple illustrative solutions for special cases are then presented, followed by numerical solutions which more directly address the above questions.

Other Constraints on Thermal History

Given the confusion described above and the paucity of observational constraints, the thermal history of the earth remains almost as obscure as ever. It seems clear that the earth must have been fairly hot fairly early in its history. The most direct constraint seems to come from the oldest known magnetization of rocks (about 2.7 b.y. old [*Fahrig and Bridgwater,* 1976; *McElhinny and Williams,* 1977; *Nairn and Ressetar,* 1978]). These imply the existence by that date of a molten core if it is assumed that the geomagnetic field is generated in the fluid core. The earth's interior must have been hot enough at some earlier time for the core to have segregated (if the earth was initially homogeneous), and core segregation would have released a considerable amount of gravitational energy, sufficient to further heat the earth by about 2000°K [*Birch,* 1965*a*; *Flasar and Birch,* 1973]. However, it is not known how long it might have taken the core to segregate, and it has been argued that the earth may have accreted heterogeneously, with the core in place [*Turekian and Clark,* 1969], so it is not known by how much the start of core segregation, if it occurred, predated the first recorded magnetic field. *Hanks and Anderson* [1969] considered conductive models of the thermal evolution of the earth between its accretion and core segregation. By requiring temperatures high enough to permit core segregation prior to the oldest known rock magnetization, and in fact prior to the oldest rock then known (3.4 b.y.), they concluded that the earth had to be strongly heated initially, and they chose to attribute this heating to a very rapid accretion. However, the initial heating may also have been due to short-lived radioactivity [*Urey,* 1956; *Lee et al.,* 1977], and the temperatures necessary for core segregation may be significantly less than they assumed if the core contains a significant proportion of sulphur, as has been subsequently suggested [*Murthy and Hall,* 1972]. High initial temperatures also seem to be characteristic of thermal history models for the other terrestrial planets [e.g., *Solomon,* 1978].

The subsequent thermal evolution of the earth is even less directly constrained. It is widely believed that the tectonic style of the Archean is indicative of steeper geothermal gradients and thinner, more deformable lithosphere [e.g., *Burke et al.,* 1976; *Windley,* 1977; *Tarling,* 1978] and that subsequent changes in tectonic style reflect a gradual lessening of geothermal gradients. However, these impressions are difficult to quantify, and the problem is compounded by the fact that, at present, geothermal gradients vary greatly with location and presumably did so in the past. *Green et al.* [1975] have deduced that an Archean komatiite (age 3.5 ± 0.2 b.y. [*Anhaeusser,* 1978]) reached the surface at a temperature of about 1650°C. If this magma is indicative of Archean upper mantle temperatures, it may be valid to compare it with modern oceanic rise magmas, which are extruded at significantly lower temperatures, typically about 1400°C [*Wyllie,* 1971; *Green,* 1972]. If this reasoning is correct, this result may be a very important constraint, since as we shall see, a change in temperature by that amount may imply a very large change in heat flux.

Heat Sources

In this study the only heat source considered is radioactivity: the calculations therefore apply only to the time since core segregation (or accretion) ceased to be a significant heat source. Consideration of the effects of core formation is deferred. Also, transient conductive near-surface cooling is ignored: if the effective conductivity of the mantle is like that of surface rocks, then only the outer few hundred kilometers of the earth could have cooled by conduction in the last 4.5 b.y. Furthermore, the surface heat flux from such transient cooling falls below the observed surface heat flux in less than 10^8 years, as is evident in the cooling oceanic lithosphere [*McKenzie,* 1967*a*; *Sclater and Francheteau,* 1970] and as was so strenuously pointed out by Lord Kelvin in the last century [*Kelvin,* 1899; *Stacey,* 1977]. We will thus be concerned with the transport of heat from the deep interior of the earth up to the conductive boundary layer. Correspondingly, the radiogenic heat produced in the continental crust is not of direct concern, since this heat is conducted directly to the surface. *Pollack and Chapman* [1977] estimate that about 40% of the observed continental heat flux is produced in the crust, and since the continents cover about 40% of the earth's surface, this accounts for about 15% of the total heat flux.

CONVECTIVE HEAT TRANSFER

Although convection is a complex phenomenon, the efficiency of heat transport by convection has been found to follow a simple rule in a variety of situations, as described below. The conditions for which this rule has been established by either experiment or theory do not include those appropriate for the mantle, but the extrapolation to mantle conditions may be justified by the fact that the mantle response must be dominated by the temperature dependence of mantle rheology, as will be seen below. While the uncertainties of this extrapolation must be acknowledged, they are probably accounted for in the present calculations by the large range of parameter values to be considered.

The relationship between the temperature difference across a convecting layer of fluid and the heat flux out of the layer has been studied experimentally [*Rossby,* 1969; *Kulacki and Emara,* 1977; *Booker,* 1976; *Booker and Stengel,* 1978], numerically [*McKenzie et al.,* 1974; *Young,* 1974], and with approximate boundary layer theory [*Turcotte and Oxburgh,* 1967; *McKenzie et al.,* 1974]. All of the results can be represented in the form

$$Nu = a(Ra/Ra_c)^p \tag{1}$$

where a and p are constants ($a \simeq 1, p \simeq \frac{1}{3}$), the Nusselt number Nu is a measure of the heat flux, the Rayleigh number Ra is a measure of the temperature difference, and Ra_c is the critical Rayleigh number for the onset of convection. Specifically, Nu is the ratio of the total heat flux q through the top of the convecting layer to the heat q_c which would be transported by conduction alone given the same temperature difference $\Delta T = (T_1 - T_u)$ across the layer:

$$Nu = qD/K\Delta T = q/q_c \tag{2}$$

where K is the conductivity of the fluid and D is the depth of the layer. The Rayleigh number is defined as

$$Ra = g\alpha D^3 \Delta T/\kappa\nu \quad (3)$$

where g is the acceleration due to gravity, α is the volume thermal expansion coefficient, $\kappa = K/\rho C_p$ is the thermal diffusivity, and ν is the kinematic viscosity: $\nu = \eta/\rho$, where η is the viscosity of the fluid and ρ is the density. C_p is the specific heat at constant pressure.

The Rayleigh number can be defined in several ways, and in particular, it can be defined in terms of either q or q_c. The value of p depends on this choice, and in this paper all results will be presented in terms of that definition, (3), which implicitly involves q_c. The relation of this to other definitions and the conversion of results are clarified in Appendix A. The relations can be obtained by using (1) and (2).

An approximate boundary layer theory [*Turcotte and Oxburgh*, 1967; *McKenzie et al.*, 1974] predicts $p = \frac{1}{3}$ for the case of bottom heating (i.e., heating at the bottom boundary) and $p = \frac{1}{4}$ for internal heating. Numerical models of two-dimensional convection in a plane layer [*McKenzie et al.*, 1974] yielded $p = 0.35$ for bottom heating and $p = 0.32$ for internal heating, and numerical models of convection in a spherical shell [*Young*, 1974] gave results consistent with these values over a limited range of Rayleigh numbers. On the other hand, experiments with bottom heating [*Rossby*, 1969] yielded $p = 0.281$, and experiments with internal heating [*Kulacki and Emara*, 1977] over a large range of Rayleigh numbers gave $p = 0.294$.

All of the above results were for fluids with constant viscosity. The most important deviation of silicates from this idealization is that their rheologies are extremely temperature-dependent at high temperatures. *Booker* [1976] and *Booker and Stengel* [1978] have experimentally investigated convection in a fluid when strongly temperature-dependent viscosity, achieving internal viscosity variations by a factor of about 300 at $Ra = 2.2 \times 10^5$. Their results are consistent with the value $p = 0.281$ obtained by Rossby with constant viscosity fluid. It thus seems that for a wide variety of situations, $p = 0.30$ to within about 10% accuracy.

