STOMMEL, H. M. (1957) A survey of ocean current theory. Deep-Sea Res. 4(3), 149-184,

A survey of ocean current theory

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(*Received* 25 *January* 1957)

INTRODUCTION

DURING the past half century there have been several independent lines of development of theory of ocean currents. The interconnections among them have not been generally recognized in the literature; but these are so interesting and instructive, especially from a physical point of view, that it seems worthwhile to attempt to describe and compare them in a brief survey article. Moreover it seemed desirable to try to isolate the basic physical ideas in each of the theoretical studies and to present them in pictorial diagrammatic form, without mathematics. The construction of the various models and of the illustrations is based, of course, on the full mathematical development.

HOUGH (1897) devoted a minor portion of his famous theoretical study of tides on a rotating globe to a discussion of the currents produced by a zonal distribution of precipitation and evaporation, ignoring friction. He found that a uniformly accelerated system of purely east-west geostrophic currents would exist, but because of the uncertainty of how to treat friction he was unable to obtain steady state solutions and thus to conclude whether precipitation and evaporation are capable of being a significant cause of real ocean currents. A more important limitation of HOUGH's ocean current model is the absence of meridional boundaries in the ocean. GOLDSBROUGH (1933) discussed a model with such boundaries and found that he could construct steady stationary current fields provided only that the integral of the precipitation-evaporation function, taken along each parallel of latitude between the two boundaries, vanishes. The steadiness of the GOLDSBROUGH solutions does not depend upon friction. However, Nature affords no guarantee that the above integral of the applied precipitation-evaporation distribution shall vanish, nor are the causes of oceanic circulation limited to precipitation and evaporation (HOUGH and GOLDSBROUGH did not imply they were so limited, but employed them for simplicity's sake).

Considered from the vantage point of the present, it is astonishing to recognize that the theories of HOUGH and GOLDSBROUGH, although as originally proposed, considered only evaporation-precipitation as a cause of currents, are actually more general than at first appears, and in fact contain the main dynamical features of our present view of oceanic circulation. Indeed, one needs merely to introduce two additional physical ideas:

(1) EKMAN's (1905) notion that the direct frictional stress of the wind is confined to a thin surface " boundary layer," and that viscous shearing stresses are unimportant dynamically in the deeper body of the ocean, excepting, perhaps, at the bottom.

(2) The concept of a narrow western boundary current of a highly frictional (MUNK and CARRIER, 1950; HIDAKA, 1950) and/or inertial (CHARNEY, 1955; MORGAN, 1956) character.

These physical ideas are applied to the HOUGH-GOLDSBROUGH models in the following way. EKMAN's frictional boundary layer provides a way of converting the field of applied wind stress into a field of vertical velocity just beneath the surface and just above the bottom. Hence the main body of ocean water can be treated as frictionless and in geostrophic motion. The analysis proceeds the same as for the model driven by precipitation-evaporation, the water being supplied or abstracted by convergent or divergent boundary layers at the top and bottom.

The second physical idea, that of the western boundary current, frees the forcing function from the above-mentioned unnatural integral constraint. Thus in GOLDSBROUGH's original model, if precipitation exceeded evaporation over a latitude circle between two meridional coasts, there would be a net transport toward the equator, and mass conservation would be violated in the steady state. It now appears physically that in fact a narrow return flow would develop along the western coast sufficient to preserve mass conservation. Today it is believed that the Gulf Stream itself is an example of such a higher-order-dynamics western boundary current. MUNK (1950), CHARNEY (1955), and MORGAN (1956) have discussed the conditions in such currents in some detail, but from the point of view of the general oceanic circulation the main interest is not in the detail, but the mere fact of their existence because they provide a means of satisfying mass conservation in oceans bounded by meridians for quite arbitrary forms of the forcing functions. From a physical point of view it seems that such meridional boundaries and their associated western boundary currents are perhaps the essential difference between the regimes of circulation in the ocean and the atmosphere.

The work of the tidal theorists was confined to homogeneous oceans. SVERDRUP (1947) showed that in the steady state vertical variations of density do not need to be taken into account explicitly providing vertically integrated velocities are employed rather than the velocities themselves. Moreover, MORGAN (1956) has shown that density stratification does not prevent the formation of western boundary currents. Thus the form of the vertically integrated velocity field in central oceanic regions (or, simply, the net transport field) is independent of the details of the density structure. This powerful simplification enables us to construct net transport fields of steady ocean current circulations caused by wind or by precipitation-evaporation quite easily. Within the same framework the thermohaline circulation appears as an internal mode, and completely vanishes when integrated vertically. There is one reservation about the method of vertical integration which should be stated explicitly: the method is strictly limited to cases where appreciable velocity does not extend to large-scale irregularities of the bottom. In the case of a homogeneous ocean this is patently not the case, but the tidal theorists have written their theories to accommodate non-uniformity of the depth. For example, LAMB (1932, p. 326) finds that non-uniformity of depth in models with uniform rotation introduces second class waves of much the same character as non-uniformity of rotation introduces on the betaplane, or spherical ocean of uniform depth. Also EKMAN (1932) recognized the analogous effects of non-uniform depth and non-uniform rotation in large-scale circulations. He called them topographic and planetary effects, but he did not construct a complete ocean current theory, evidently because of the want of the western boundary current.

Another example of the close connection of the work of the tidal theorists with more recent studies in oceanography and meteorology is the topic of large-scale transient motions. Oceanographers and meteorologists distinguish between long inertio-gravitational waves on one hand, and quasi-geostrophic waves (e.g. planetary waves in the westerlies) on the other ; but this distinction is already implicit in the tidal theorists' classification of waves of the first and second class (LAMB, 1932, p. 350). The recent study of free and forced motions of a two-density layered ocean on the beta-plane by VERONIS and STOMMEL (1956) is really a study of these same waves of both external and internal modes. It is found that the transient response of the ocean to time-variable forces depends essentially upon the vertical density structure.

The current systems of HOUGH *and* GOLDSBROUGH

In order to compare more closely the solutions obtained by HOUGH and GOLDSBROUGH we take a particularly simple pattern of distribution of precipitation and evaporation: precipitation over one hemisphere, evaporation over the other. The two cases are distinguished by choosing these hemispheres as the northern and southern respectively in the HOUGH case, and as the western and eastern in the GOLDSBROUGH case. The only difference between the two cases is that the distribution of precipitation-evaporation has been rotated 90°. The motion in both cases is regarded as very slow so that inertial effects are negligible ; also friction is neglected. The rain does not impart momentum to the ocean; the ocean is homogeneous in density, uniform in undisturbed depth, and covers the entire globe.

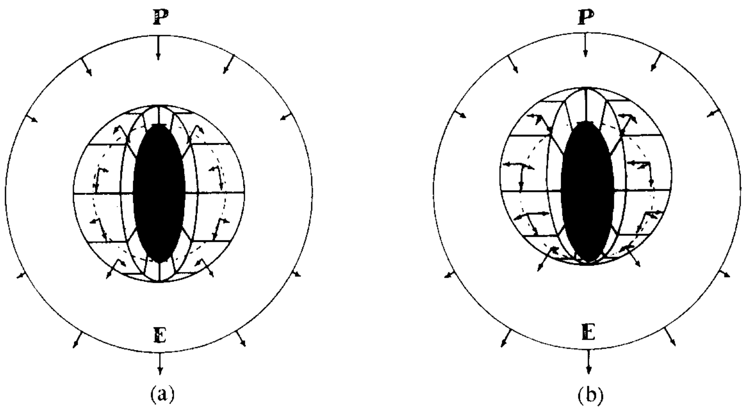


Fig. 1 (a and b). Two successive stages of the HOUGH-type circulation pattern, driven by precipitation distributed over the northern hemisphere (P) and evaporation distributed over the southern hemisphere. The hovering arrows indicate the distribution of precipitation-evaporation. The arrows drawn on the surface of the spheres are velocity components. The zonal currents grow with time. The central solid portion of the earth is shown as shaded in the cut-away mid-section.

First consider the HOUGH case, illustrated in Figs. 1 (a) and 1 (b), with precipitation in the northern hemisphere, and evaporation in the southern hemisphere. The solid lines are contours of equal precipitation-evaporation rate. The circulation is not steady, hence Figs. 1 (a) and 1 (b) represent two successive stages in the development of the field of flow. The elevation of the free surface and the zonal velocity component vary linearly with time as shown by comparing the two figures, but the meridional component of velocity and the vertical component of velocity are independent of time. The horizontal components of velocity are independent of depth ; the vertical component of velocity varies linearly from the surface to zero at the bottom. At the surface the vertical component of velocity (measured positive from the centre of the globe) is the sum of the rate of increase of the elevation of the free surface and of the evaporation. Pressure is determined hydrostatically. As can been seen from the cut-away portions of Figs. 1 (a) and 1 (b), as time progresses not all of the water which is precipitated upon the northern hemisphere crosses the equator to the evaporation hemisphere, but some is used to build up an ever-increasing high-pressure region centred at the North Pole, and an ever-depressed low-pressure region grows around the South Pole. Thus meridional pressure gradients increase with time; and associated with them are two geostrophic zonal currents that also grow linearly with time. The zonal current is zero at the poles and at the equator. The meridional component of velocity is directed southward everywhere, and reaches its maximum where it crosses the equator. Although it flows from a region of high to a region of low pressure this meridional component is not driven by the pressure gradient, but is related through the Coriolis force only to the acceleration of the zonal current (remember there is no friction). Of course, a uniformly accelerated system of this kind persisting throughout all time is wholly unphysical – at some stage the velocities would no longer be small, our premise of slow motion would be violated, and at some stage the regime should become unstable or the effects of friction should intrude. Although instability of the flow pattern may be more plausible physically as a means of braking the motion, and indeed does seem to occur in the atmosphere, there is reason to believe, as we will see below, that in low latitude at least, the ocean circulation is stable. A simple braking of the currents in the HOUGH case can also be introduced by putting in a large virtual turbulent eddy viscosity, which leads to a steady state circulation pattern qualitatively similar to the pattern shown in Fig. 1 (b), as though the HOUGH solution were suddenly arrested. Introduction of large horizontal eddy viscosities is convenient mathematically, but the physical justification is doubtful.

The chief interest of the HOUGH case lies in the remarkable contrast which is produced by simply rotating the distribution of the precipitation-evaporation distribution 90°, so that the precipitation-evaporation maxima lie on the equator instead of at the poles. This is a special case of the GOLDSBROUGH type solution, shown in different views in Fig. 2. Here, even without friction, the field of pressure and flow is steady. Both horizontal velocity components are completely geostrophic – the flow is along isobars. The vertical component of velocity is zero at the bottom and varies linearly with depth up to the surface where it equals the precipitation-evaporation rate. The pressure field (obtained hydrostatically from the elevation of the free surface) is a rather complicated one. On the eastern edge of the hemisphere of precipitation there are two low-pressure cells; on the western edge, two high-pressure cells. The general pattern of flow is therefore a flow across both polar regions from the region of evaporation into the region of precipitation, and a flow in tropical and sub-tropical latitudes in the opposite direction. The difference of these two transports is, of course, equal to the precipitation in the western hemisphere.

The fact that there can be a net transfer of mass from one hemisphere to another by geostrophic flow in a closed system of isobars is simply a result of the fact that the

Coriolis parameter varies along the meridian between the hemispheres. The lower the latitude the greater the transport between two isobars.

Geostrophic flow with a meridional component, in a homogeneous ocean of uniform depth, must exhibit a horizontal divergence – positive if directed toward the equator, negative if directed poleward. This is readily seen from Fig. 3 (a) in which a portion of a homogeneous, uniform depth ocean, bounded by two isobars

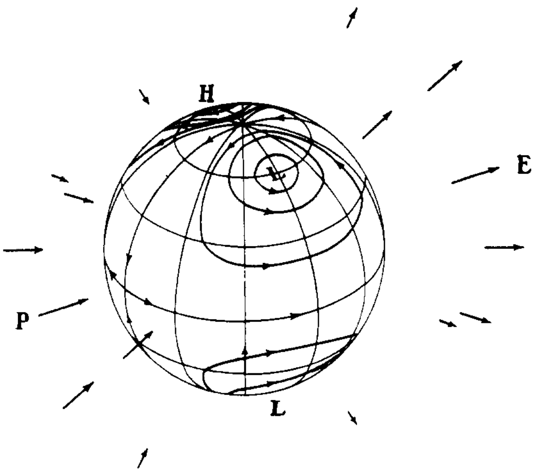


Fig. 2. The GOLDSBROUGH-type steady solution driven by precipitation over the western hemisphere, and evaporation over the eastern. The curve lines with arrows on them are isobars. The centers of high- and low-pressure cells are indicated by H and L.

