Features of the Large-scale Atmospheric Circulation

From Figures 11.1 through 11.3 we see or infer:

- ⋆ A pole-equator temperature gradient that is much smaller than the radiative equilibrium gradient.
- ⋆ A troposphere, in which temperature generally falls with height, above which lies the stratosphere, in which temperature increases with height. The two regions are separated by a tropopause, which varies in height from about 16 km at the equator to about 6 km at the pole.
- ⋆ A monotonically decreasing temperature from equator to pole in the troposphere, but a weakening and sometimes reversal of this above the tropopause.
- ⋆ A westerly (i.e., eastward) tropospheric jet. The time- and zonally-averaged jet is a maximum at the edge or just polewards of the subtropics, where it is associated with a strong meridional temperature gradient. In mid-latitudes the jet has a stronger barotropic component.
- ⋆ An E-W-E (easterlies-westerlies-easterlies) surface wind distribution. The latitude of the maximum in the surface westerlies is in mid-latitudes, where the zonally-averaged flow is more barotropic.

11.1.4 Summary

Some of the main features of the zonally averaged circulation are summarized in the shaded box above. We emphasize that the zonally-averaged circulation is not synonymous with a zonally symmetric circulation, and the midlatitude circulation is highly asymmetric. On the other hand, the large-scale tropical circulation of the atmosphere is to a large degree zonally symmetric or nearly so, and although monsoonal circulations and the Walker circulation are zonally asymmetric these are relatively weaker than midlaitude weather systems. Indeed the boundary between the tropics and midlatitude may be usefully defined by the latitude at which such zonal asymmetries become dynamically important on the large scale and this boundary, at about 30° on average, is quite sharp. We thus begin our dynamical description with a study of the low latitude zonally symmetric atmospheric circulation.

11.2 A STEADY MODEL OF THE HADLEY CELL

11.2.1 Assumptions

Let us try to construct a zonally symmetric model of the Hadley Cell.[3] Such a model is likely applicable mainly to the tropical atmosphere, this being observed to be more zonally symmetric than the midlatitudes. We will suppose that heating is maximum at the equator, and our intuitive picture, drawing on the observed flow of Fig. 11.3, is of air rising at the equator and moving poleward at some height H, descending at some latitude ϑ_H, and returning equatorwards near the surface. We make three major assumptions about this circulation:

(i) That it is steady.

(ii) That the polewards moving air conserves its axial angular momentum, whereas the zonal flow associated with the near-surface, equatorwards moving flow is frictionally retarded and is weak.

(iii) That it is in thermal wind balance.

We also assume the model is symmetric about the equator (an assumption we relax in section 11.4). These are all reasonable assumptions, but they cannot be rigorously justified — in other words, we are constructing a *model* of the Hadley Cell, schematically illustrated in Fig. 11.4. The model defines a limiting case — steady, invisicid flow — that cannot be expected to quantitatively describe the atmosphere, but that can be analysed fairly completely. Another limiting case is described in section 11.5. The real atmosphere may defy such simple characterizations, but they provide in invaluable benchmark of understanding.

11.2.2 Dynamics

We seek, then, to determine the strength and poleward extent of the Hadley circulation in the steady model. For simplicity we will work with a Boussinesq atmosphere, but this is not an essential aspect. Neglecting friction, the zonally-averaged zonal momentum equation is

$$\frac{\partial \overline{u}}{\partial t} - (f + \overline{\zeta})\overline{v} + \overline{w}\frac{\partial \overline{u}}{\partial z} = -\frac{1}{\cos^2\vartheta}\frac{\partial}{\partial \vartheta}(\cos^2\vartheta\,\overline{u'v'}) - \frac{\partial \overline{u'w'}}{\partial z}. \tag{11.4}$$

where $\overline{\zeta} = -(a\cos\vartheta)^{-1}\partial_y(\overline{u}\cos\vartheta)$. If we neglect the vertical advection and the eddy terms on the right-hand-side, then a steady solution, if it exists, obeys

$$(f + \overline{\zeta})\overline{v} = 0. \tag{11.5}$$

Presuming that the meridional flow $\overline{v}$ is nonzero (an issue we address in section 11.2.6) then $f + \overline{\zeta} = 0$, or equivalently

$$2\Omega\sin\vartheta = \frac{1}{a}\frac{\partial \overline{u}}{\partial \vartheta} - \frac{\overline{u}\tan\vartheta}{a}. \tag{11.6}$$

At the equator we shall assume that $\overline{u} = 0$, because here parcels have risen from the surface where, by assumption, the flow is weak. Eq. (11.6) then has a solution of

(2)

$$\boxed{\overline{u} = \Omega a\frac{\sin^2\vartheta}{\cos\vartheta} \equiv U_M}. \tag{11.7}$$