Following *McKenzie and Weiss* [1975], (1) can be rewritten using (2) and (3), as

$$q/q_0 = (T/T_0)^{1+p}(\nu/\nu_0)^{-p} \quad (4)$$

where subscript 0 denotes a reference value and $T = \Delta T$. *Davies* [1979] noted that at high temperatures the temperature dependence of the effective viscosity of olivine can be approximated by

$$\nu/\nu_0 = (T/T_0)^{-n} \quad (5)$$

where n is large (probably about 30). This is discussed further in the next section. Combining (4) and (5), we obtain

$$q/q_0 = (T/T_0)^m \quad (6)$$

where $m = 1 + (n + 1)p$ is of the order of 10, as is discussed in the next section. With such large values of m, (6) implies that large changes in heat flux are accompanied by relatively small changes in temperature in the convecting fluid.

Equation (1) is not accurate for $Ra < Ra_c$, since there is then no convection and $Nu = 1$ (in steady state). Because of the long thermal time constant of the earth, however, steady state is probably never approached, and the amount of heat conducted out of the deep interior is negligible. Thus (1)–(6) predict the right behavior for the wrong reasons in this range of Ra. In comparison with the large range of Ra to be considered below, the range in which (6) is inaccurate is not important and will only cause a slight blurring of the transition from conduction to convection.

THERMALLY ACTIVATED RHEOLOGY

The value of the exponent n in (5) can be estimated from the results of rock deformation experiments, and it will be shown here that n is simply related to the relevant activation enthalpy. Since the activation enthalpy increases with pressure, it is possible that n increases significantly with depth in the mantle: arguments will be presented that in fact n does not vary much through the mantle.

The deformation of mantle rocks may involve either Nabarro-Herring or Coble creep, involving vacancy diffusion [*McKenzie*, 1967*b*], or power law creep, involving dislocation climb [*Weertman*, 1970]. In either case the rate-limiting process at high temperatures is the diffusion of vacancies, either to grain boundaries or to dislocations [*Stocker and Ashby*, 1973; *O'Connell*, 1977]. Vacancy diffusion is a thermally activated process controlled by a diffusion coefficient of the form

$$D = D_0 \exp(-H/RT) \quad (7)$$

where D_0 is a constant, H is the activation enthalpy, and R is the gas constant. The effective viscosity at high temperatures can be written in the form [*Weertman*, 1970]

$$\nu = bT \exp(H/RT) \quad (8)$$

where b is a material parameter which may depend on stress. Equation (8) can be differentiated to yield

$$n = -\frac{\partial \ln \nu}{\partial \ln T} = H/RT - 1 \quad (9)$$

The value of n obviously depends on T, but for a restricted range of T it can be reasonably assumed to be constant. If n is constant, then the first equality in (9) integrates to yield (5). Thus the exponent n in (5) is simply related to the activation enthalpy of the deformation process.

The activation enthalpy is

$$H = E^* + PV^* \quad (10)$$

where E^* and V^* are the activation energy and volume, respectively, and P is pressure. This shows that H will increase with depth in the earth, and the increase may be substantial [*McKenzie*, 1967*b*; *O'Connell*, 1977]. This in turn means that, on the one hand, higher temperatures will be required to achieve a given viscosity and, on the other hand, the viscosity will be more strongly temperature-dependent because n will be larger. In fact, a simple argument suggests that n may not change much with depth. Current interpretations of postglacial rebound data [*Cathles*, 1975; *Peltier*, 1976] indicate that the earth's mantle has a fairly uniform viscosity of about 10^{22} P through most of its depth. This implies, through (8), that the temperature increases with depth at a rate which offsets the increase of H with depth, so that H/RT is approximately constant. But then (9) shows that n would also be approximately constant.

This effect is illustrated in Figure 1, which is a logarithmic plot of viscosity versus temperature for a series of increasing values of H (and thus of depth). The curves are calculated from (8) using parameter values appropriate for olivine, which

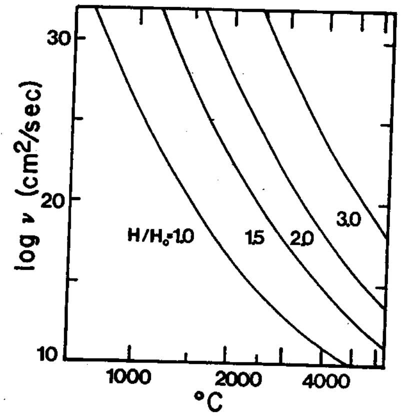

Fig. 1. Kinematic viscosity ν versus temperature for different activation enthalpies H (equation (10)). Parameters are appropriate for olivine: $E^* = 460$ kJ/mole, $\nu = 10^{20}$ cm^2/s at 1500°C. The range of values of H is appropriate for the mantle if $V_0^* = 12$ cm^3/mole and V^* is constant or decreases slowly with increasing pressure [*O'Connell,* 1977]. $H_0 = E^*$ (see (10)).

are discussed below. The effect of increasing H on the viscosity-temperature relation is to shift a curve in Figure 1 horizontally to the right. If T is held constant while H is increased, then both viscosity and n (the negative of the slope of the curve) increase rapidly. On the other hand, if T increases in proportion to H, so that viscosity is constant, the slope of the curve is unchanged. In the intermediate case where there is a moderate increase of viscosity with depth, n increases slightly, and the temperature increases less rapidly.

This argument applies almost equally to changes in activation energy E^* brought about by phase transitions in the mantle. The inference from postglacial rebound observations that the viscosity of the mantle is fairly uniform (averaged over significant depth ranges) implies that any increase in E^*, and hence in H, is compensated by an increase in T. The compensation may not be exact in this case because the material parameter b in (8) may change, but this will not be an important effect unless b changes by orders of magnitude.

The activation energy of the deformation of dry olivine at high temperatures has been estimated from several series of experimental observations. These are summarized in Table 1. The zero-pressure value of the exponent n has been calculated from (9) assuming a temperature of 1700°K, which is appropriate for the upper mantle. These values and the corresponding values of m in (6) are included in Table 1, where it can be seen that n is in the range 29–36 and m is in the range 10–12. The activation energy of oxygen self-diffusion in MgO [*Narayan and Washburn,* 1973] is included in Table 1: the similarity of the values supports the inference that the deformation of both materials is rate limited by the same process at high temperatures.

Ross et al. [1979] have reported an experimental determination of the activation volume V^* of high-temperature creep in olivine, obtaining a value of 13 ± 4 cm^3/mole. They note that this is comparable to the activation volume for self-diffusion of oxygen, 11.6 cm^3/mole, which again supports the inference that this is the rate-limiting process: diffusion of larger complexes, such as SiO_4, would be expected to have activation volumes of 40 cm^3/mole or greater.

O'Connell [1977] pointed out that V^* can be expected to decrease considerably under pressure, so that, assuming an initial value appropriate for oxygen self-diffusion, the calculated effect of pressure on rheology is much less than was previously estimated.

To summarize this section, the exponent n probably has a value of at least 30 in the earth's upper mantle. It may have a larger value deeper in the mantle, but the inference that the apparent viscosity of the mantle is fairly uniform requires that n also does not vary much. Thus the appropriate value of m (equation (6)) for a convecting mantle is probably 10 or a little greater.

RADIATIVE HEAT TRANSFER

Since lattice conductivity in minerals seems quite inadequate to remove the observed heat flux from the earth, it was long suspected that the effective conductivity might be enhanced by radiative transfer at high temperatures [e.g., *MacDonald,* 1964]. This would give a K proportional to T^3. If the diffusion time scale is not too long for steady state flow to be achieved, we could then write

$$q = K\Delta T/D \propto T^4 \qquad (11)$$

which is in the form of (6) with $m = 4$.

In fact, experiments by *Schatz and Simmons* [1972] have shown that a strong iron absorption band in olivine (and presumably in other iron-bearing mantle minerals) broadens into the relevant wavelength range at high temperatures, with the result that K is proportional only to T. Again, if the diffusion time scale is not too long, this would imply

$$q = K\Delta T/D \propto T^2 \qquad (12)$$

Even with ideal radiative transfer the diffusion time scale of the mantle is probably too great for (11) or (12) to be appropriate, but the effect of radiative transfer may be at least crudely described by taking $m = 2$ or 4 in (6), and it is instructive to compare these to the convective case.