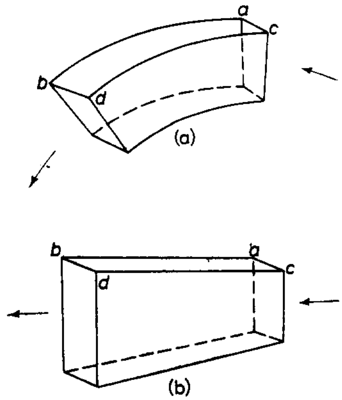


Fig. 3. (a) Divergence of geostrophic flow associated with change of latitude. (b) Divergence of geostrophic flow associated with non-uniform depth.

ab (high pressure) and *cd* (low pressure) is envisaged as flowing toward the equator. The pressure difference across section *ac* is thus the same as that across *bd*, independent of how wide these sections may be, but the Coriolis parameter is less at the latter, hence the geostrophic transport across *ac* is less than across *bd*. Inasmuch as no geostrophic flow can occur across *ab* and *cd* the excess of water must be supplied from somewhere – and in the steady system envisaged by GOLDSBROUGH this excess is provided by precipitation. In Fig. 2 the reader will note that the flow is entirely equatorward in the precipitation hemisphere. Similar remarks apply to poleward flows in the evaporation hemisphere. From a physical point of view the GOLDSBROUGH type circulation is constructed simply by choosing the particular system of geostrophic flow whose distribution of divergence (or convergence) is everywhere sufficient to absorb (or give up) the water locally precipitated (or evaporated). For certain types of precipitation-evaporation distribution there are no geostrophic flow patterns that can operate in this fashion, and it is under these circumstances that HOUGH-type solutions occur. Similar considerations apply to a plane ocean of variable depth (Fig. 3b), the geostrophic transport across *ac* being less than that across *bd* on account of lesser depth rather than larger Coriolis parameter.

Several further features of the GOLDSBROUGH solution should be mentioned here. First, in Fig. 2, it is clear that a coastal barrier could be placed along any complete isobar without affecting the solution, for example, along the meridional circle passing through the centres of the precipitation and evaporation hemispheres, or as another example the equator itself could be replaced by a coastal barrier.

There is an important restriction upon the kinds of precipitation-evaporation distribution that allow of GOLDSBROUGH-type solutions. Since there can be no net steady geostrophic transport across a latitude circle it is necessary that the integral of the imposed distribution of precipitation or evaporation along a latitude circle shall vanish. This is a very severe restriction which no natural distribution of precipitation-evaporation necessarily fulfills.

Extension to the wind as a causal agent and to more general ocean-basin shapes

For the sake of simplicity we retain at this stage of the discussion the homogeneity of the ocean and its uniformity of depth, and inquire into more general causes of oceanic circulation and the influence of the introduction of various forms of coastal boundary.

EKMAN (1905) (or LAMB, 1932, p. 593) has shown that the currents directly produced by the stress of the wind are confined to a thin surface layer of less than 100 m thickness. Non-uniformity of the wind stress over the ocean causes a convergence or divergence within the EKMAN layer, and this integrated over the depth of the layer is independent of the details of the turbulent eddy structure of the layer. In the steady state, by conservation of mass, these distributions of net horizontal divergence produce an impressed vertical velocity on the top of the body of water immediately

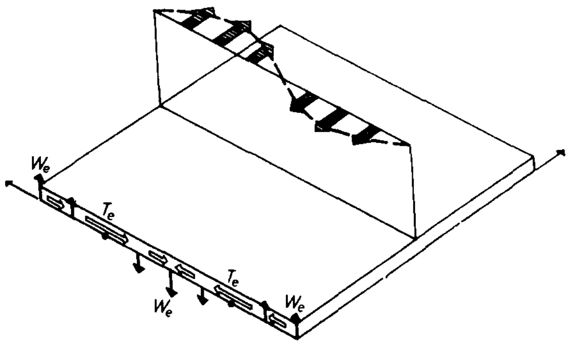


Fig. 4. The surface EKMAN layer, produced by a wind-field with sinusoidal pattern (the shaded arrows hovering above the surface). The EKMAN layer transports to the right of the wind are indicated by the open arrows labelled T_e. The vertical velocities produced by convergence and divergence of the EKMAN transports, and thus impressed upon lower layers, are indicated by solid arrows labelled w_e.

below the EKMAN layer. A particular example is illustrated in Fig. 4. The applied wind stress is purely zonal. The total transport of the wind-driven layer to the right (in the northern hemisphere) of the wind stress is indicated by the horizontal arrows labelled T_e. It is worth stating that the vertical scale in the diagrams is generally greatly exaggerated. Since these EKMAN transports are opposed, and since we can rule out an escape of water across the top surface, the water must escape the EKMAN layer from the bottom with a vertical component labelled w_e. Moreover, within the linearized dynamical framework under consideration, it does so without carrying any horizontal momentum with it. Thus it is immediately evident that so far as the deep frictionless layer is concerned it might just as well be operated upon by a precipitation-evaporation distribution equivalent to w_e. We are thus permitted to investigate the transports of the deep layer within the framework of the HOUGH-GOLDSBROUGH-type solutions*.

The magnitude of w_e depends upon the magnitude of the curl of the quotient of the wind-stress and the Coriolis parameter. The mean pattern of wind-stress over the real oceans is predominantly a zonal one, and were the real ocean unbounded by coasts, would produce a HOUGH-type circulation, which, as we have seen, in the absence of friction or instability, does not admit a steady solution. However, if a meridional boundary is introduced it is possible to construct a steady solution starting at this boundary and working toward the west. SVERDRUP (1947) has shown that this is possible even in a density-layered ocean, but he does not explain why the development cannot be made eastward from a coast, nor does he indicate how the other side of the ocean is to be treated. Before attempting to explain these points, it is perhaps best to first describe a very simple SVERDRUP-type system: a homogeneous

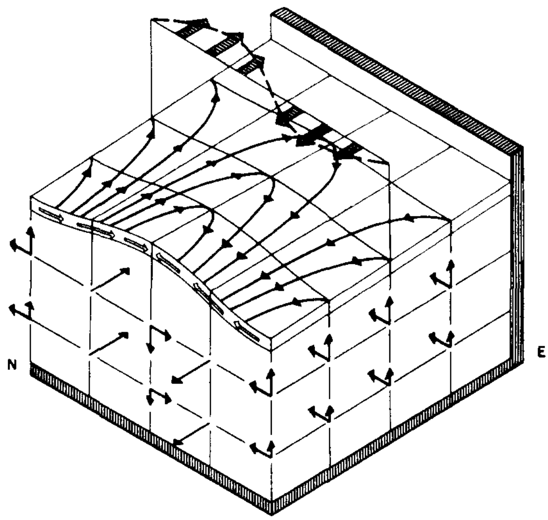


Fig. 5. SVERDRUP-type solution in homogeneous ocean of uniform depth, bounded by meridional coastal wall on the east. The wind system and EKMAN layer are the same as in Fig. 4. The curved lines with arrows on them are isobars ; the arrowheads indicate direction of the geostrophic horizontal flow (independent of depth). The components of velocity at a number of subsurface depths are shown by the solid arrows.

ocean, bounded by an eastern coast, and acted upon by a zonal wind stress. Fig. 5 shows the type of solution which applies in this case. At the surface there is a zonal wind stress similar to the distribution of the Northern Hemisphere westerlies and Trades over the real ocean. The mathematical solution from which the figure is drawn was obtained on the so-called beta-plane – a plane system of co-ordinates in which the Coriolis parameter is treated as a constant except where differentiated in the meridional direction. The transport of the EKMAN layer is indicated by the open arrows everywhere proportional to and to the right of, the wind. There is a vertically downward velocity imposed in central regions of the diagram: between the maximum

*Dr. K. YOSHIDA has pointed out to me that he and Dr. HAN-LEE MAO used these ideas in an unpublished (1954) paper on coastal upwelling off California.

westerlies and maximum easterlies. Outside this zonal belt the impressed vertical velocity is upward. From the bottom of the EKMAN layer to the bottom of the ocean this vertical velocity component diminishes linearly to zero. This water is absorbed, so to speak, by the divergence of the meridional component of the geostrophic velocity. Thus at midlatitudes where the vertical velocity is downward, the meridional component of geostrophic velocity is southward. The zonal components of the geostrophic transport are introduced to preserve mass continuity. At the latitude of maximum westerlies, for example, where there is no impressed vertical velocity the geostrophic flow is entirely zonal, diminishing linearly toward the coast. The contours of surface height, which determine the pressure field associated with the geostrophic flow field, are also shown in Fig. 5. The reader will remember that we are here dealing with a specially simplified case of the SVERDRUP-type model. We have constructed a geostrophic current system, consistent with the eastern coastal boundary condition which matches by its field of divergence and convergence (due to the meridional variation of the Coriolis parameter) the impressed field of convergence and divergence of the thin surface EKMAN wind-driven layer. We have obtained a steady-state solution. But it is also evident that we cannot satisfy conditions at the

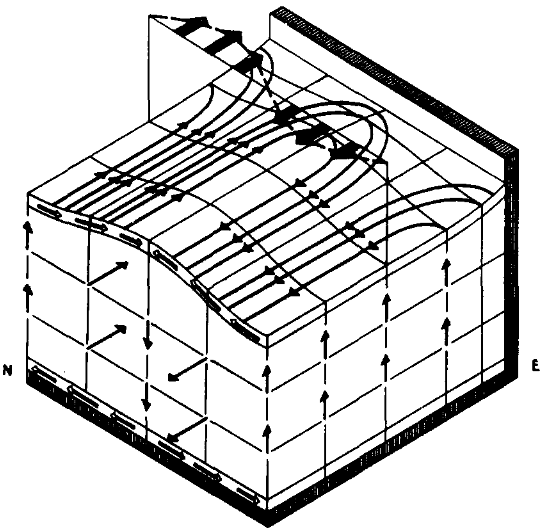


Fig. 6. EKMAN-type solution for the same wind field and shape ocean as shown in Fig. 5. In this case, with no divergence associated with geostrophic meridional currents, the compensating divergence occurs in a bottom frictional layer.

western coast unless some additional physical feature is introduced. But before looking further into this question, let us first consider what is involved if the eastern coast is removed. The there will be no possibility of an east-west pressure gradient, no meridional component of geostrophic flow, and the solution would accelerate. EKMAN has already discussed this case. He suggests that one may introduce a bottom frictional layer, with transports just opposite to those in the surface wind-driven layer. The final current system consists of three layers: the surface EKMAN layer, the intermediate layer of geostrophic flow, and a bottom EKMAN layer. The vertical velocity is constant with depth within the geostrophic layer (which being purely zonal has no horizontal divergence). The amplitude of the geostrophic current at mid-depths depends entirely upon the size of the frictional coefficients of the bottom layer. If there were no variation of the Coriolis parameter with latitude a meridional wall would cause the zonal current to turn around within a region more or less as wide as the width of the zonal current (Fig. 6), and the amplitudes of the motion would not be affected markedly. Far from the coast the flow is the same as in the absence of the coastal barrier. In Fig. 5, which has both the wall and variation of Coriolis parameter with latitude the situation is quite different: the flow never becomes purely zonal no matter how far removed from the coast one may be. A number of further comparisons between the regimes indicated in Fig. 5 and 6 can be seen by studying the illustrations: note for example the different dependence of the vertical velocity component upon depth in the two cases, the different form the surface elevation, etc. There are also marked quantitative differences in the transports in the two models: for example if the depth is increased in Fig. 5 the geostrophic transport remains constant, but the velocity decreases, but in Fig. 6 the geostrophic transport increases and the geostrophic velocity remains constant.