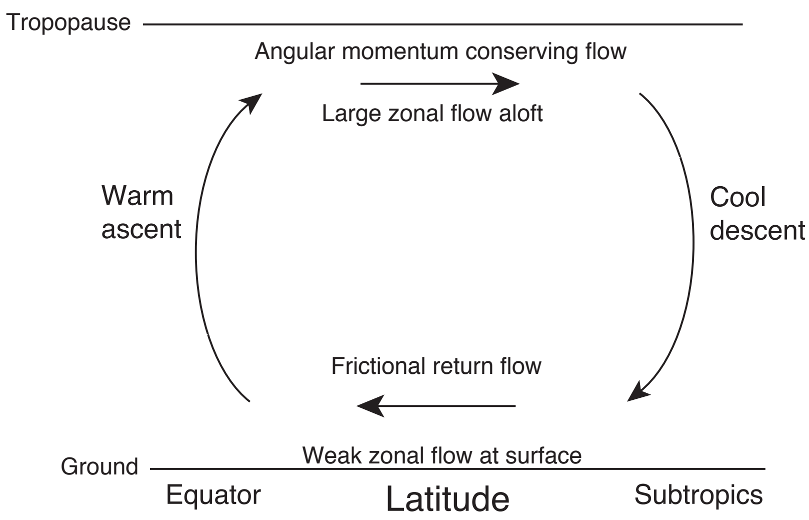

Fig. 11.4 A simple model of the Hadley Cell. Rising air near the equator moves polewards near the tropopause, descending in the subtropics and returning near the surface. The polewards moving air conserves its axial angular momentum, leading to a zonal flow that increases away from the equator. By the thermal wind relation the temperature of the air falls, slowly, as it moves poleward, and to satisfy the thermodynamic budget it sinks in the subtropics, so defining the polewards edge of the cell. The return flow at the surface is frictionally retarded and small.

This gives the zonal velocity of the polewards moving air in the upper branch of the (model) Hadley Cell, above the frictional boundary layer.

We can derive this result directly from the conservation of axial angular mometum, m, of a parcel of air at a latitude ϑ. In the shallow atmosphere approximation we have

(1) [c.f., (2.65) and equations following] The orbital angular momentum of a point mass = cross product of its position vector (from the chosen origin) and its linear momentum: L=r×p=L=m v r_⊥

$$\overline{m} = (\overline{u} + \Omega a \cos\vartheta) a \cos\vartheta, \tag{11.8}$$

and if $\overline{u} = 0$ at $\vartheta = 0$ and if $\overline{m}$ is conserved on a polewards moving parcel, then (11.8) leads to (11.7). It also may be directly checked that

$$(f + \overline{\zeta}) = -\frac{1}{a^2 \cos\vartheta} \frac{\partial \overline{m}}{\partial \vartheta}. \tag{11.9}$$

Thus, if eddy fluxes and frictional effects are negligible, the polewards flow will conserve its angular momentum, and the zonal flow in the earth's rotating frame increases with latitude (see Fig. 11.5). If we do assume this, our model is zonally symmetric and we drop the overbars over the variables.

If (11.7) gives the zonal velocity in the upper branch of the Hadley Cell, and that in the lower branch is close to zero, then the thermal wind equation can be used to infer the vertically averaged temperature. Although the geostrophic wind relation is

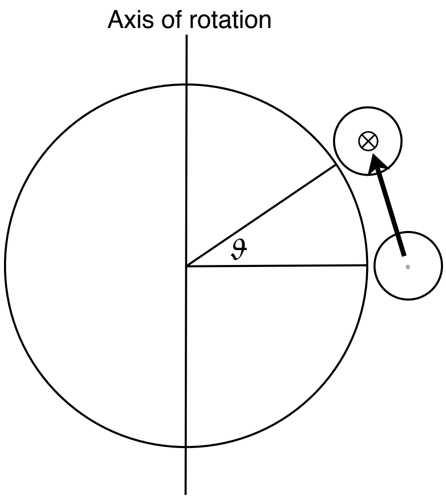

Figure 11.5 If a ring of air at the equator moves polewards it moves closer to the axis of rotation. If the parcels in the ring conserve their angular momentum their zonal velocity must increase; thus, if $m = (\overline{u} + \Omega a \cos\vartheta) a \cos\vartheta$ is preserved and $\overline{u} = 0$ at $\vartheta = 0$ we recover (11.7).