THERMAL HISTORY MODEL AND EQUATION

For a preliminary exploration of the implications of the foregoing relations for the thermal evolution of the earth a very simple model was assumed. The thermal state of the earth at a given time is represented by a single characteristic internal temperature. The difference between this and the surface temperature (about 0°C) is then related through (6) to the surface heat flux. The rate of change of internal temperature is simply related to the difference between the rate of internal heat generation and the surface heat flux. This approach is essentially the same as that briefly considered by

TABLE 1. Activation Parameters of High-Temperature Creep in Olivine and MgO

	E^*, kJ/mole	$n = E^*/RT - 1$ (T = 1700°K)	$m = 1 + p\,(n + 1)$ (p = 0.3)
	Olivine		
Kirby and Raleigh [1973]	418	29	10
Carter and Ave'Lallemant [1970] (cited by *Carter* [1976])	464	32	11
Kohlstedt and Goetze [1974]	526	36	12
Post [1977]	526	36	12
	MgO		
Narayan and Washburn [1973]	460	32	11

McKenzie and Weiss [1975], except that the use of (6) in place of (4) and (8) simplifies the mathematics and leads to some simple analytic solutions for special cases.

Characterizing the thermal state of the earth's interior by a single temperature is reasonable if both the mantle and the core are convecting, so that temperatures are relatively homogeneous. It will obviously not be a good approximation, for example, soon after accretion, when the interior may have been relatively cold. However, the calculations will apply only to post–core segregation, since gravitational energy is being neglected, and it is reasonable to expect that core segregation will have thoroughly stirred the interior and homogenized temperatures, apart from a modest vertical adiabatic gradient. For the subsequent evolution of the earth, then, the assumption is that the temperatures at all depths in the earth vary approximately in the same proportion. The only place in the following calculations where an absolute value of temperature is required is in the calculation of a characteristic cooling time; otherwise only relative temperature changes through time are involved.

Another possible limitation of this approach is that the heat flow–temperature relation (equation (4)) was derived from steady state situations but is being applied to a time-dependent calculation. This should not be unreasonable in this case, since the time scales of thermal change are considerably longer than the convective overturn time of the mantle: at present these are probably about 2×10^9 and 3×10^8 years, respectively. If the earth was hotter in the past, both time scales would have been shorter, as will be seen, but they should still have been different by at least an order of magnitude, since convective velocity varies approximately as the square of heat flux [*Davies*, 1979].

To facilitate conversion to dimensionless variables, some dimensional variables will henceforth be denoted by primes: T', q', and t' will denote dimensional temperature, heat flow, and time, respectively.

The total heat content of the earth is $Q = MC_pT' = cT'$, where M is the total mass, C_p is the average specific heat, and T' is an average temperature. If the rate of internal heat generation is A and the surface heat flow is q', then

$$\frac{\partial Q}{\partial t'} = c\frac{\partial T'}{\partial t'} = A - q' \tag{13}$$

Defining the reference heat flux q_0' and reference temperature T_0' (but leaving their values to be specified later) and the following dimensionless variables,

$$T = T'/T_0' \qquad q = q'/q_0' \qquad h = A/q_0' \qquad t = t'/\tau \tag{14}$$

where

$$\tau = cT_0'/q_0' \tag{15}$$

and invoking (6), (13) becomes

$$\partial T/\partial t = h - T^m = h - q \tag{16}$$

Except for the case $m = 1$ this differential equation is nonlinear, which leads to some interesting complications in its solutions. Equation (16) is the basis of the discussion in the following sections, in which solutions are described for various situations.

It is important to consider the uncertainty in the value of m, which is a critical parameter in (16). Unfortunately, it is difficult to make any formal estimate of uncertainty, because the relevance of the existing constraints on n and p can be questioned. The preceding discussions indicate a scatter of about 10% each in the available determinations of n and p. For later calculations, three values of m (7, 10, and 15) will be used: this range seems to include generous allowance for both experimental uncertainties and unforeseen complications, such as larger values of n in the lower mantle or different values of p for fluids with extremely temperature-dependent rheology.

At this stage it is worth noting the way in which the dimensionless variables rescale. Suppose a different reference heat flux $q_0^* = aq_0'$, where a is a constant, is chosen. Then the new values of the dimensionless variables, and the time scale, are related to the old ones as follows:

$$h^* = h/a \qquad T^* = Ta^{-1/m} \qquad t^* = ta^{1-1/m} \qquad \tau^* = a^{1/m-1} \tag{17}$$

In particular, the new time scale is not simply related to the old one except in the case $m = 1$, in which case the time scale is unchanged.

Most of the following results use the present thermal state of the earth as a reference. The relevant quantities are listed in Table 2. The average temperature of the earth's interior is dominated by the temperature of the mantle, which has an average temperature probably not much over 2000°K. Allowing for the heat content of the core, the appropriate characteristic temperature is probably between 2000° and 2500°K with an upper limit of about 3000°K. These values yield time scales of 13, 16, and 20 b.y., respectively. A time scale of 15 b.y. will be used to interpret the results of the calculations.

RESULTS FOR SPECIAL CASES

No Heat Sources

Equation (16) has simple analytic solutions when $h = 0$. Assuming that $t = 0$ when $T = 1$, these are, for $m = 1$,

$$T = q = e^{-t} = e^{-t'/\tau} \tag{18}$$

and, for $m \neq 1$,

$$1/T^{m-1} = 1 + (m - 1)t \tag{19}$$

Thus for $m = 1$ the temperature of an initially hot body simply decays exponentially with a time scale of τ. The heat flux is linearly related to the temperature in this case (equation (6)), and so it also decays exponentially. The time scale is about 15 b.y., which is, of course, much longer than the age of the earth.

Equation (19) also describes a cooling body, but it is no-

TABLE 2. Parameters Relevant to the Thermal Regime of the Earth

Parameter	Value
Mass of earth*	5.97×10^{24} kg
Mass of mantle*	4.0×10^{24} kg
Surface area of earth*	5.10×10^{14} m^2
Mean temperature of interior*	2000–3000°K
Specific heat of mantle*†	700–1200 J/kg °K
Specific heat of core*†	450 J/kg °K
Surface heat flux‡	$4.1 \pm 0.4 \times 10^{13}$ W

**Stacey* [1977].
†High-temperature Dulong-Petit limit is 25 J/mole °K.
‡*Davies* [1980].

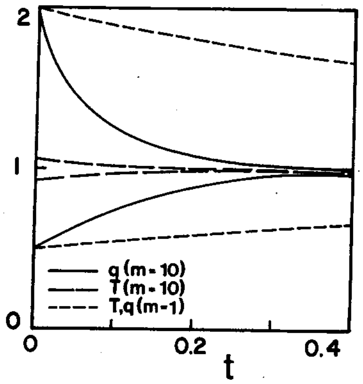

Fig. 2. Comparison of approaches to steady state when $m = 1$ and $m = 10$. Heat generation rate h is constant, and starting heat flows q deviate from steady state by factors of 2. All variables are nondimensionalized according to equations (14) with $q_0' = A$ and $T_0' = T'$ (steady state).

table for having a singularity at $t = -1/(m - 1)$, that is, at $t' = -\tau/(m - 1)$. This immediately implies that if the mantle is convecting (so that $m = 10$), the earth must have significant radioactive heat sources, since otherwise a thermal catastrophe (or at least very high temperatures) about 1.5 b.y. ago would be implied. This will be explored in more detail below. It is also notable that the time between the singularity and $t = 0$ is significantly less than τ for $m \neq 1$. This expresses the fact that the rate of heat loss increases much faster than T at high T.

These simple solutions thus demonstrate a very important effect of a convecting mantle and of the temperature dependence of mantle rheology: the characteristic cooling time is likely to be an order of magnitude less than τ, which is just the time it would take to cool the earth to 0°K at the present rate of heat loss.

Constant Heat Sources

Some important effects of including heat sources can be qualitatively illustrated by the case for which the heat generation rate is constant. When $m = 1$, the general solution to (16) is

$$Te^t = \int he^t\,dt + C \qquad q = T \tag{20}$$

where C is a constant of integration. If h is constant and $T = T_0$ at $t = 0$, this becomes

$$T = q = h + (T_0 - h)e^{-t} \tag{21}$$

These solutions show that the time scale τ is characteristic of the general case $m = 1$. When h is constant, the solution exponentially approaches the steady state $T = h$ (i.e., $q' = A$) with the time scale τ.