Further complications of the simple homogeneous model are, of course, possible. Non-uniform rotation and bottom friction may be both introduced simultaneously, also lateral friction, and even bottom topography of various simple forms. MUNK (1950) has given an indication of the possible role of lateral friction near the eastern coast. Quantitative comparison of the transports in simple models and as inferred from hydrographic data suggests that in the middle regions of sub-tropical and equatorial oceans, at least, topographic and frictional influences play a negligible role, and that the dynamical framework shown in Fig. 5 is more akin to that of the real ocean than that of Fig. 6. Lack of friction places an upper limit on the realizable transport in ocean currents, and this seems to be realized (e.g. SVERDRUP, 1947 ; REID, 1948, etc).

The fact that higher order physical processes might be important only at the western boundary of the oceans was first suggested by MUNK's (1950) grouping of the terms in his solution for a wind-driven, laterally viscous ocean. CARRIER (MUNK and CARRIER 1950) was apparently the first to recognize that this was tantamount to treating the intense western current by the mathematical technique of the boundary layer; that is to say, for certain values of the physical parameters determining ocean circulation the higher order effects of friction and inertia can be neglected everywhere in the ocean except in the theory of the narrow high velocity current along the western coast of the ocean. For further details about the theories and observational material on the viscous western boundary current, and the more recent inertial theory, I refer the reader to my book on the Gulf Stream (STOMMEL, 1957).

The main point that ought to be made here is that we can always fit a narrow western boundary current to a SVERDRUP- or GOLDSBROUGH-type solution. And we can do it symbolically – there is little to gain, from the point of view of the large-scale mid-ocean general circulation, by attempting to discuss the detailed structure of the western boundary current. We regard the western boundary current as a device for satisfying mass conservation and applying a physically realistic western boundary condition. In a word, it frees us from the crippling restriction upon the distribution of the driving agency already indicated by GOLDSBROUGH (above).

It would be misleading to give the impression that the theory of the western boundary current is complete. There is a complete theory of the purely lateral-viscous western boundary current (MUNK 1950; MUNK and CARRIER, 1950), but the theory, so far developed, of the inertial boundary current applies only to the growth region, or region of formation of the western current – where the transport of the stream increases in the down stream direction. The remainder of the inertial boundary current, where it must gradually leave the boundary and broaden out to join the sluggish interior solution, has not been studied. Evidently in nature, there is a rather complicated "decay region," meanders and breakdown of the western current, and a tendency of the boundary current to pull away from the coast bodily and thus to penetrate quite far into the interior without at once diminishing its narrow intense character (FOFONOFF's (1954) free solutions are enlightening in this respect).

The chief ingredients of the ocean current as outlined here are as follows :

(1) The simple precipitation-evaporation distribution may be replaced by an EKMAN-type wind-driven surface layer.

(2) A meridional boundary converts an otherwise accelerated Hough-type solution to a steady GOLDSBROUGH-type one, developed westward from the boundary.

(3) A western boundary current preserves mass conservation and satisfies the boundary conditions at the western boundary of the ocean.

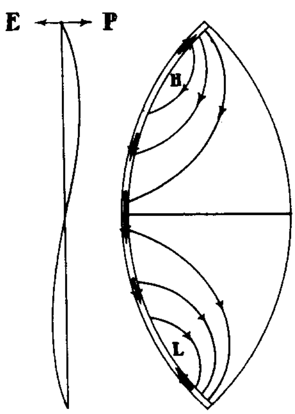


Fig. 7. The steady circulation produced in a uniform depth ocean bounded by meridional coasts 60° apart under the influence of precipitation in the northern hemisphere and evaporation of the southern hemisphere. The curve lines with arrowheads are isobars. The western boundary current is drawn schematically within the double line on the western coast.

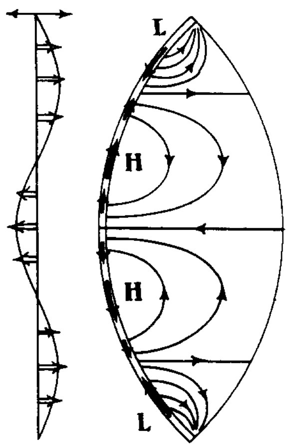


Fig. 8. The steady circulation produced in an ocean of same shape as that in Fig. 7, but acted upon by a particular distribution of zonal winds, shown at the left. The western boundary current is shown schematically by the double line at the western coast, and its transport is indicated by heavy arrows.

Further, as SVERDRUP (1947) has shown in the steady-state vertical density stratification does not materially affect the conclusion. SVERDRUP's well-known curl-equation merely states that the meridional component of the vertically integrated geostrophic transport at each point in the ocean is precisely that required to counterbalance the divergence impressed by the Ekman layer. The zonal component of the vertically integrated transport is obtained from the continuity equation, and the boundary condition of no normal transport through the eastern coast. The discussion of internal modes of circulation, whose vertically integrated transport vanishes, is reserved until later.

In order to demonstrate these ideas implemented in two special cases, Figs. 7 and 8 have been prepared. In each case the ocean is bounded by meridians 60° apart. In Fig. 7 the northern hemisphere is one of precipitation, the southern of evaporation. The distribution of precipitation-evaporation is somewhat similar to that shown in Fig. 1, except near the poles where it has been altered to avoid infinities, but otherwise it would be expected, in the absence of the meridional barriers, to lead to an accelerated HOUGH-type system. The presence of the meridional barriers permits pressure gradients along the latitude circles, and a steady geostrophic flow field with meridional components. The impressed precipitation or evaporation is absorbed or given up by the meridional geostrophic flow at each point in the ocean. The isobars, or lines of equal elevation of the sea-surface are shown. The flow is along these lines, but they are not transport lines, of course. The eastern coast and equator are isobaric. At the equator, where there is no evaporation or precipitation in Fig. 7, there is no meridional component of geostrophic flow; hence all of the water precipitated in the northern hemisphere must cross the equator at the western coast as a western boundary current. Slightly north and south of the equator there are meridional components of flow in the central portions of the ocean, but these are not sufficient to carry all the flow across the latitude circle that is required by mass conservation, so that much of this flow likewise occurs in the western boundary current. At sufficiently high latitudes the meridional geostrophic transport is just sufficient to carry away the mass added further northward: there is no western boundary current at these latitudes. Poleward of these points the mid-ocean geostrophic transports exceed the available supply of water added by precipitation to the northward, and hence the western boundary currents reverse, and actually flow toward the north.

In Fig. 8 a similar model is portrayed, except that here the causal agent is a zonal wind-stress distribution (indicated to the left) blowing upon an ocean, also with meridional boundaries 60° a part. Between 30° latitude and the poles prevailing westerlies are depicted; the trades extend across the equator from 30°S to 30°N latitude. This is, of course, only a very crude representation of the real wind system over the globe. The lines are isobars parallelling the geostrophic flow. In this case there is a certain amount of EKMAN wind-drift transport in the surface layers which ought to be added to the geostrophic transport field to give an accurate picture of the total transport field, but this is omitted in the figure to preserve simplicity of presentation. Here we obtain, by a series of arguments similar to those used in justifying Fig. 7, a system of gyres and western currents resembling that obtained in the complete theory of MUNK (1950) and HIDAKA (1950). The boundaries between the gyres correspond to latitudes of no EKMAN layer convergence. The regions of maximum geostrophic meridional flows in each gyre correspond to the latitudes of maximum convergence (equatorward flow) or divergence (poleward meridional flow) of the EKMAN wind-driven layer. The zonal geostrophic transports are constructed by continuity from the eastern coast, and western boundary currents are fitted where needed to conserve mass.

Some of the features of the oceanic circulations described in this section have been qualitatively reproduced by VON ARX (1954) in a circular rotating basin using a density-homogeneous water layer of non-uniform depth. VON ARX's model is driven by applied wind-stress. In experimental work it is not possible to avoid bottom friction altogether, but evidently, with sufficiently deep layers the divergence of meridional geostrophic flows dominates over that of the bottom frictional layer. To date, quantitative experimental studies have not been made.

Some integral relations pertaining to thermohaline and internal mode circulations and application to interpretation of the atlantic ocean circulation

Before proceeding to the topic of the next section, where an effort is made to visualize the physical factors governing the vertical density distribution in the ocean, it is perhaps worthwhile to indicate here that formally, the simple ideas described above, and applied to a homogeneous or vertically integrated ocean, can also be employed to elucidate certain features of internal modes of circulation in the ocean by simply dividing the ocean into two layers by a *level surface* at mid-depth, say 1500 or 2000 m and specifying the vertical mass transport across this level surface as a function of geographical position. What is taken from one layer is added to the other. A pattern of geostrophic flow can be constructed in each layer to absorb

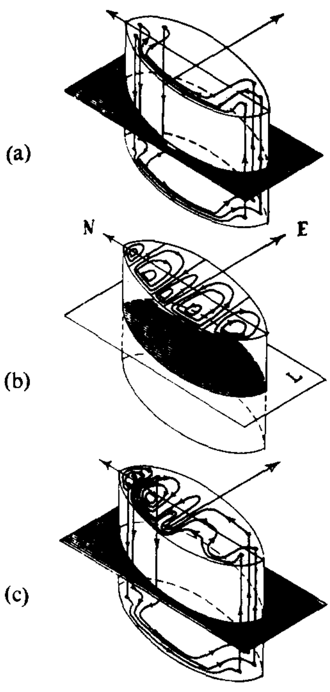
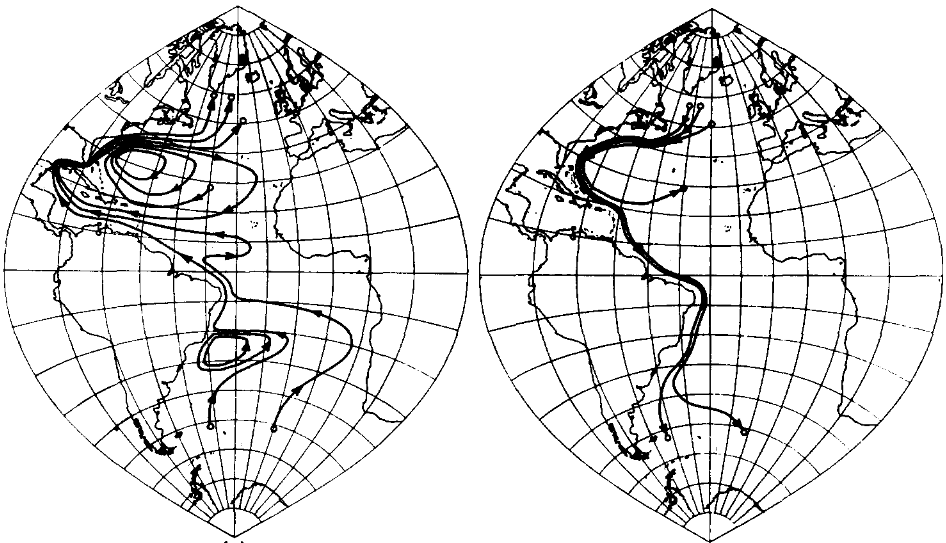


Fig. 9. A schematic interpretation of the circulation in the Atlantic Ocean constructed by superposition of an internal mode associated with flow across a level surface L at mid-depth (a) and a purely wind-driven circulation in the surface layers (b). The sub of these two is shown in figure (c).

(or give up) the water required for the vertical transport across the level surface, and a western boundary current fitted where needed in each layer. The vertically integrated transport over both layers taken together vanishes: the circulation is a pure internal mode, except perhaps where topographic effects dominate. These considerations, and the pictures derivable from them, are not *theories*. They merely provide an interpretative tool, and indicate what flow patterns must be associated with various distributions of vertical transport across level surfaces. As an illustration of the use of this tool, a rudimentary interpretation of the Atlantic Ocean circulation is presented here.