not valid at the equator (a more accurate balance is the so-called cyclostrophic balance, $fu + u^2 \tan\vartheta / a = -a^{-1}\partial\phi/\partial\vartheta$) the zonal wind is in fact geostrophically balanced until very close to the equator, and at the equator itself the horizontal temperature gradient in our model vanishes, because of the assumed interhemispheric symmetry. Thus, conventional thermal wind balance suffices for our purposes, and this is

$$2\Omega \sin\vartheta \frac{\partial u}{\partial z} = -\frac{1}{a}\frac{\partial b}{\partial \vartheta} \tag{11.10}$$

where $b = g\delta\theta/\theta_0$ is the buoyancy and $\delta\theta$ is the deviation of potential temperature from a constant reference value θ_0. Vertically integrating from the ground to the height H where the outflow occurs, and substituting (11.7) for u yields

$$\frac{1}{a\theta_0}\frac{\partial \theta}{\partial \vartheta} = \frac{2\Omega^2 a}{H}\frac{\sin^3\vartheta}{\cos\vartheta}, \tag{11.11}$$

where $\theta = H^{-1}\int_0^H \delta\theta \,\mathrm{d}z$ is the vertically averaged potential temperature. If the latitudinal extent of the Hadley Cell is not too great we can make the small-angle approximation, and replace $\sin\vartheta$ by ϑ and $\cos\vartheta$ by one, then integrating (11.11) gives

$$\boxed{\theta = \theta(0) - \frac{\theta_0 \Omega^2 y^4}{2gHa^2}}. \tag{11.12}$$

where $y = a\vartheta$ and $\theta(0)$ is the potential temperature at the equator, as yet unknown. Away from the equator, the zonal velocity given by (11.7) increases rapidly polewards and the temperature correspondingly drops. How far poleward is this solution valid? And what determines the value of the integration constant $\theta(0)$? To answer these questions we turn to thermodynamics.

Thermodynamics

In the above discussion, the temperature field is slaved to the momentum field in that it seems to follow passively from the dynamics of the momentum equation. Nevertheless, the thermodynamic equation must still be satisfied. Let us assume that the thermodynamic forcing can be represented by a Newtonian cooling to some specified radiative equilibrium temperature, θ_E; this is a severe simplification, especially in equatorial regions where the release of heat by condensation is important. The thermodynamic equation is then

LHS includes advection, not zero in a steady state

$$\frac{D\theta}{Dt} = \frac{\theta_E - \theta}{\tau}, \tag{11.13}$$

where τ is a relaxation timescale, perhaps a few weeks. Let us suppose that θ_E falls monotonically from the equator to the pole, and that it increases linearly with height, and a simple representation of this is

$$\frac{\theta_E(\vartheta, z)}{\theta_0} = 1 - \frac{2}{3}\Delta_H P_2(\sin\vartheta) + \Delta_V\left(\frac{z}{H} - \frac{1}{2}\right), \tag{11.14}$$

where Δ_H and Δ_V are nondimensional constants that determine the fractional temperature difference between equator and pole, and the ground and the top of the fluid, respectively. P_2 is the second Legendre polynomial, and it is usually the leading term in the Taylor expansion of symmmetric functions (symmetric around the equator) that decrease from pole to equator; it also integrates to zero over the sphere. $P_2(y) = (3y^2 - 1)/2$, so that in the small angle approximation and at $z = H/2$, or for the vertically averaged field, we have

$$\frac{\theta_E}{\theta_0} = 1 + \frac{1}{3}\Delta_H - \Delta_H\left(\frac{y}{a}\right)^2 \tag{11.15}$$

or

$$\theta_E = \theta_{E0} - \Delta\theta\left(\frac{y}{a}\right)^2, \tag{11.16}$$

where θ_{E0} is the equilibrium temperature at the equator, $\Delta\theta$ determines the equator-pole radiative-equilibrium temperature difference, and

$$\theta_{E0} = \theta_0(1 + \Delta_H/3), \qquad \Delta\theta = \theta_0\Delta_H. \tag{11.17}$$

Now, let us suppose that the solution (11.12) is valid between the equator and a latitude ϑ_H where $v = 0$, so that within this region the system is essentially closed. Conservation of potential temperature then requires that the solution (11.12) must satisfy

$$\int_0^{Y_H} \theta\, dy = \int_0^{Y_H} \theta_E\, dy, \tag{11.18}$$

where $Y_H = a\vartheta_H$ is as yet undetermined. Poleward of this, the solution is just $\theta = \theta_E$, and

Furthermore, we may demand that the solution be continuous at $y = Y_H$ — without temperature continuity the thermal wind would be infinite — and so

$$\theta(Y_H) = \theta_E(Y_H). \tag{11.19}$$

The constraints (11.18) and (11.19) determine the values of the unknowns $\theta(0)$ and Y_H. A little algebra (excercise 11.1) gives

$$Y_H = \left(\frac{5\Delta\theta g H}{3\Omega^2\theta_0}\right)^{1/2}, \tag{11.20}$$

and

$$\theta(0) = \theta_{E0} - \left(\frac{5\Delta\theta^2 g H}{18a^2\Omega^2\theta_0}\right). \tag{11.21}$$