When m $\neq$ 1, (16) no longer yields simple solutions, but it can be readily integrated numerically. Figure 2 shows a comparison of the approach to the steady state $T = h = 1$ for the cases $m = 1$ and $m = 10$, starting from heat flows greater than or less than h by a factor of 2. The much more rapid approach to steady state when $m = 10$ is again evident. The rule of thumb that the time scale is reduced by the factor $1/(m - 1)$ again seems appropriate, although the approach is a little slower from $q_0 = 0.5$ than from $q_0 = 2$. These results are quite analogous to those presented by *McKenzie and Weiss* [1975, Figure 3].

The results shown in Figure 2 can be thought of as a consequence of the smaller adjustment to the total heat content Q when $m = 10$. When $m = 10$, Q, which is proportional to T, must change by less than 10% to reach steady state, whereas when $m = 1$, Q must change by a factor of 2, and much more heat must be transported to reach steady state.

Figure 3 shows the warming history of initially cold bodies with constant heat sources, again for the two cases $m = 1$ and $m = 10$. For $m = 1$ the usual exponential approach to steady state occurs. For $m = 10$ the body initially warms steadily with essentially zero heat loss. This persists until the temperature has reached about 80% of its steady state value, at which point the heat flux rises relatively rapidly toward the steady state value, and the temperature levels off.

The above results demonstrate quantitatively two points which have been repeatedly emphasized by Tozer [e.g., *Tozer*, 1972]: (1) that a planet with sufficient heat sources and inefficient internal conduction and radiation must eventually become hot enough for convection to become the dominant heat transport mechanism and (2) that in a planet which has reached this stage the temperature is strongly buffered against changes in its heat budget by the strong temperature dependence of the rheology of solids.

Let us now consider the history, rather than the future, of bodies with constant heat sources, concentrating for the moment on the case $m = 10$. If the solution for $q_0 = 0.5$ in Figure 2 is followed back in time, it clearly will reproduce the behavior shown in Figure 3, except for changes of scale. On the other hand, the heat flow is rising rapidly with age for the case $q_0 = 2$, suggesting that another thermal catastrophe will be encountered in the past. Figure 4 shows the results of a series of integrations backward in time. The solutions in this case are scaled such that at $t = 0$ the dimensionless heat flux is $q_0 = 1$, and solutions are obtained for different values of present (i.e., $t = 0$) dimensionless heat generation h_0. Cases with $h_0 > 1$ are presently warming and thus have cooler pasts, analogous to the case in Figure 3. Cases with $h_0 < 1$ are cooling and have hotter pasts, and singularities are indeed encountered if they are followed back far enough. The limiting case, $h_0 = 0$, is just the solution given by (19) for the case of no heat sources, and the singularity is at $t = -\frac{1}{9} = -0.111$. With $h_0 = 0.5$ the singularity occurs at about $t = -0.15$. Any other solution with $0 < h_0 < 1$ can be obtained from that for $h_0 = 0.5$ (or, equivalently, from the solution for $q_0 = 2$ in Figure 1) through the scaling relations, equations (17): this establishes that a singularity will always be encountered if $h_0 < 1$.

The singularities in these solutions are not of great signifi-

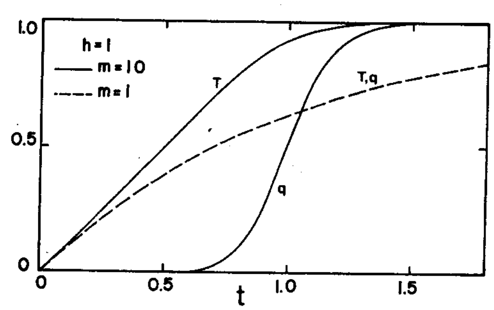

Fig. 3. Comparison of the warming histories of initially cold bodies for the cases $m = 1$ and $m = 10$. Dimensionless internal heating rate is $h = 1$. T, q, and t are dimensionless temperature, surface heat flux, and time, respectively.

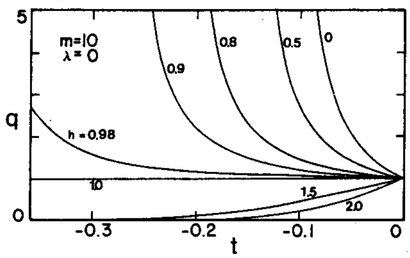

Fig. 4. Histories of surface heat flux q from integrations back from $t = 0$ of bodies with constant internal heating rate h. Singularities occur for $h < 1$. All quantities are nondimensionalized according to equations (14), with $q_0' = q'(t = 0)$, $T_0' = T'(t = 0)$.

cance in themselves, of course, especially since they would not occur if the power law dependence of viscosity on temperature (equation (5)) had not been substituted for the exponential form (equation (8)). Nevertheless, they are useful as readily identifiable features indicative of unreasonably high temperatures. If the singularity is to occur before $t = -0.2$ (i.e., $t' = -3$ b.y., assuming $\tau = 15$ b.y.), then h_0 must be greater than about 0.8. In other words, if the rate of heat generation in the earth had been constant and any hot stage in its history occurred no later than 3 b.y. ago, then no more than 20% of its present surface heat flux could be attributed to that hot stage, and the balance would have to be attributed to the internal heat generation.

Radioactivity and Other Heat Sources

Of course, if the earth's internal heat generation is due to radioactivity, then it has not been constant, and part of the present surface heat flux may be a result of the greater radiogenic heating in the past. This question will be addressed in the next section.

The major possible sources of the earth's surface heat flux are radioactivity, gravitational energy released by differentiation, and stored internal heat, including latent heat of crystallization of the core. The stored internal heat is explicitly included in (13) and (16), and the latent heat of melting of the core is a minor contributor to this [*Stacey,* 1977]. The greatest release of gravitational energy occurred during accretion of the earth and (if it occurred after accretion) the separation of the core, and both events may have heated the earth considerably [*Birch,* 1965*a; Flasar and Birch,* 1973; *Stacey,* 1977]. The release of gravitational energy will not be considered in this paper. The relevance of the following calculations will thus cease at the time in the past when the release of gravitational energy was last a significant heat source, which is presumed to have been early in the earth's history and probably predates the oldest known magnetized rock, which is about 2.7 b.y. old [*McElhinny and Williams,* 1977]. *Gubbins* [1977] and *Loper* [1978*a, b*] have suggested that the gravitational segregation of the outer and inner core may be driving the geomagnetic dynamo and hence that this may be a continuing source of heat, but it constitutes only about 1% of the total observed heat flux.

The long-lived radioactive isotopes ^{238}U, ^{235}U, ^{232}Th, and ^{40}K have probably produced most of the radiogenic heat in the earth's history [e.g., *Urey,* 1956]. Shorter-lived 'extinct' isotopes, notably ^{26}Al, may have produced considerable early heating [*Lee et al.,* 1977], but this would be indistinguishable from accretional heating in the present context, and so they will not be considered.

The absolute abundances of U, Th, and K in the earth are still rather uncertain, and so the present rate of radiogenic heating will here be treated as a free parameter. The major remaining uncertainty is then the abundance of potassium relative to the other elements. The isotopic ratios of uranium are well known [e.g., *Stacey,* 1977, Appendix H], and the mass ratio of thorium to uranium, Th/U, averages 3.5–4 for many rocks and meteorites [*Wasserburg et al.,* 1964; *Stacey,* 1977]. In this study it is assumed that Th/U = 4. Estimates of the mass ratio K/U range from 10^4 to about 8×10^4, depending on whether crustal rocks, chondrites, carbonaceous chondrites, or the sun are believed to be more indicative of the potassium content of the earth [e.g., *Wasserburg et al.,* 1964; *Ringwood,* 1975; *Goettel,* 1976].