Consider the schematic ocean drawn in Fig. 9 (a), extending from pole to pole across the equator and bounded by meridional boundaries roughly 60° apart. A level surface L which divides the ocean into an upper and a lower layer is also shown. Now suppose that some thermohaline process, into the nature of which we do not here inquire, causes a sinking of water across the level surface in subarctic latitudes, and a rising of water across the level surface in subantarctic latitudes, as indicated by the vertical transport lines drawn in Fig. 9 (a) across the level surface. Our argument is that having specified the quantity and location of the vertical transport across level L, the rest of the transport picture in each layer is completely determined, in fact looks very much like that shown in Fig. 7 for a different physical situation.



(a) (b)

Fig. 10. A similar interpretation of the total transports in the surface layer (a) and bottom layer (b) but drawn on a chart of the Atlantic Ocean. Points of sinking and upwelling across the level surface are indicated by the little circles.

Thus for example, in the subarctic regions of the mid-oceanic portions of the upper layer the horizontal flow field is convergent, hence it must have a northward directed meridional component. The zonal component is determined by continuity, starting at the eastern boundary. Similar reasoning is applied to the lower layer and then to both layers in the subantarctic region. In subtropical and tropical latitudes we have specified no vertical transport across the level surface, and there is no geostrophic in the mid-oceanic regions of either layer. The transport of water between hemispheres which is required for conservation of upper layer and lower layer water separately must be accomplished by a narrow western boundary current. The field of motion is entirely internal; its vertically integrated transport vanishes everywhere. Fig. 10 (a) portrays the resulting circulation pattern.

Now consider separately a wind-driven circulation in the upper layer maintained by the impressed vertical velocity field due to an EKMAN layer as shown in Fig. 9 (b). It looks much like that shown in Fig. 8, except that there is an additional gyre just north of the equator (after MUNK, 1950) which is presumably caused by the existence of a doldrum area in the winds just north of the equator. Within the simple framework of the linear mechanics of the models under consideration these two circulations (Figs. 10 a and 10 b) are additive. The sum is shown in Fig. 9 c. Note the absence of the Brazil Current, the very intense Gulf Stream. The picture looks much more realistic than the simple gyre system (Fig. 9 b) alone. A slightly different interpretation is drawn in Fig. 10 upon two charts of the Atlantic Ocean, the right figure representing the lower layer, the left the upper layer, and I have placed the diving reference level at 1500 metres. It seems probable that over subtropical and tropical latitudes there must be certain small upward flux of mass across the level surface to maintain the observed vertical increase of density with depth against eddy diffusion, so I have drawn the figure showing one transport line ascending through the reference level in midlatitudes. Each transport line corresponds to about 10 million m^3/sec. There are a number of features in this diagram which should be noted. For example, the vertically integrated transports of the Brazil Current and the Gulf Stream are the same, but according to the interpretation suggested here the current in the deep layer opposes the Gulf Stream, whereas it has the same direction as the Brazil Current. The opposite is true in the upper layer: the Gulf Stream is re-enforced by the thermohaline component, the Brazil Current so weakened that it almost disappears. The number of solenoids (intersections of surfaces of equal pressure and specific volume) in the Gulf Stream is very much greater than in the Brazil Current, thus by ordinary oceanographic reasoning the Gulf Stream would look like a current of very much greater transport than the Brazil Current. We identify the deep current with North Atlantic Deep Water. The reader will find it interesting to examine in some detail such features as the vanishing of the western current in the region of latitude of the Equatorial Counter-current, the curious spiral in the subarctic region of Fig. 9 (c), etc. This picture of the circulation of the Atlantic is certainly not to be dignified by calling it a theory. It is simply offered as an interesting example of how far reaching the consequences of specifying the vertical velocity at the surface and some mid-depth level can be.

There is a further reason for focussing attention on the vital role played by the vertical velocity distribution, and hence the field of horizontal divergence in the ocean: namely, to emphasize the danger of making the assumption that because the vertical velocity is small it may be neglected – or even arbitrarily set equal to zero. For example, HIDAKA (1955) has recently worked out an elaborate numerical theory of the general circulation of the Pacific Ocean, interpreting it as a purely wind-driven phenomenon and assuming that the vertical velocity vanishes exactly everywhere. As HIDAKA himself has commented there is no clear physical explanation or interpretation of his numerical results ; indeed, from the point of view of this survey, it could hardly be otherwise. As I see it, the chief way that the Ekman layer affects deeper portions of the ocean is by imposing a vertical velocity upon them at the top, and chief agent of the thermohaline circulation is the slow vertical advection of density which balances the vertical diffusive loss of density to the surface.

Another example is NEUMANN's attempt (1955) to identify DEFANT's intuitive choice of reference level (DEFANT, 1941) in the Atlantic Ocean with a rigid impenetrable surface through which no water can pass. As I have indicated elsewhere (STOMMEL, 1956) the DEFANT level may actually be the level of no horizontal divergence in the ocean, in other words the level of maximum vertical velocity. This review is no place to attempt a detailed critique of the above studies of HIDAKA and NEUMANN. One can only point out that, on physical grounds, the field of vertical velocity is so intimately connected with the large scale circulation patterns that arbitrary assumptions about the vertical velocity structure in the ocean are attended by the most far-reaching implications concerning the form of the horizontal current patterns; and that unless this is fully recognized in setting up theoretical models, the results may be disastrous.

Extension to density-layered models in the steady state

The fact that SVERDRUP's theory gives the field of vertically integrated transport makes it possible to construct certain elementary models in which the density layering

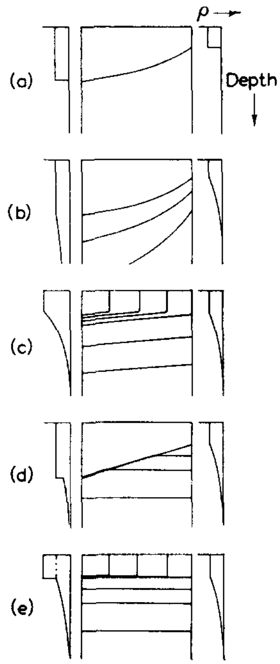


Fig. 11. These five vertical profiles (a, b, c, d, e) show the density structure which different models obtain in the plane $AA'B$ of Fig. 12. See text for explanation.

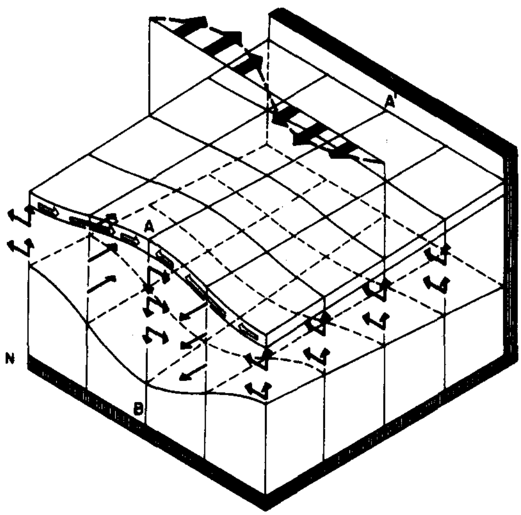


Fig. 12. The circulation in a SVERDRUP-type model such as shown in Fig. 5, but with two immiscible layers of slightly different density. The interface is shown by dashed lines.

is specifically taken into account. The most elementary model is one in which the density is essentially inactive – that is to say, the amount of water of each density is specified to begin with; no particle of water can change its density. One thus considers density as an immiscible property – various portions of the ocean remain unmixed, like layers of oil and water. This is, of course, a considerable oversimplification of the state of affairs in nature.

For example one might consider an ocean consisting of two separate layers each of uniform density, the top layer being slightly less dense than the lower layer. A vertical sounding (of density against depth) in such an ocean would look like one of the curves in Fig. 11 (a). If there is no friction across the interface, and the ocean bottom is of uniform depth, the deep layer cannot move, and the slope of the interface must entirely counterbalance the horizontal pressure gradient associated with the geostrophic flow in the upper layer. If the distribution of wind is the same as that shown in the homogeneous model depicted in Fig. 5, the state of the two-layer system is as depicted in Fig. 12. The vertical section of the density field along the latitude of maximum wind-stress curl (the $AA'B$ plane) is shown in Fig. 11 (a). The densities of the two layers are taken as constants in the model. The depth of the interface at the eastern coast is also an arbitrary constant. The functional dependence of the depth of the interface on position, subject to the constraint of the SVERDRUP theory, is then completely determined. Since there must be the same geostrophic transport to the south across each unit width of the section, and the depth of the interface increases by a marked fraction of its mean value, the geostrophic velocities and slope of the interface must diminish somewhat with distance from the wall.

REID (1948) has given another model which exhibits much similarity with reality, based upon an arbitrary vertical distribution of density sketched in Fig. 11 (b). Here the ocean is covered everywhere by a thin homogeneous layer of uniform density, but of variable depth, and is underlain by a region in which the density increases to a fixed bottom density by an exponential law – the coefficient of depth in the exponential function being inversely proportional to the depth of the homogeneous upper layer. This is, of course, a very special form of density law, but it must be said that it closely resembles the form of density distribution in the Eastern Equatorial Pacific quite well. As in two-layer model it contains only one dependent variable. Again the functional form of this variable is completely specified subject to SVERDRUP's constraint on the distribution of vertically integrated transport. The analog of the vertical density profile along the latitude of maximum curl of the wind stress for REID's density model is shown in Fig. 11 (b).

STOCKMANN (1953) has worked with a different type of density field. Suppose that a density anomaly is defined by subtracting from actual densities the bottom density, and at the eastern wall a density anomaly vs depth curve is drawn of any arbitrary or observed shape (Fig. 11 c). The density structure at other stations is now assumed to be formed by multiplying the given curve by a constant factor (constant with depth at any one station, but variable from station to station). This again is an attempt to describe the density field in terms of a single dependent variable. The density field along $AA'B$ according to STOCKMANN's model is sketched in Fig. 11 (c).

It is important to note, at this point, that in the models of REID and STOCKMANN, the fluid cannot be regarded as inactive or immiscible. There is in both models an unstated but implicit assumption about transfer of mass across isopycnic surfaces. The deeper layers are supposed, in the section $AA'B$ to be moving geostrophically in a meridional direction, and this can only occur with divergence. In the top layer this horizontal divergence is balanced by the impressed vertical velocity from the

EKMAN surface layer ; in deeper layers by a flow across isopycnic surfaces. Thus strictly speaking an explicit assumption of immiscibility would lead to profiles of the form shown in Figs. 11 (d) and 11 (e), instead of those of 11 (b) and 11 (c). Indeed it ought to be possible to compute the amount of mixing in nature from these considerations (STOMMEL, 1956).

The models mentioned above suffer being highly artificial, highly specified in form of the density field, and even more disconcerting, they involve implicit assumptions about the nature of the mixing processes. It would be very interesting to invert the problem: to make explicit assumptions about the nature of the mixing processes and, subject to integral constraints of the SVERDRUP kind, to find the resulting density fields. One might even hope to find a partial explanation of the main thermocline.

The pioneer effort in this direction is reported in an important Russian paper by LINEYKIN (1955). The model, and its implications, are best described in terms of the

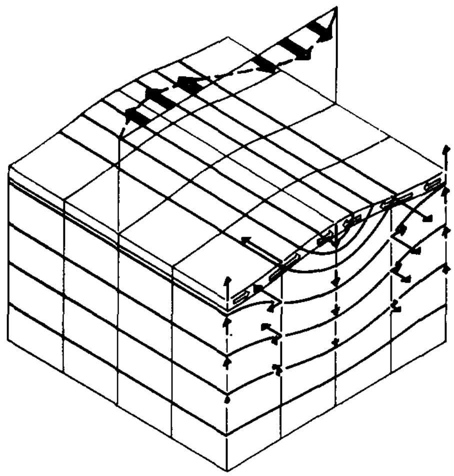


Fig. 13. LINEYKIN's explanation of the depth to which the baroclinic geostrophic currents produced by the wind extend in a stable ocean. The isopycnal surfaces are shown by the wavy nearly horizontal solid lines. The velocity components are shown by solid arrows.

illustration in Fig. 13. One begins by considering a basic state in which there is no motion – and no wind blowing over the ocean. There is, however, a uniform vertical stability maintained by vertical eddy conduction. We might suppose for example that the ocean is heated from above, and cooled from below, thus maintaining a constant and uniform heat flux downward, and a stable stratification of density. This is, it must be admitted, a rather artificial way to begin – the only excuse for it is that this conductive stable state is used as a basic state upon which to build a linearized perturbation theory when the surface is subjected to an infinitesimal wind-stress distribution of the form indicated by the large arrows hovering over the diagram. In the surface there is an EKMAN transport to the right of the wind, and an impressed vertical velocity field just beneath it. LINEYKIN found that if the Coriolis parameter is taken as a constant, and for such choice of the parameters as make the scale of the phenomenon truly oceanic, this vertical velocity perturbs the density field by vertical advection of heavy in such a way as to produce a perturbed field as portrayed by the lines in Fig. 13. Isopycnals are depressed in regions of downward flow, and vice versa.