A useful nondimensional number that parameterizes these solutions is

$$R \equiv \frac{gH\Delta\theta}{\theta_0\Omega^2 a^2} = \frac{gH\Delta_H}{\Omega^2 a^2}, \tag{11.22}$$

which is the the square of the ratio of of the speed of shallow water waves to the rotational velocity of the earth, multiplied by the fractional temperature difference from equator to pole. In terms of this we have

$$\boxed{Y_H = a\left(\frac{5}{3}R\right)^{1/2}}, \tag{11.23}$$

and

$$\boxed{\theta(0) = \theta_{E0} - \left(\frac{5}{18}R\right)\Delta\theta}. \tag{11.24}$$

The solutions are plotted in Fig. 11.6 and Fig. 11.7. Perhaps the single most important aspect of the model is that it predicts that the Hadley Cell has a *finite* meridional extent, *even for an atmosphere that is completely zonally symmetric.* The baroclinic instability that does occur in midlatitudes is not necessary for the Hadley Cell to terminate in the subtropics, although it may be an important factor, or even the determining factor, in the real world. More specifically, the model predicts the the latitudinal extent of the Hadley cell is: (i) proportional to the square root of the meridional temperature gradient; (ii) proportional to the square root of the height of the outward flowing branch; (iii) inversely proportional to the rotation rate Ω.

Zonal wind

The angular momentum conserving zonal wind is given by (11.7), which in the small angle approximation becomes

$$U_M = \Omega\frac{y^2}{a}. \tag{11.25}$$

This holds for $y < Y_H$. The zonal wind corresponding to the radiative-equilibrium solution is given using thermal wind balance and (11.16), which leads to

$$u_E = \Omega a R. \tag{11.26}$$

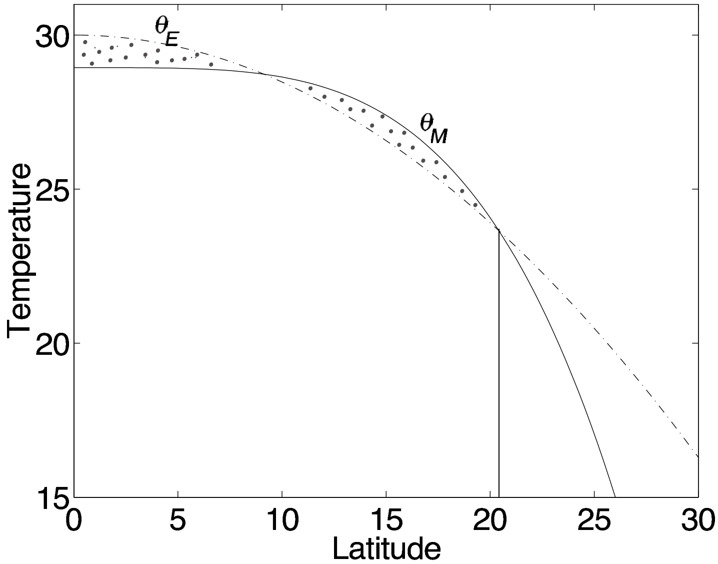

Fig. 11.6 The radiative equilibrium temperature (θ_E, dashed line) and the angular-momentum-conserving solution (θ_M, solid line) as a function of latitude. The two dotted regions have equal areas. The parameters chosen which give the solution here are: $\theta_{EO} = 303\,\mathrm{K}$, $\Delta\theta = 50\,\mathrm{K}$, $\theta_0 = 300\,\mathrm{K}$, $\Omega = 7.272 \times 10^{-5}\,\mathrm{s}^{-1}$, $g = 9.81\,\mathrm{m\,s}^{-2}$, $H = 10\,\mathrm{km}$. These give values of $R = 0.076$ and $Y_H/a = 0.356$, with a corresponding latitude for the edge of the Hadley Cell of 20.4°.

That the radiative equilibrium zonal wind is a constant follows from our choice of the second Legendre function for the radiative equilibrium temperature and is not a fundamental result; nonetheless, for most reasonable choices of θ_E the corresponding zonal wind will vary much less than the angular momentum conserving wind (11.25). The winds are illustrated in Fig. 11.7. There is a discontinuity in the zonal wind at the edge of the Hadley Cell, and of the meridional temperature gradient, but not of the temperature itself.

11.2.3 Properties of solution

We can see that the model predicts that the latitudinal extent of the Hadley Cell is:

⋆ Proportional to the square root of the meridional radiative equilibrium temperature gradient — the stronger the gradient, the farther the circulation must extend to achieve thermodynamic balance via the equal area construction in Fig. 11.6;

⋆ Proportional to the square root of the height of the outward flowing branch — the higher the outward flowing branch, the weaker the ensuing temperature gradient of the solution (via thermal wind balance), and so the further polewards the circulation must go;