The variation with time of the heat production of the four isotopes is shown in Figure 5, based on the half-lives and energy productions given by *Stacey* [1977, Appendix H]. The proportions are adjusted to sum to 1 at present, assuming K/U = 10^4. Since the relative abundances of U and Th are fairly well constrained, it is useful to sum them, yielding the U + Th curve shown, and to compare this to the ^{40}K curve. Two other sums are shown in Figure 5: U + Th + K, assuming K/U = 10^4 and K/U = 8×10^4 (the latter scaled to yield 1 at present). The importance of the uncertainty in this ratio is evident in Figure 5: the heat productions of the two curves at $t = -4.5$ b.y. differ by almost a factor of 2. These two curves presumably bracket the likely range of radiogenic heat production through most of the earth's history.

It is convenient to represent the variation in heat production shown in Figure 5 as a single exponential decay with an appropriate mean half-life, as was done by *McKenzie and Weiss* [1975]. In fact, their approximations are reproduced in Figure 5 and can be seen to be adequate approximations to the curves derived above. The same curves will therefore be used here: their half-lives are 1.54 and 2.25 b.y. for K/U = 8×10^4 and K/U = 10^4, respectively. For inclusion in the calculations these can be converted to dimensionless half-lives or, more conveniently, decay constants, defined as

$$\lambda = (\ln 2)\tau/\tau_R \quad (22)$$

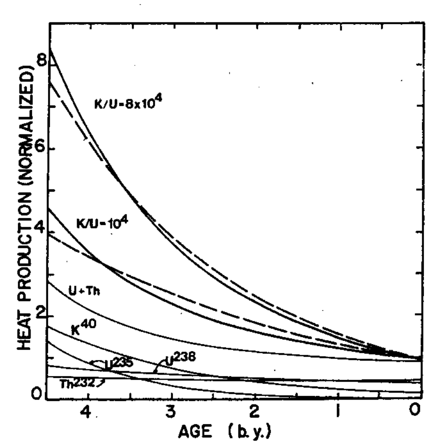

Fig. 5. Radiogenic heat production curves for U-Th-K mixtures with K/U = 8×10^4 and 10^4, both normalized to 1 at $t = 0$. For the latter case, contributions of individual isotopes are shown. Th/U = 4 for both cases, and the U + Th sum is included. Dashed curves are exponential approximations to the sums, with parameters from *McKenzie and Weiss* [1975].

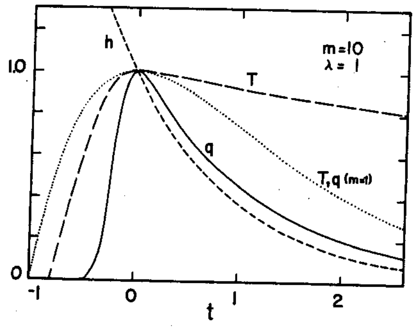

Fig. 6. Illustrative thermal history of an initially cold body with decaying heat sources. Heat generation rate h, heat loss rate q, temperature T, and time t are nondimensionalized according to equations (14), with $q_0' = q'(t = 0)$, $T_0' = T'(t = 0)$. Solution for the case $m = 1$ is included for comparison ($T = q$ for this case).

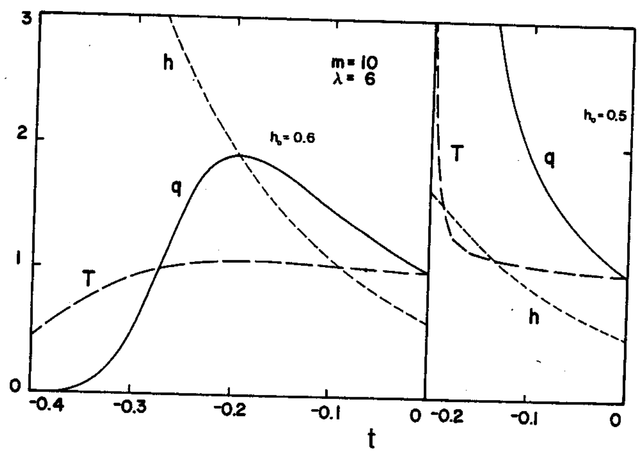

Fig. 8. Two cases from Figure 7 ($h_0 = 0.5$ and 0.6) showing all variables: temperature T, surface heat flux q, and heat generation rate h; $h_0 = 0.5$ implies a 'hot' history, while $h_0 = 0.6$ implies a 'cool' history.

where τ is the time scale defined by (15) and τ_R is the mean radioactive half-life. Using the value $\tau = 15$ b.y. arrived at earlier, (22) yields $\lambda = 4.6$ and 6.8 for the two radioactive abundance models. If τ is as large as 20 b.y., these values would become 6.2 and 9.0, respectively. The following calculations have been made using the representative values $\lambda = 6$ and 9. The latter value is rather high but might increase the range of validity of solutions to include a period in which some residual gravitational differentiation persisted beyond the main core segregation phase.

SOLUTIONS WITH DECAYING HEAT SOURCES

An Initially Cold Body

Qualitative aspects of the thermal history of an initially cold body with exponentially decaying heat sources are illustrated by the solution shown in Figure 6, which was obtained by integrating in both directions from the time when $T = q = h = 1$ for the case $m = 10$ and $\lambda = 1$. (It should be noted that the scaling in this case is different from that in all other figures because a different reference state has been used.) The high initial heat production warms the body fairly rapidly, and there is initially little heat loss because of the low temperatures, as was seen in the case of constant heat production. As higher temperatures are achieved, the heat flux rises even more rapidly until it intersects the heat production curve. At this point the temperature, and therefore the heat flux, is a maximum, since h and q are equal, but then the continuing decline of h reduces it below q, and the body begins to cool. Thereafter the heat flux follows the declining value of h but with a time lag which is a fraction of the decay half-life of h. The corresponding solution for $m = 1$ is included in Figure 6 for comparison, and it can be seen that in this case, q follows the decline of h much less closely. Thus it is again seen that the adjustment to changes in heat production is much more rapid when $m = 10$ than when $m = 1$. Nevertheless, when $m = 10$ and h has declined to a fraction (0.1–0.2) of its initial value, q may be as much as 30% greater than h.

Integrations Backward Through Time

Let us now consider the results of integrating backward through time from the present, as was done in deriving Figure 4 for the case of constant heat sources. Figure 7 shows the variation of dimensionless heat flow q for a series of values of h_0, the ratio of the present heat production to the present observed heat flow, and for the case $m = 10$, $\lambda = 6$. The solution with $h_0 = 0$ is again that given by (19) and shown in Figure 4. As h_0 is increased, the singularity again moves further into the past until, for h_0 greater than about 0.6, there is a transition to solutions of the type shown in Figure 6. For $h_0 = 1$, q increases monotonically with time up to $t = 0$: even though the body is not presently warming, $h_0 = 1$ implies a cooler past. Solutions for $h_0 > 1$ are not shown but would be qualitatively similar to those shown in Figure 4 and would, of course, imply that the body is presently warming.

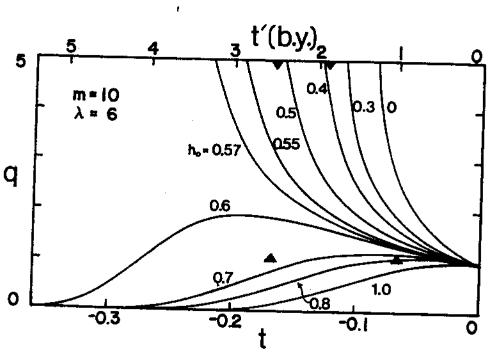

Fig. 7. Histories of surface heat flux q integrated back from $t = 0$ with different heat production rates h scaled by $h_0 = h(t = 0)$. Heat sources decline with decay constant $\lambda = 6$ (cf. Figure 6). The exponent in the temperature–heat flux relation, (6), is $m = 10$. All variables are nondimensionalized according to equations (14), with $q_0' = q'(t = 0)$; h_0 is given as a fraction of q_0. A dimensional time scale $t' = \tau t (\tau = 15$ b.y.) is included for comparison. Triangles mark possible bounds on the earth's surface heat flux (see text).

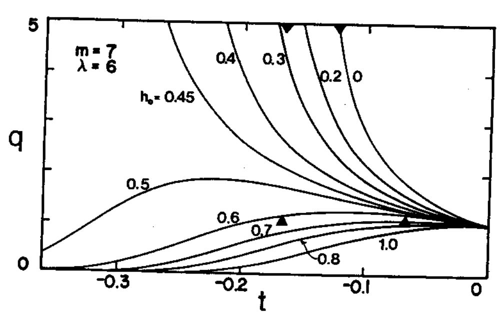

Fig. 9. Histories of surface heat flux q. Symbols and variables are the same as in Figure 7.