Where the isopycnals slope most strongly the geostrophic subsurface currents are strongest. As is shown by the velocity arrows, this subsurface geostrophic current is generally in the direction of the surface wind, and there is a small non-geostrophic component (below the EKMAN layer but associated with friction) across the isopycnals directed from the region of downwelling toward the region of upwelling. This non-geostrophic flow at mid-depth compensates for the divergence of the EKMAN surface layer.

The most interesting feature of LINEYKIN's model is the fact that the geostrophic currents decrease exponentially with depth at a rate completely determined by the physical parameters of the problem (and not by arbitrary assumption). LINEYKIN has treated the problem rather generally, retaining non-isotropic viscosity and diffusivity, etc. But for the purposes of this exposition we ignore large-scale lateral turbulent exchanges, and hence, on account of the small vertical scale of the phenomenon (remember the vertical scale in the diagrams is greatly exaggerated) only vertical viscous and diffusive transports need be considered. It may be shown that if the turbulent coefficients of viscosity and diffusivity are roughly of the same magnitude – which there is generally reason to suppose is the case and, as in fact LINEYKIN does – the depth to which the geostrophic currents extend is independent of the eddy coefficients. This LINEYKIN-depth depends only upon the geometry of the system, the unperturbed vertical stability, and the Coriolis parameter. If the LINEYKIN-depth be defined as that depth at which the amplitude of the density perturbation is reduced to exp $(-\pi)$ of its service amplitude, the ratio of the LINEYKIN-depth to the half-wavelength of the wind system is equal to the ratio of the Coriolis frequency f to the VAISALA or BRUNT frequency, $\sqrt{gs}$, where s is the mean stability of the model. In midlatitudes this ratio must be taken somewhere between 0·1 and 0·01, but in the neighbourhood of the equator it is certainly much less. One can see, therefore that although the LINEYKIN depth may be a fair representation of, and model for, the depth of the baroclinic layer in a small basin such as the Caspian Sea, it is several orders of magnitude too great to provide a reasonable explanation of the depth of the main thermocline in the oceans in midlatitudes. This, of course, does not detract the fact that LINEYKIN's paper has been a most stimulating and interesting one.

It seems likely that it is necessary to construct a somewhat different version of the LINEYKIN model to account for the density field in the ocean. In particular one would like to introduce the variation of the Coriolis parameter with latitude, since this has already been demonstrated to produce a marked tendency toward limiting currents to the upper layers (VERONIS and STOMMEL, 1956).

VERONIS and I have shown (VERONIS and STOMMEL, 1957) in a short contribution to the theory of thermally driven circulations based upon the LINEYKIN stable model, how the introduction of the variable Coriolis parameter modifies the depth obtained in uniform rotation (the LINEYKIN depth). I will give here a slight modification of these published results to make the physics as clear as possible. Consider (Fig. 14) a horizontal layer of fluid conducting heat downward by eddy conduction from a fixed hot surface temperature, to a fixed cold bottom temperature. Now suppose we can add water to the top, or take it away, without altering the fixed surface temperature. We can effect this, for example, by imposing a distribution of bands of precipitation and evaporation parallel to meridians (indicated in Fig. 14 by the hovering arrows), or probably equally well by putting a thin wind-driven EKMAN layer on top. Thus in Fig. 14 the vertical velocity imposed at the top surface is a function of longitude only, downward in the mid portions of the diagram, and upward along the outer east and west edges. At some distance below the surface the temperature field is perturbed by the vertical advection of heat, which in turn is held steady by conductive reaction of the perturbed temperature field. The model shown in Fig. 14 is computed in a beta-plane, so that vertical gradients of the vertical velocity are intimately related to the horizontal meridional components of the geostrophic sub-surface currents, and in addition the vertical gradient of the horizontal component of velocity is related to the horizontal gradients of the temperature field through

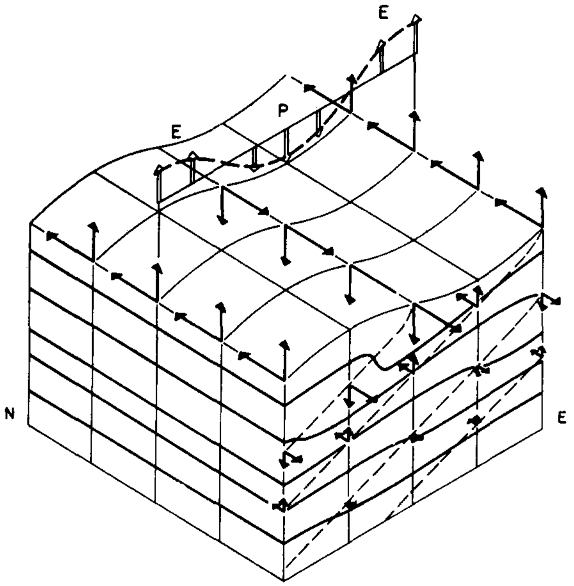


Fig. 14. Modification of the LINEYKIN model caused by introduction of a variable Coriolis parameter. The sloping dashed lines indicate the limits of the regions of positive and negative perturbation temperature (hot and cold cells) ; the rest of the diagram is labelled in the same manner as Fig. 13, but for convenience the model is driven by meridional bands of precipitation and evaporation (the hovering open arrows).

the well-known thermal wind relation. These necessary relations are sufficient to define the entire temperature and velocity fields as shown in Fig. 14. It is interesting to note the asymmetrical distortion of the isothermal surfaces; the maximum perturbation of temperature moves toward the west with increasing depth. If the east-west dimensions of the bands of precipitation-evaporation, or of wind stress, have a wavelength of 6000 km, the vertical eddy diffusivity is 5 cm^2 sec and the mean vertical temperature gradient is 5°C/km, the depth of the maximum amplitude of perturbed temperature lies at about 1000 metres in midlatitudes, and decreases markedly toward the equator. Comparison of different latitudes on the beta-plane is possible by treating the Coriolis parameter parametrically. There is an indication of a rudimentary thermocline in Fig. 14, at about this depth (the thermocline tilts downward toward the west, and one also gets the impression of a tongue-like protrusion of warm water downward and westward, between cold tongues moving upward and eastward. However this is only an illusion, and not at all like a true tongue or

" Kernschicht " because there is no east-west component of velocity anywhere in the model. All motion is confined to vertical meridional planes. Although this model contains eddy diffusivity, it does not contain eddy viscosity.

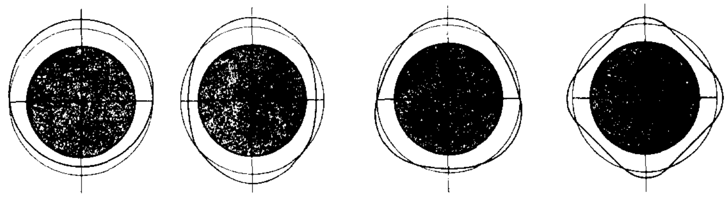
The physical setup of this model is contrived to make the mathematics easy, consequently the regime does not look much like naturally occurring ones. The distribution of causal agent in parallel meridional bands is clearly an artifice. Over real oceans winds and heating and cooling are much more nearly functions of latitude than of longitude. The LINEYKIN model (Fig. 13) is completely indifferent to a shift of axes of the causal agent, but the model on the beta-plane (Fig. 14) is very sensitive to a shift of axes. If in the latter case the perturbing forces are made functions of latitude rather than of longitude, the meridional components of geostrophic flow vanish, and the flow pattern deepends and is transformed to the LINEYKIN one. In such a case the variation of Coriolis parameter with latitude would enter only parametrically. However, the introduction of a meridional barrier into the model would force strong meridional components of geostrophic flow by making the zonal turn around, and we can anticipate that this would permit the variation of Coriolis parameter to play an important role over a large distance from the coast, a distance possibly as great as the entire width of terrestrial oceans. Thus it seems possible that the vertical distribution of density and currents in a more realistically setup theoretical model (with zonal winds, the beta-plane, and a meridional boundary) might resemble that in the extremely artificial model of Fig. 14. To date it has not been possible to work out the mathematics of such a happier model.

There are more serious drawbacks to these models, however, than superficial similarity to nature. First, the strong vertical stability, uniform with depth, is merely a contrivance that permits a linear perturbation analysis ; it has no counterpart in the ocean. Actually all the heat advective terms in the heat transfer equation must be significant. Secondly, the parametric treatment of the eddy coefficients is exceedingly primitive and misleading. Even if oceanic turbulence at mid-depths is not directly related to the mean flow, it is desirable to have some physical understanding as to what causes this turbulence. At present we simply regard these coefficients as given arbitarily, at best we infer them from water-mass analysis. Our models are, so to speak, externally stirred. And perhaps they actually are, through short internal waves generated by storms. But this is all sheer speculation.

Time variable systems : Free Waves on a Homogeneous Ocean of Uniform Depth

The purpose of this section is to give a general description – in physical terms – of the types of free wave motion which are possible on a rotating globe. In order to avoid the almost endless variety of small-scale waves that are discussed in the general hydrodynamic literature, and have no special significance for phenomena of the oceanographic scale, only waves in which hydrostatic equilibrium exists at all times are considered. Thus we confine our attention to dynamical systems akin to those discussed in LAMB's chapter on tidal waves. The reader will note that the term " waves " is used in a somewhat general way, to embrace not only those transient phenomena which are customarily considered examples of hydrostatic wave motion (tides, seiches, long-period internal waves, surges, etc.) but also to include certain transient ocean current systems which propagate themselves in a wave-like manner. Thus the term is used more in the meteorologists' sense – it can include quasi-geostrophic disturbances of large-scale circulation systems – or in certain contexts, the largest scale systems themselves, there being no implicit upper limit to the length of the period admitted. On the other hand, certain familiar types of waves, such as ocean surface waves, being of too high a frequency, too small a scale, and not at all hydrostatic, are not considered at all.

First let us consider some of the results of the tidal theorists (HOUGH, as summarized by LAMB, 1932, pp. 349-350) in particular the case of free waves on a homogeneous ocean of uniform depth on a rotating globe. In order to limit discussion only the case of uniform depth of 14,520 feet is considered here, a particular depth being



(a) (b) (c) (d)

Fig. 15. Four different symmetrical modes of oscillation of the free surface (heavy line) about the undisturbed sea level (light line) on a globe. The figure represents a section through the globe's axis of rotation.

chosen so that numerical values for periods of various waves can be given. In Figs. 15 (a, b, c, d) the deformation of the elevation of the free surface are independent of longitude. In Fig. 15 (a) there is only one nodal circle at the equator, the elevation of the surface seesaws from north to south across the equator in a huge standing oscillation. For the actually rotating earth with depth 14,520 feet the free period of this standing wave across the equator is 25 hr 28 m, but if the earth did not rotate the period would be 41 hr 55 m – it is striking how much the rotation of the earth alters the free period of this large-scale oceanic standing wave, or seiche.

The second case shown, in Fig. 15 (b), has two nodal latitude circles at about 30°N and 30°S. The elevation of the free surface alternates between polar regions and tropical regions. The free period, as might be expected is shorter than for the case with one nodal circle, namely 15 hr 11 m (23 hr 12 m without rotation). Similarly higher numbers of nodal circles lead to shorter periods : thus in Fig. 15 (c), with three nodal circles the period is 11 hr 54 m (16 hr 08 m without rotation) ; or in Fig. 15 (d) with four nodal circles the period is 10 hr 01 m (12 hr 23 m without rotation). As the number of latitudinal nodal circles is increased the effect of the rotation on the period becomes negligible. For example, the case with twelve latitudinal nodal circles has a period of 4 hr 12 m (4 hr 21 m without rotation). This is an illustration of the more general fact that the earth's rotation does not apparently affect wave motions with a frequency much higher than the rotational frequency of the earth (2π radians a day).

Now it is an important thing to note at this point that any one of the configurations of the free surface associated with the axisymmetrical waves drawn in Fig. 15 can also be maintained, within the framework of this simple frictionless dynamical system, with infinite period by an appropriate system of steady zonal geostrophic currents. For example, in Fig. 15 (b) westward flowing geostrophic zonal currents in both hemispheres would balance the particular deformation of the free surface there depicted.

We will now proceed to describe wave forms which are not axisymmetrical and will find similar geostrophic current systems, which are functions of longitude as well as of latitude, and whose periods are not infinite, but are quite long relative to the period of inertio-gravitational waves with the same shape of the free surface. They are more like wavy current systems such as one encounters in the atmosphere.

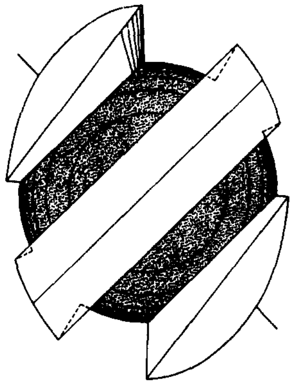


Fig. 16. Antisymmetrical mode of oscillation.

In Fig. 16 a wave is illustrated in which one complete meridian and the equator are nodes. The ocean is 14,520 feet deep. Consider waves moving east or west : waves in which the equatorial node remains fixed, but the meridional node moves around the earth. The wave toward the east has a period of 19 hr 16 m ; the wave toward the west, 12 hr 51 m (without rotation both waves have the period 23 hr 12 m). Waves of this kind – whose period approaches a finite value as the rotation is decreased toward zero – are called waves of the First Class and correspond physically to what one usually has in mind when he thinks of long tidal waves (this is also true of the finite period waves in Fig. 15).

In addition to the high-frequency First Class waves, there is a nearly geostrophic wave of much the same form (the actual form shown in Fig. 16 is computed from HOUGH's expansion, 1897, II, p. 167) which travels only toward the west) there is no eastward counterpart) and has a period of 11 days. These waves are called Second Class waves by the tidal theorists and are distinguished by the fact that they become stationary (their period becomes infinite) if the rotation of the globe is reduced to zero. As we have seen, in the special case of axisymmetric waves (Fig. 15) the period of of the Second Class motions is always infinite. From a physical point of view these waves of the Second Class are moving current systems. In the presence of a basic zonal current waves of the Second Class may be identified with ROSSBY's planetary waves (ROSSBY, 1939). The Second Class waves are essentially the building blocks from which the general forced wave motions characterizing the transient ocean current theory are constructed. One more example of free wave motions on the sphere may be of interest to mention. GOLDSBROUGH (1933) has obtained the free periods of waves in a homogeneous ocean of uniform depth (12,880 ft) bounded by meridional coasts 60° apart. In addition to two First Class waves (rotary tidal waves with periods

12 hr 25 m and 10 hr 18 m) he has also found a Second Class wave with a period of 7·2 days. This wave might be spoken of as a free quasi-geostrophic seiche. One supposes that the Atlantic Ocean must be somewhat resonant to weekly periods of atmospheric disturbance. The successive elevations of the free surface characterizing this Second Class standing oscillation of GOLDSBROUGH are sketched in Fig. 17 : only the first quarter period is illustrated.

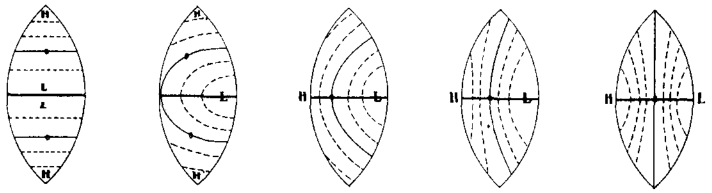


Fig. 17. Sketch of successive contours of surface level in GOLDSBROUGH's 7·2 day geostrophic standing wave (Second Class) in an ocean bounded by meridians 60° apart ; only the first quarter period is depicted.

It is much more convenient, in trying to obtain a clear physical understanding of the differences between the First and Second Class waves, and in working out simple mathematical examples, to abandon the co-ordinate system of the sphere, and to make use of a plane system of co-ordinates, introduced by ROSSBY (1939) in which the variation of the Coriolis parameter is taken as a constant over the plane, and the Coriolis parameter itself is also taken as a constant wherever it occurs in undifferentiated form. This so-called beta-plane has been widely adopted in dynamical meteorology on account of analytical advantages and the simplicity of physical interpretation of the phenomena which it exhibits. On the other hand, one should keep in mind the fact that the beta-plane is not a perfect analogue of this sphere.

Consider now free harmonic waves propagated along the east-west direction with horizontal crests and troughs parallel to the north-south axis, as sketched in Fig. 18 (a-f). The undisturbed depth is uniform and the water homogeneous. Hydrostatic equilibrium prevails; the pressure gradients are independent of depth and depend only upon the instantaneous slope of the disturbed surface (remember the vertical scale in the diagrams is much exaggerated). Thus the horizontal components of velocity are independent of the vertical co-ordinate. The vertical component of velocity varies linearly with depth in such a manner as to vanish at the bottom and to equal to time rate of change of surface elevation at the top of water layer. For any wavelength there are three possible such waves on the beta-plane. Two are First Class; one, Second Class. A so-called frequency diagram is sketched in Fig. 19. The ordinate is frequency, the abscissa wave number or reciprocal wave length. THOMPSON (1952) has given a very clear review of the theory of these different waves. We will content ourselves here with a simple physical description. The curve marked I in Fig. 19 corresponds to First Class Waves which can move either towards the east or the west. They have relatively high frequency, and are simply the familiar long gravitational-inertial waves. For short wavelengths (region 1 in Fig. 19) the First Class waves are unaffected by the earth's rotation, are ordinary long gravitational or tidal waves, and are non-dispersive. The particle orbits are indicated in Figs. 18 (a) and 18 (b). Particles move in straight-line segments (neglecting the very small vertical displacements) alternately toward the west and east, the maximum horizontal velocity being toward the direction of wave propagation under a crest, and in the opposite direction under a trough. For longer wave lengths (region 2, Fig. 19) the First Class waves are dispersive, and the period approaches half a pendulum day. These waves were first discussed in detail by SVERDRUP (1927), in connection with tides on the North Siberian Shelf. In addition to the back-and-forth particle motion in the direction of wave propagation, there are also components of horizontal motion transverse to the direction of wave propagation, so that the particles execute elliptical orbits,

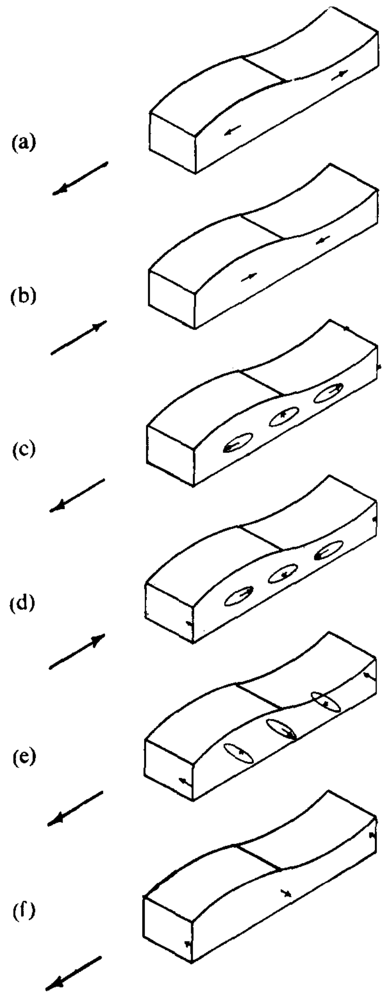


Fig. 18. Sketch of surface perturbation, instantaneous particle velocity (arrows), hodograph (ellipses) for different kinds of free long waves. The arrows on the left show the direction of propagation. By convention we take left ward as westward.

(Fig. 18c, d) which, as the frequency approaches half a pendulum day, tend to become circular – the limit particle motion in pure inertial circles. Density stratification introduces free internal modes, which, for a given wavelength are much more affected by the earth's rotation (VERONIS and STOMMEL, 1956), but it would be too much of a digression from the main theme of this review to consider First Class waves much further here.

The second curve in Fig. 19, marked II, gives the frequency vs. wave length relation for Second Class waves in the homogeneous ocean on the beta-plane (ROSSBY 1939). There are two distinct regions to the curve : a long wave length region (region 3 in Fig. 19) with long periods, the waves being non-dispersive, and another region (region

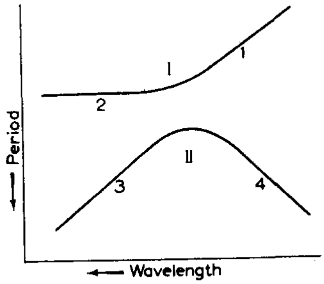


Fig. 19. Period-wave length relation for First Class (I) and Second Class (II) waves in a homogeneous layer on the beta-plane.

4 in Fig. 19) of shorter wavelengths, also of long period. These short waves are highly dispersive, and interesting enough are of such a nature as to have very little horizontal divergence. The two regions 3 and 4 are connected by a shorter period transition region. The minimum period of Second Class waves appears in this transition region. The free waves of Second Class move only to the west. The orbits are essentially horizontal ellipses with major axes parallel to the crests, as shown in Fig. 18 (e). The particle orbits become more and more eccentric as the period increases, for both long and short wavelengths (Fig. 18f). It is clear that the Second Class waves are essentially geostrophic current systems which because of the planetary geostrophic divergence inherent in the beta-plane cannot remain stationary. Consider the Second Class wave shown in Fig. 18 (f). The current is toward the south and the Coriolis force is balanced by the eastward horizontal pressure gradient due to the slope of the free surface. The divergence of this geostrophic current can be balanced in two ways : (1) first, the free surface can drop over southward currents and rise over northward ones (northern hemisphere) – which leads to the non-dispersive westward propagation of the wave form indicated by region 3 in Fig. 19; (2) the southward current is subject to an increase in relative vorticity, the elevation of the free surface producing negligible divergence, and northward current suffer a decrease in relative vorticity, thus shifting the vorticity pattern toward the west as indicated by the region 4 in Fig. 19. The transition region in Fig. 19 combines both these effects.

Two free progressive Second Class waves of the same frequency can also be combined to permit reflection from a meridional barrier (ARONS and STOMMEL, 1956). Both the waves move in the same direction of course, but the group velocity is opposite for the incident and reflected wave, and there is no net energy flux. If the homogeneous water layer is bounded on both the east and west by meridional walls there are only certain discrete free frequencies allowed – an eigenvalue problem which gives the free Second Class periods of closed basins.

Free Second Class waves in a density-stratified system

The introduction of vertical density stratification in the model permits internal or baroclinic modes of both First and Second Class. The periods for a given wavelength are much longer than for the external or barotropic mode. Since the frequency

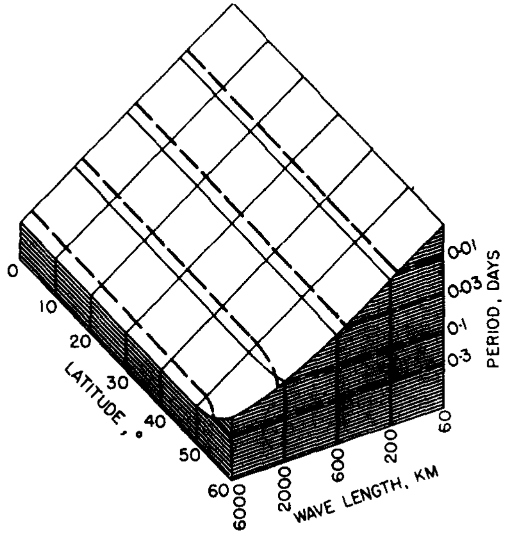


Fig. 20. Periods of First Class barotropic ocean waves.