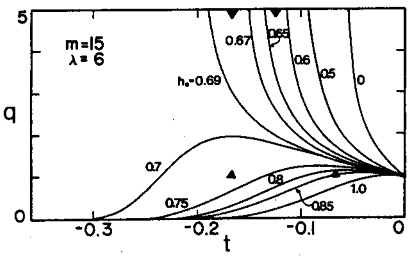

Fig. 10. Histories of surface heat flux q. Symbols and variables are the same as in Figure 7.

The reason that the transition from 'hot' to 'cool' pasts occurs for $h_0 < 1$ is that although the heat flux increases with age (i.e., negative time), the heat production increases more rapidly until a time in the past is reached when they were equal and q was a maximum, as in Figure 6. This is illustrated in Figure 8, which shows the variation of all three variables (h, q, T) for the two cases $h_0 = 0.5$ and $h_0 = 0.6$ of Figure 7. The transition can be approximately predicted from the condition that the time derivatives of h and q be equal at $t = 0$. From (16) this condition occurs for

$$h_0 = m/(m + \lambda) \tag{23}$$

For the case of Figure 7 this yields $h_0 = 0.625$, which is slightly larger than the true transition value but quite close. The reason for the discrepancy is that the solution for q is not exponential.

It is important to test the sensitivity of these solutions to the parameters m and λ. Figures 9–13 show solutions for $\lambda = 6$ and 9 and $m = 7$, 10, and 15; these values bracket the range of reasonable values, according to the preceding discussion. The same qualitative behavior as that in Figure 7 is evident. A larger value of λ reduces the value of h_0 for which the transition occurs: this is because q lags further behind h because of the faster decrease in h. A larger value of m, on the other hand, increases the transition value of h_0 because the response time of the system is shorter and q tracks h more closely.

An important feature of all of the solutions is their sensitivity to h_0 near the transition from hot to cold pasts, especially for large values of m and λ: in Figure 12, changing h_0 from 0.4 to 0.6 swings the solution from a relatively recent 'thermal catastrophe' to a relatively cool history. This means, of course, that it is very difficult to estimate the thermal history of the earth on the basis of independent estimates of h_0. On the other hand, it means that any independent geological constraint on the earth's thermal history may yield a very strong constraint on the present value of h. This will be taken up in the next section.

The solutions shown in Figures 7–13 are probably appropriate if convection is the dominant heat transport mechanism in the mantle, but if conduction or radiation is dominant, then at least lower values of m must be used, if not a different governing equation, as was discussed earlier. For comparison with the above solutions, and with the caution reiterated that the implicit approximations are probably not very good, solutions are shown in Figure 14 for the case $m = 3$, $\lambda = 6$. The character of these solutions is dominated by the much slower response time of q to changes in h, and the range of past values of q is quite restricted. Thus, no thermal catastrophe is likely within the age of the earth, and even heat flows significantly greater than those at present are precluded for h_0 greater than about 0.3. If h_0 approaches 1, then the surface heat flux would have increased steadily through most of the earth's history. Given the slow response time, the neglect of the energy released by core segregation is probably very important here, since that energy might dominate the present surface heat flux.

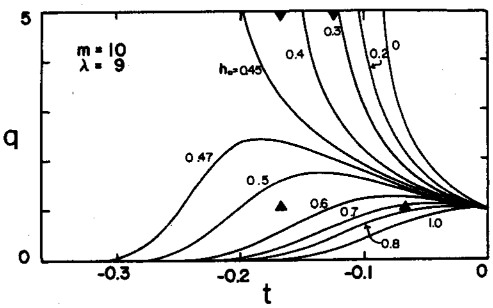

Fig. 12. Histories of surface heat flux q. Symbols and variables are the same as in Figure 7.

CONSTRAINTS ON RADIOGENIC HEAT PRODUCTION IN THE EARTH

The total abundance in the earth of radioactive heat sources is an important parameter not only because of its immediate relevance to the thermal evolution of the earth but also because it is an important part of the larger question of the composition and origin of the earth and the terrestrial planets [e.g., *Urey*, 1956; *Wasserburg et al.*, 1964; *Birch*, 1965*b*; *Ganapathy and Anders*, 1974; *Ringwood*, 1975; *Goettel*, 1976]. Thus the suggestion made in the preceding section that independent geological constraints on the earth's thermal history would yield constraints on radioactive heat source abundance suggests that this in turn would constrain theories of the origin of the terrestrial planets. In this section a series of constraints on h_0 will be derived from possible constraints on the earth's thermal history, starting with the widest plausible bounds and proceeding to tighter, but more debatable bounds.

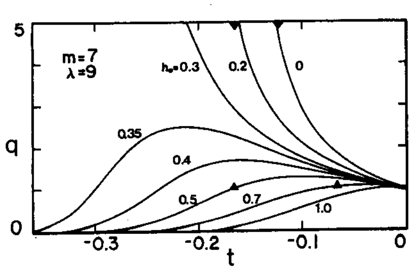

Fig. 11. Histories of surface heat flux q. Symbols and variables are the same as in Figure 7.

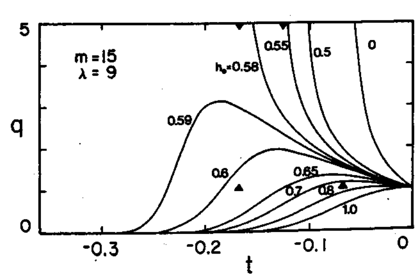

Fig. 13. Histories of surface heat flux q. Symbols and variables are the same as in Figure 7.

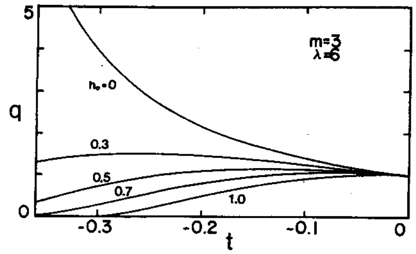

Fig. 14. Histories of surface heat flux q. Symbols and variables are the same as in Figure 7.

It will be convenient to present the dependence on m and λ of the resulting preliminary or potential bounds on h_0 in graphical form (Figure 15). For comparison, Figure 15 also includes the value of h_0 corresponding to the transition from hot to cold history and the estimate of this value derived from (23). It can be seen that (23) consistently overestimates the transition value by a small amount but that it gives a general indication of the dependence of the transition value and the various 'bounds' on m and λ.

Hot History

The surface heat flow q' probably cannot have been greater than about 5 times the present heat flow q_0' more recently than about 2.5 b.y. ago. Core segregation may have maintained such a high heat flow, but it is unlikely that this lasted beyond 3.0 b.y. ago. Without other major heat sources, Figure 4 and the scaling equations (17) show that a hypothetical body cooling from a singularity would reach $q' = 5q_0'$ within an interval $\Delta t \simeq 0.03$, that is, $\Delta t' \approx 0.4$ b.y. Using the larger reasonable value of $\tau = 20$ b.y., the latest reasonable value of t for which we can have $q \geqslant 5$ is $t = -0.125$. This constraint gives an upper bound on 'primordial' heat currently escaping from the earth and hence a lower bound on h_0. The lower bounds obtained from Figures 7–13 are shown in Figure 15. For $m \leq 7$ no useful bound is obtained. For the preferred range of m of 10–12 the lower bound is 0.3–0.4.

If the more likely value $\tau = 15$ b.y. is used, then the requirement is that $q \leq 5$ at $t = -0.167$. For $m \geq 7$ this yields (Figure 15) $h_0 \geq 0.2$, and for the preferred range $10 \leq m \leq 12$, $h_0 \geq 0.4$.