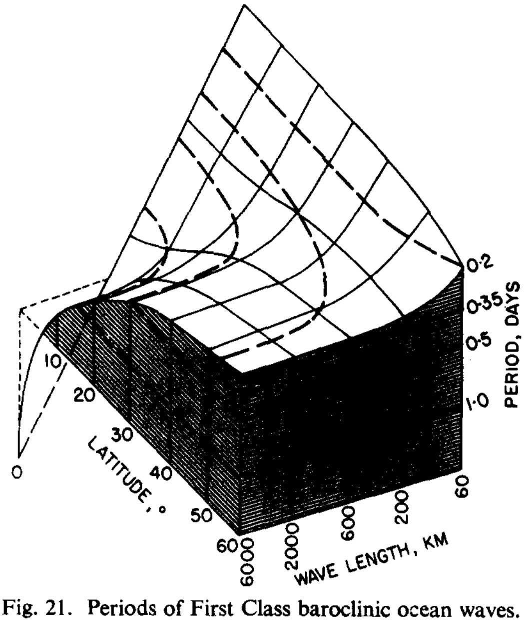


Fig. 21. Periods of First Class baroclinic ocean waves.

diagram of these different classes and modes is partly a function of latitude, and of density structure, a single curve will not be very helpful. Therefore I have prepared four block diagrams that give periods of free waves as a function of latitude and wave length for the cases : First Class barotropic, Fig. 20 ; First Class baroclinic, Fig. 21 ; Second Class barotropic, Fig. 22 ; and Second Class baroclinic, Fig. 23. The baroclinic diagrams are prepared by using a two density layer model (VERONIS and STOMMEL, 1956) with thickness of the layers and density difference chosen to

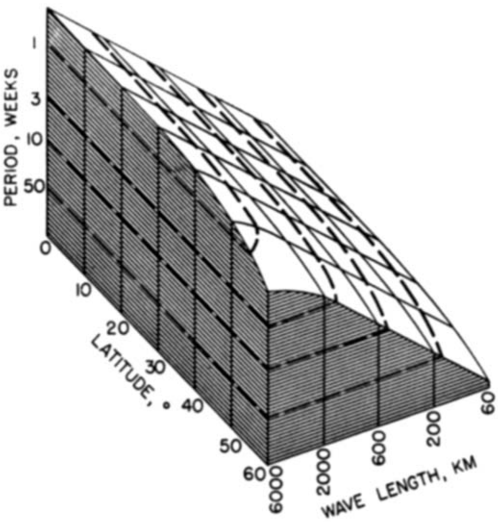


Fig. 22. Periods of Second Class barotropic ocean waves.

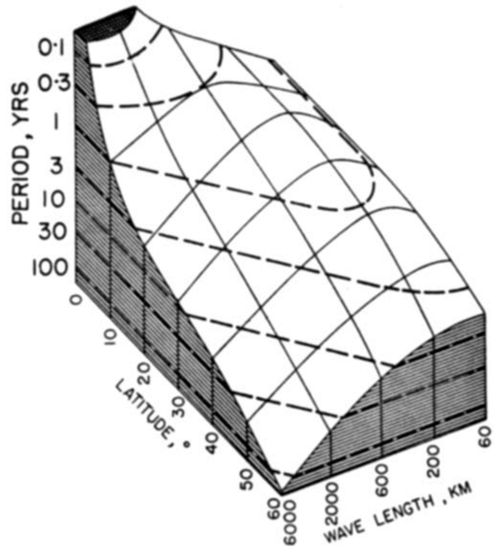


Fig. 23. Periods of Second Class baroclinic ocean waves.

match as nearly as possible that of the North Atlantic for each latitude. The barotropic diagrams are prepared using representative depths of the North Atlantic. Hence the diagrams are not quite as smooth looking as they would be if computed entirely from a single geometry and density difference.

The Second Class barotropic waves have periods, for wavelengths corresponding to oceanic dimensions, of about a week. Second Class baroclinic waves of oceanic dimension have very long periods ; only near the equator is there a period as short as a year, however it should be remembered that the approximations in the beta-plane are bad at the equator, so that the results there must be received with caution.

Transient response of the ocean to a variable driving force

The properties of free waves in a two-layer stratified fluid on the beta-plane are helpful in understanding physically the response of a stratified system to changes in the driving system. Quantitative theoretical studies of the response to changing forces are rather involved, and it seems fruitless to try to explain them verbally. A set of references to work in this field is given in another place (VERONIS and STOMMEL, 1956). In this brief review article we must be satisfied by a brief statement of the main results of the theories.

First of all, very short period or abrupt changes in the driving forces such as earthquakes, or sudden storms over shallow seas excite mainly the First Class barotropic mode. VERONIS (1956) has shown that when the interval during which an applied wind stress is changed from one value to another exceeds a half pendulum day most of the work done by the wind stress upon the sea goes into Second Class waves, that is, into currents. If this interval during which the wind changes is smaller, energy goes into First Class wave motions. Within the Second Class motions themselves the distribution of energy between barotropic and baroclinic modes depends very much upon whether the period of the forcing function is closer to the barotropic free period corresponding to the wave length of the forcing function. Thus in midlatitudes a large-scale storm of dimensions 600-2000 km and duration of three to five days will excite the barotropic mode of the Second Class. The Second Class baroclinic mode is also excited, but its amplitude is much smaller than would be expected were the storm to become stationary and to blow constantly for a long time. This can be seen by comparing Figs. 22 and 23. The currents produced will be largely independent of depth (beneath the surface friction layer of course) and the thermal structure of the ocean will not be much influenced. On the other hand in low latitudes a large scale fluctuation of the winds with a period of a year (for example, the Indian Ocean Monsoon) will nearly be in resonance with the Second Class baroclinic mode, as can be seen from Fig. 23. Hence in this case the currents responded chiefly in the upper layer, and the density structure changes markedly to accommodated them. In high latitudes seasonal changes in the winds are not so near to resonance with the Second Class baroclinic mode ; for example one might expect the Antarctic Circumpolar Current to show little seasonal change of deep thermal structure due to seasonal wind changes.

In a simple two-density layered ocean in a closed basin the steady state vertically integrated transport is given by the SVERDRUP solution. If there is no mass transport or mixing across the interface, all the steady motion is in the upper layer. If the wind-stress distribution is now changed, from one pattern to another, on an oceanic scale, the vertically integrated transport distribution adjusts itself to that corresponding to a new steady state distribution within a few weeks ; but the vertical distribution of velocities is very different from that of the final steady state. The simple two-layer theory indicates that several years or decades (in midlatitudes) are necessary before the vertical distribution of velocities approaches that of the steady state solution with all the velocity in the upper layer. Thus we see that the total vertically integrated transport responds rapidly to changes in applied wind stress, but the internal density field responds quite slowly. The response of the internal density field is not governed by the period of inertial-gravitational (First Class) waves, as is sometimes thought by hydrographers, but by the much longer period Second Class internal waves.

As another illustration, this time a hypothetical one, imagine a two-layer ocean completely at rest. The surface and the interface are horizontal. Now suppose that a constant convergent EKMAN wind-driven layer with maximum convergence at the midlatitude of the diagram (Fig. 24 a, b, c) is superposed at the surface in such a manner that the response of the ocean is mostly in the Second Class baroclinic mode – in particular, the large wavelength non-dispersive portion of the frequency diagram.

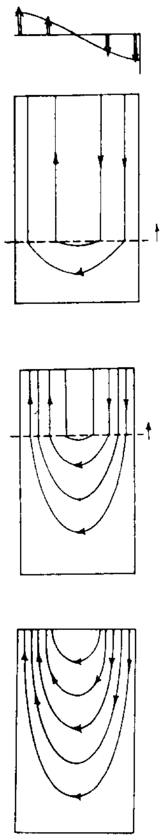


Fig. 24. Three successive stages of the slow development of the steady deformation of the thermocline from rest by a wind stress distribution as shown at the left. The dashed line is a " front " moving slowly (a mile a day) from the eastern coast to the western coast. On the left of the front the thermocline steadily deepends; on the right it is stationary in the SVERDRUP manner (Fig. 12).

Under these circumstances the first response of the interior of the ocean is a simple deepening of the interface at a constant rate equal to the supply of water at the surface from the EKMAN layer. Near the coast there is a sharp transition from this simple response of the interior to a configuration of the interface characteristic of the steady state solution which would exist had the wind blown forever (Fig. 12). The region of transition from one type of solution to the other slowly moves westward until the entire ocean is covered by the steady state solution. Two successive stages of this development and motion of the discontinuity between types of solution are drawn in Fig. 24 (a, b, c). One assumes that the whole process is so slow that at every stage of development there is a western current equilibrium at the western boundary that is able to cope with the need for a western boundary condition.

The stability of the solutions and the dilemma of the Antarctic circumpolar current

One of the most unsatisfactory aspects of the theoretical solutions so far developed in oceanography is the fact that they have not been exmined for stability. Dynamical meteorologists are convinced that a steady state model does not apply to the atmosphere except possibly in the Trade Wind regions. Elsewhere large quasigeostrophic wave disturbances appear and disappear spontaneously in the zonal wind systems and play a very important dynamical role in the general circulation of the atmosphere. Some investigators even believe that disturbances maintain the zonal flows. Should this be the case, it is obvious that steady state models incorporating large arbitrary lateral eddy coefficients would scarcely be very enlightening. By analogy with developments in the study of meteorology, oceanographers are warned to employ steady-state theories of the ocean with caution – especially models which employ large-scale eddy coefficients. In the tropics and subtropics the SVERDRUP theory seems to apply quantitatively to the ocean circulation, although there is not sufficient observational material for a completely convincing demonstration one way or the other. This suggests that the SVERDRUP model is essentially stable in the tropics, but we are far from a mathematical demonstration of its stability. In general it seems to me that we might expect theoretical models of oceanic circulation to be much more stable than those of the atmosphere on account of the presence of the meridional barriers because the high velocity zonal currents characteristic of instability in the atmosphere are suppressed by the coasts. This is, of course, sheer speculation.

A numbers of authors (e.g. MUNK and PALMEN, 1951) have pointed to the Antarctic Ocean as an example of an ocean without meridional barriers, and as such one in which a SVERDRUP type solution cannot be constructed. In an ocean without meridional barriers there can be no net meridional component of geostrophic flow, and we are forced back to circulations like the HOUGH regimes. A very crude model of the Antarctic Circumpolar Current can be constructed by employing lateral viscosity. Several Japanese authors (HIDAKA and TSUCHIYA, 1953, and others) have constructed models of the Antarctic Circumpolar Current as essentially a laterally viscous zonal stream driven by the southern hemisphere westerlies. In order to limit the total transport of the Circumpolar Current to about 100×10^6 m^3/sec a lateral eddy viscosity of more than 10^{10} cm^3/sec is required – several orders of magnitude greater than that generally envisaged in other oceanic currents. If vertical viscosity is employed very large coefficients are also required. On the other hand, the fact that the deep warm water in the Circumpolar Current preserves its Atlantic characteristics even into the Pacific Ocean suggests that the lateral diffusivity can hardly exceed 10^7 cm^2/sec, nor the vertical diffusivity be much greater than 10 cm^2/sec. Thus the implications of water-mass analysis are not compatible with the large turbulent coefficients required in the purely zonal and frictional theory.

There have been several other mechanisms suggested for the Antarctic Circumpolar Current (MUNK and PALMEN, 1951; FOFONOFF, 1955) all of which regard it as a purely zonal phenomenon. The cross section of the channel between the Cape of Good Hope and Antarctica is quite broad and deep (Fig. 25a), and a section straight from Cape Horn and Graham Land is also quite adequate (Fig. 25b) to permit the

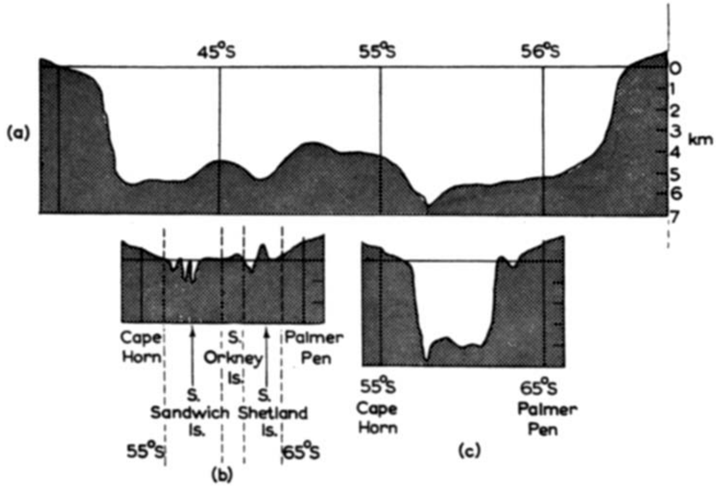


Fig. 25. Depth profiles across the Southern Ocean : (a) From Cape of Good Hope to Antarctica ; (b) From Cape Horn to Palmer Peninsula ; (c) Minimum depth of each latitude circle.

Circumpolar Current to pass through. However, if one plots the minimum depth for each complete latitude circle, it is found that the latitude circles that pass through Drake Passage are blocked by the island arc somewhat to the east. The minimum depths are plotted in Fig. 25 (c). It is seen that nowhere in the Antarctic Water-Ring is there a latitude with a deeper threshold than 1000 m. The Antarctic Circumpolar Current therefore cannot be purely zonal. In fact (DEACON, 1937) there seems to be a general southward component of the current over the entire southern ocean except just after passing through Drake Passage, where it is deflected sharply north by about 10° of latitude in a very limited narrow region, and possibly also to some extent just after passing New Zealand. Now if there is anything to this picture it suggests that the Antarctic Circumpolar Current is essentially amenable to treatment by the SVERDRUP theory; that the current is essentially frictionless except in a narrow region just after it passes through Drake Passage. In this one limited region we anticipate that all the dissipation of energy, any instability, and higher order processes occur. The remarks which follow cannot be considered to be a theory; they are merely suggestive of one, and they do illustrate some of the physical ideas described in this survey.

Before attempting to interpret the internal modes of circulation, we first consider only the vertically integrated flow – it is only necessary to consider a simple homogeneous ocean of uniform depth. In order to proceed simply we first consider that Drake Passage is closed (Fig. 26a). This barrier is indicated by the heavy radial line in the figures, joining the schematic Antarctic continent. The assumed zonal wind system is shown in Fig. 26 (a). Trades from the Equator to 30°S, westerlies from 30°

to a little over 60°S, and further to the south a small zone of Easterlies. The EKMAN drift is northward in the westerlies, southward in the easterlies. South of about 50°–55°S the EKMAN drift is divergent, in more northerly latitudes it is convergent. According to our simple physical notions we expect this divergence to be compensated by the planetary divergence of meridional components of geostrophic flow. There is no difficulty in obtaining such a flow in the ocean of Fig. 26 (a) because there is a

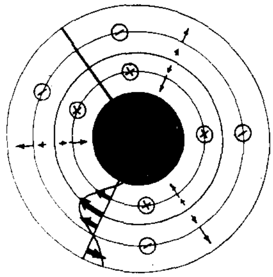


Fig. 26 (a). The schematic Southern Ocean. Antarctica is the solid black circle. The meridional barrier extending northward from Antarctica is represented by the solid heavy black vertical line. The schematic wind system (purely zonal) is depicted by the heavy arrows on the lower left. The concentric circles are latitude circles. Latitudes of EKMAN convergence and sinking at the surface are indicated by minus signs, latitudes of EKMAN divergence and upwelling are indicated by plus signs. The direction of the required meridional geostrophic flow is indicated by light radial arrows.

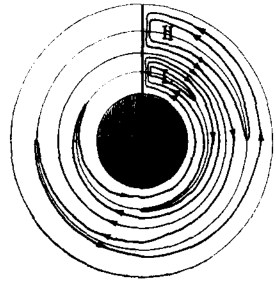


Fig. 26 (b). Transport lines of the solution for the model depicted in Fig. 26 (a). The western boundary currents are to be interpreted schematically.

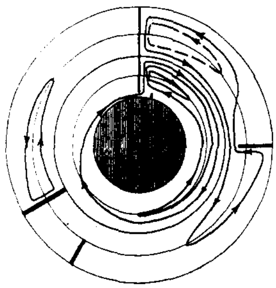


Fig. 26 (c). Modification of the transport field produced by introduction of other meridional barriers corresponding to Africa, Australia, and New Zealand, and by breaking the American-Antarctic barrier so as to admit a very constricted Davis Straits.

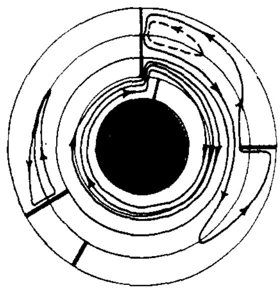


Fig. 26 (d). Hypothetical form of the solution that results from rupturing the American-Antarctic barrier in such a way as to permit water to flow through, but to obstruct all latitude circles.

complete meridional barrier. The necessary meridional components are shown by heavy arrows in Fig. 26 (a). The full transport field is developed by continuity from the eastern side of the ocean (western coast of South America) and fitted with an intense western boundary current at the western coast of the ocean. There are two immense gyres in Fig. 26 (b). wrapped around the world parallel to latitude circles. A simple calculation shows that the transport in the southern gyre amounts to somewhat more than 100×10^6 m^3/sec, the northern gyre somewhat less. Thus each line in Fig. 26 (b) is roughly 30 × 10^6 m^3/sec. Actually the northern gyre is broken up by the African and Australia-New Zealand continents so that it must be modified in some such way as indicated in Fig. 26 (c). In this figure a small breach has been made in the South America to Antarctica barrier, enough to permit one transport line to pass through, otherwise the general shape of the southern gyre is not much altered. If the barrier is broken in a manner indicated in Fig. 26 (d), so that there is no unbounded latitude circle, but there is a pass for water to flow through to the north, it seems to me that the pattern of flow must then look something like drawn. This picture is obtained by keeping all mid-ocean transport lines north of the latitude of the passage just the same as in Fig. 26 (b), but south of this latitude by developing the transport function from the western coast of the ocean (the east coast of the Palmer Peninsula) toward the east. The flow through the passage itself is something of a mystery, but doubtless models can be devised to describe it. Due to the extreme elongation of the flow pattern illustrated in Fig. 26 (d) the current looks like a purely zonal one at first glance. The over-all agreement of this picture with the lines of equal dynamic height in DEACON's study (1937) is fairly striking. Thus we see that there is some reason to suppose that the Antarctic Circumpolar Current is not such a dynamical misfit as originally it was thought to be. One need

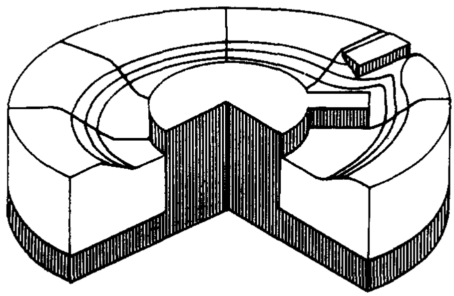


Fig. 27. Surface contours (or isobars) of the hypothetical Circumpolar Current shown in Fig. 26 (d).

merely recognize that it is not a zonal current after all. Fig. 27 shows the homogeneous model in perspective. The transport lines are parallel to the surface height contours everywhere except in the narrow passage where they cross isobars in a downhill direction. A water particle does not drop the full dynamic height difference across the Antarctic Circumpolar Current each time it makes a circuit around the world, but requires from three to five circuits on the average it drops the full dynamic height and rises to the surface, and is carried away to the north across the latitude of maximum westerlies in the surface EKMAN layer. The EKMAN northward transport is non-geostrophic, and distributed all around the southern ocean ; it is indicated by the radial arrows in Fig. 28. For entire 55°S latitude circle it must amount to at least 20 × 10^6 m^3/sec. This is the latitude of maximum westerlies, and there is no mid-ocean meridional flow where the EKMAN layer is non-divergent. There is a western current on the western coast of the ocean (eastern South America) setting southward, and just to the north of Drake Passage. Thus it is that the water which replenishes that lost to all the oceans north of 55°S by the EKMAN layer, is drawn mostly from the South Atlantic western trough. There is reason to suppose that this supply is largely Atlantic Deep Water.

The actual Antarctic circulation is partly baroclinic. We now proceed to make a crude interpretation of the internal structure of the flow. In Fig. 28a schematic diagram of the southern ocean is presented, showing the Antarctic continent as the central cylinder, and the American Barrier with Drake Passage indicated to the right. Two level surfaces intersect the body of the ocean: L_1 at a depth of approximately 600 and L_2 at a depth of about 3500 to 4000 meters. Let us now specify the vertical transfer across these levels. Across level L_1 there is a general upward flux of mass from the warm deep layer, apparently almost enough to compensate for the water lost by the upper layer to the divergent EKMAN layer on the very top. Across level L_2 there is very little vertical transfer, except near the Weddell Sea where some Antarctic Bottom Water sometimes sinks across it. The Bottom Water cannot flow anywhere to speak of except up the western coast of the ocean in a narrow western boundary current beneath L_2, because the rest of the layer is non-divergent. The whole of the large body of warm saline water between level L_2 and L_1 is apparently convergent on account of the flux upwards across L_1. As we have seen, a system of this sort can develop a strong geostrophic component of flow everywhere in mid-ocean by spiralling in towards the Antarctic Continent except for the abrupt northward displacement that occurs in Drake Passage. The supply of water for this main body of the Circum-

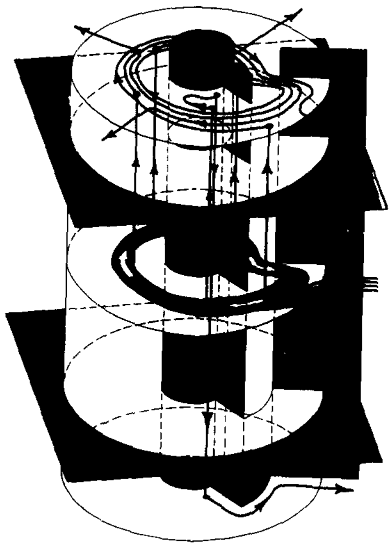


Fig. 28. Schematic three-dimensional diagram illustrating the hypothetical schematic internal structure of the Circumpolar Current (see text for detailed explanation). Two reference levels, L_1 and L_2, intersect the body of the ocean. The central shaded cylinder is the Antarctic Continent. The vertical plane attached to it at the forward right is Palmer Pensula, and the detached vertical plane at the extreme right is South America. The curved lines with arrow heads are transport lines. In order to avoid complicating the diagram, the more realistic spiralling motion is not shown.

polar Current comes from a narrow stream of Atlantic Deep Water flowing, as western boundary current, down the west side of the South Atlantic Ocean. In the layer between 600 m and the bottom of the EKMAN layer it is not clear just how to interpret the state of affairs. If this layer is slightly convergent it should move more or less along with the deep water ; if it is non-divergent – that is, loses as much to the EKMAN layer as it gains from the warm deep layer, it may move more nearly zonally than the deep water, because it is not so completely blocked by the island arc from attaining zonal flow, or if it is divergent it may drift northward everywhere with the surface EKMAN drift. The simple picture shown in Fig. 28 does not extend north of the Antarctic Convergence, so no attempt is made to interpret the formation of Antarctic Intermediate water, etc.

Finally I should like to make an apology for ending this survey article with such a sketchy and incomplete interpretation of the Antarctic Circumpolar Current as I have just given above. I should have hesitated to do so were it not for the fact that it so clearly illustrates both the usefulness and weakness of the physical ideas reviewed above.

Acknowledgements—I am particularly indebted to Mr. GALE PASLEY for drafting some of the intricate diagrams in this article, to Dr. G. VERONIS and Dr. G. MORGAN for many discussions during the past year, and to the Office of Naval Research for underwriting my studies.

Woods Hole Oceanographic Institution. *Contribution No.* 917 *from the Woods Hole Oceanographic Institution*

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