Cool History

It is difficult to imagine that Archean tectonics can be explained without a geothermal flux at least as great as today's, but it is conceivable that the Archean heat flux may have been derived from the outer layers of the earth (for example, from a residual of accretional heating) in a way not accounted for in the present models. On the other hand, the present plate tectonic regime is intimately related to the present heat flux, and plate tectonics seems to have been operating in the present manner at least through the Phanerozoic [e.g., *Dewey and Bird,* 1970], and there is significant paleomagnetic evidence for continental drift well back into the Proterozoic [*McElhinny and Williams,* 1977]. Even if plate tectonics involves convection only in the upper mantle, there is neither sufficient heat nor sufficient heat sources in the upper mantle to have maintained the current heat flux for 1 b.y. or more [*O'Nions et al.,* 1978]. The weakest reasonable constraint would seem to be that the heat flux from the deep interior had reached a significant fraction of the present surface flux by 1 b.y. ago. Figures 7–13 show that all of the models meet this constraint with $h_0 = 1$.

Although it may not be possible to rule out such a cool history, it does leave the considerable tectonic and magmatic activity of both the Archean and the Proterozoic [*Windley,* 1977] unaccounted for. The strong heating of the earth which would accompany core segregation [*Birch,* 1965*a*; *Flasar and Birch,* 1973] also is not likely to be consistent with this cool history, since this would have raised the average internal temperature to at least 50% of its present value. Quantitative lower limits on the heat flux in the past are difficult to arrive at, so for the sake of definiteness, two representative constraints will be illustrated. The less restrictive is that q should have been at least equal to the present heat flux 1.0 b.y. ago. Using $\tau = 15$ b.y., this requires $q \geq 1$ at $t = -0.067$. A more restrictive requirement is that $q \geq 1$ at $t' = -2.5$ b.y. (i.e., at $t = -0.167$). The resulting constraints on h_0 are shown in Figure 15. The stronger constraint requires that $h_0 \leq 0.7$.

A Plausible Warm History

The Archean komatiite reported by *Green et al.* [1975] to have reached the surface at a temperature of about 1650°C may not have been typical of such extrusives about 3.5 b.y. ago, but there seem to be no modern analogues produced at such high temperatures, so it seems reasonable to conclude that Archean upper mantle temperatures were significantly higher than those of today. Modern ridge basalts are produced at between 1350° and 1450°C [*Wyllie,* 1971; *Green,* 1972]. Taking these temperatures at face value and using (6) with $m = 10$, they imply that the heat flux 3.5 b.y. ago was between about 4 and 8 times the present heat flux. This would seem to be consistent with the strong heating early in the earth's history produced by the combination of core segregation and high radioactivity. Recently, *Gubbins* [1977] and *Loper* [1978*a, b*] have argued that the geomagnetic dynamo may be driven by the gravitational settling of solidifying material from the outer core to the inner core. While this is still speculative, it suggests that the earliest occurrence of rock magnetization (about 2.7 b.y. ago [*McElhinny and Williams,* 1977]) may have coincided with the beginning of solidification of the

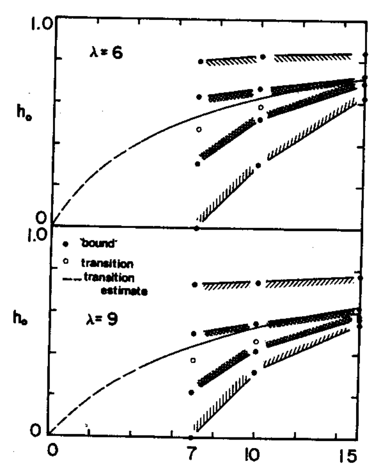

Fig. 15. Possible bounds in h_0 derived from Figures 7–13 as functions of heat flow–temperature exponent m (equation (6)) and for two heat source decay constant values: $\lambda = 6, 9$. Open circles are values of h_0 which separate 'hot' histories from 'cool' histories. The dashed curve is an estimate of this transition value (equation (23)).

TABLE 3. Heat Loss Rates in the Earth

Parameter	Value		
Total heat flux	4.1×10^{13} W		
Convected heat flux, Q_c	3.6×10^{13} W		
Mass of earth, M_E	5.97×10^{24} kg		
Mass of mantle, M_M	4.0×10^{24} kg		
Percentage of Q_c	Q, 10^{13} W	Q/M_E, pW/kg	Q/M_M, pW/kg
100	3.6	6.0	9.0
85	3.1	5.1	7.7
65	2.3	3.9	5.9
45	1.6	2.7	4.1
30	1.1	1.8	2.7

core, which would require that the earth was cooling at that time. There are many indications that Archean tectonics involved a higher heat flux [*Windley,* 1977], and the most straightforward interpretation of Proterozoic tectonics is that it represents a transition between Archean and present tectonic styles and heat fluxes.

If these arguments are tentatively adopted, they combine to suggest that at $t' = -3$ b.y. ($t = -0.2$), q' was between 2 and 5 times its present value. In all cases shown in Figures 7–13 this range brackets the transition between hot and cool solutions, and so the corresponding values of h_0 are close to the transition values shown in Figure 15. It could thus be concluded that h_0 is constrained to the range 0.35–0.70. If the preferred values of m (between 10 and 12) are taken, then for $\lambda = 6$, h_0 would be between 0.55 and 0.65, while for $\lambda = 9$, h_0 would be between 0.45 and 0.55.

RADIOACTIVE HEAT SOURCE CONCENTRATIONS

The total abundance of radioactive heat sources in the earth has been the subject of much discussion, especially since *Urey* [1956] noted the chondritic coincidence: the heat loss per unit mass of the earth is approximately equal to the radiogenic production per unit mass of chondritic meteorites. Since meteorites and the planets are the presumed derivatives of a protosolar nebula, the chondritic coincidence supports the idea that chondrites are representative of the original bulk composition of the earth, at least for the relatively involatile elements. In order to assess the implications of the preceding results for the bulk composition of the earth they must be converted to absolute dimensional values.

Estimates of the present total surface heat flux of the earth have been reviewed by *Davies* [1980]. Recent estimates by others, some taking into account recently increased estimates of the contribution from cooling oceanic lithosphere which was partly obscured by hydrothermal circulation in the oceanic crust, ranged from 3.0 to 4.3×10^{13} W, but it seems likely that the actual value is between 3.7 and 4.5×10^{13} W after account is taken of some inconsistencies and updated parameter values. A representative value of 4.1×10^{13} W will be used here. Of this, about 0.5×10^{13} W is generated in the continental crust [*Pollack and Chapman,* 1977] and is therefore carried to the surface by conduction. Thus the amount of heat which is transported from the deep interior of the earth either to the ocean floor or to the base of the continental lithosphere is probably about 3.6×10^{13} W. This converts to a rate of heat loss per unit mass of the earth of 6.0 pW/kg (6.0×10^{-12} W/kg). The various fractions of this which were estimated in the last section to represent the present rate of radiogenic heat production in the earth are listed in Table 3. Since there is currently some debate as to whether there are significant radioactive heat sources in the core [e.g., *Oversby and Ringwood,* 1973; *Goettel,* 1976], these values are given both per unit mass of the earth and per unit mass of the mantle.

Some representative radiogenic heat production rates, calculated from measured K, U, and Th concentrations, are given in Table 4 for various chondritic meteorites and terrestrial basalts. Most chondrites have U concentrations within about 30% of 1.3×10^{-8} g/g [*Morgan,* 1971], and K/U mass ratios range from about 2×10^4 for carbonaceous chondrites up to about 8×10^4 for some ordinary chondrites. The estimated heat production rates of all chondrites are equal to or less than the heat loss rate of the earth and comparable to the estimates of the earth's present heat production rate made in the last section. (The last column of Table 4 gives the estimated heat production rate as a percentage of the heat loss rate per unit mass of the earth.) The best agreement is for the lower K/U ratios, more characteristic of carbonaceous chondrites. It has been noted that the K/U mass ratios of terrestrial rocks are usually close to 10^4, consistently lower than those for chondrites [*Wasserburg et al.,* 1964; *O'Nions et al.,* 1978], and it has been suggested that this is because the earth was depleted in K during its formation [*Ringwood,* 1975]. Therefore a hypothetical 'K-depleted chondrite' has been included in Table 4 with a K/U mass ratio of 10^4: its heat production rate

TABLE 4. Radiogenic Heat Production Rates

Material	[U], 10^{-9} g/g	Q(U + Th), pW/kg	K/U, 10^4 g/g	[K], 10^{-6} g/g	Q(K), pW/kg	Q_{total}, pW/kg	$Q_T M_E/Q_c$, %
Chondrite (ordinary)	13	2.7	8	1040	3.6	6.3	105
Chondrite (ordinary)	13	2.7	6	780	2.7	5.4	90
Chondrite (carbonaceous)	13	2.7	2	260	0.9	3.6	60
Chondrite (K depleted)	13	2.7	1	130	0.4	3.1	52
MORB	15	3.1	1	150	0.5	3.6	60
MORB	150	31.0	1	1500	5.0	36.0	600
MORB/5	3	0.6	1	30	0.1	0.7	12
MORB/5	30	6.1	1	300	1.1	7.2	120
Model earth 1	8*	1.6	8	640*	2.3	3.9	65
Model earth 2	11*	2.3	4	450*	1.6	3.9	65
Model earth 3	14*	2.9	2	280*	1.0	3.9	65
Model earth 4	16*	3.3	1	160*	0.6	3.9	65

For chondrites, concentrations are representative of data from *Morgan* [1971], *Goles* [1971], and *Stacey* [1977]. For midocean ridge basalts (MORB), values represent the range of data compiled by *O'Nions et al.* [1978]. A Th/U mass ratio of 4 is assumed. Heat production rates are from *Stacey* [1977, Appendix H].

*Concentrations are per unit mass of the earth; concentrations per unit mass of the mantle are 50% greater.

is near the low end of the preferred range of terrestrial heat production rates.

Terrestrial heat production rates are poorly constrained at present by analyses of terrestrial rocks. Heat production rates for midocean ridge basalts (MORB) are not likely to be representative of the bulk earth. However, if it is assumed that they are produced by about 20% partial melting of the underlying mantle and that virtually all K, U, and Th are partitioned into the basalt [e.g., *Ringwood,* 1975], then the heat production in MORB source regions can be estimated as being about 20% of that in the basalts. The different values given in Table 4 for MORB reflect the large variations in K and U concentrations from different parts of the ridge system [*O'Nions et al.,* 1978]. The range of estimates for the MORB source regions (MORB/5 in Table 4) is correspondingly large, implying that total mantle heat production might be anything from much less than half to more than the total internal heat production of the earth. Since there is increasing evidence that the mantle has large-scale chemical heterogeneities [e.g., *Sun and Nesbitt,* 1977; *O'Nions et al.,* 1978], it would not seem to be possible at this stage to reach any definite conclusions concerning the level of mantle heat productions based on analyses of terrestrial rocks.

To facilitate comparisons, four 'model earth' abundances are included in Table 4 which have different K/U ratios but which all produce 3.9 pW/kg. If 0.8 pW/kg of this is allowed for partitioning into the continental crust, then 3.1 pW/kg is left for the interior, which is 52% of the estimated total heat loss rate from the interior and within the preferred range of heat production estimates obtained in the last section.

Thus according to the estimates presented here the chondritic coincidence applies only for ordinary chondrites with the highest K/U ratios. On the other hand, the present models show that if the earth is cooling by convection at very plausible rates, the implied present heat production rate is very similar to that of carbonaceous chondrites or of K-depleted chondritic material. Of course, such comparisons are complicated by the fact that the correspondence between chondrites and the earth is uncertain. For example, it is presumably necessary to remove volatiles from carbonaceous chondrite to obtain a bulk earth composition [*Ringwood,* 1975], which would increase the concentrations of U and Th and possibly of K: the U/Si atomic ratios are about 30% higher for carbonaceous chondrites than for ordinary chondrites [*Morgan,* 1971]. On the other hand, it is possible that extra radiogenic heat sources are buried in deep-earth reservoirs, that is, the lower mantle or the core: in particular, some of the excess K (relative to U) of chondrites might be in the core [e.g., *Goettel,* 1976]. Furthermore, the present rate of heat loss from the earth may be significantly lower than the average over the past 200 m.y. or so, since some major midocean ridge segments have been subducted in the past 40 m.y. and the average sea floor spreading rate may have been higher during parts of the Mesozoic [*Davies,* 1980].

To summarize this section, the terrestrial heat production rates deduced in this study are most consistent with those of carbonaceous chondrites, but a K-depleted chondritic material with $K/U = 10^4$ g/g is also acceptably within the uncertainties of the estimates. On the other hand, the heat production rate of ordinary chondritic material does seem to be too high to be acceptable.

CONCLUSIONS

There is probably a significant imbalance between present heat production and heat loss in the earth: as much as half of the present surface heat flux may be heat which was generated over the past few billion years. Assuming that heat transport is by convection throughout the earth's interior, using current determinations of the temperature dependence of mantle rheology, and applying minimal geological constraints to the calculated thermal histories require the present heat production rate to be between about 30% and 85% of the present rate of heat loss. Applying plausible stronger geological constraints restricts the heat production to between 45% and 65% of the heat loss. Thus previous assumptions that the rates of heat production and heat loss are equal [*McKenzie and Weiss,* 1975; *Davies,* 1979] may not have been very accurate. Likewise, the significance of the chondritic coincidence (the rate of radiogenic heat production per unit mass in chondrites is similar to the rate of heat loss per unit mass of the earth) must be reevaluated. However, it seems that the inference that the rate of radiogenic heat production per unit mass in the earth is similar to that in some chondrites is not invalidated, carbonaceous chondrites (perhaps with some K depletion) being the most appropriate.

The calculations on which these conclusions are based are dominated by the effect of the strong temperature dependence of mantle rheology. Thus although it will be important to undertake more rigorous experimental and theoretical testing of the approximations used, it does not seem likely that the character of the solutions will be strongly affected by more refined analyses. At this stage it would seem most desirable to test the relation between temperature difference and convective heat flux for fluids with large Rayleigh numbers and large viscosity variations, but this is a very difficult problem to tackle, either experimentally or theoretically. Continuation of efforts to characterize the rheology of likely mantle constituents is also very desirable.

The great advantage of such very simplified models and calculations as used here is that they permit rapid exploration of a large range of solutions, so that more elaborate analyses can be applied to better defined problems. It will be easy, for example, to include the energy released by core segregation in these calculations, and they can be readily adapted to apply to other terrestrial planets and satellites, as has been independently demonstrated by *Reynolds and Cassen* [1979] and *Peale et al.* [1979]. In spite of the great simplifications used in these calculations, however, the results obtained to date indicate that some rather strong conclusions can be reached through them.

APPENDIX A

This appendix reviews an alternative form of (1) in terms of alternative Rayleigh numbers to that defined in (3). Equation (3) can be rewritten in terms of the conducted heat flux $q_c = k\Delta T/D$:

$$Ra = g\alpha D^4 q_c/\rho C_p \kappa^2 \nu \quad \text{(A1)}$$

An analogous Rayleigh number Ra_q can be defined in terms of the total heat flux q [*McKenzie et al.,* 1974]:

$$Ra_q = g\alpha D^4 q/\rho C_p \kappa^2 \nu \quad \text{(A2)}$$

From the definition (2) of Nu it follows that

$$Ra_q/Ra = Nu \quad \text{(A3)}$$

When $Ra = Ra_c$, $Nu = 1$ (equations (1) and (2)), so that $Ra_{qc} = Ra_c$. Combining (1) and (A3) and using these results,

$$Nu = a(Ra_q/Ra_{qc})^{p/p+1} \quad \text{(A4)}$$

Thus when, for example, $p/p + 1 = \frac{1}{3}$ in (A4), then $p = \frac{1}{2}$ in (1).

McKenzie et al. [1974] have suggested a further modification of the definition of the Rayleigh number in the case of internal or partial internal heating. They define

$$Ra_I = (1 - \tfrac{1}{2}\mu)Ra_q \quad \text{(A5)}$$

where μ is the ratio of internal heating to total heat flux. The additional factor takes account of the fact that in the internally heated conductive reference state the average temperature gradient across a fluid layer is only half that for bottom heating with the same heat flux. This definition results in Ra_{Ic} being almost independent of μ [*McKenzie et al.*, 1974].

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