Axisymmetric Circulations Forced by Heat and Momentum Sources: A Simple Model Applicable to the Venus Atmosphere

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(Manuscript received 27 January 1984, in final form 13 July 1984)

ABSTRACT

The parametric behavior of an axially symmetric circulation induced by heat and momentum sources is analyzed in the context of a simple Boussinesq model. Implications for the Venus atmosphere are examined in the light of recent data.

For nearly inviscid flows in a stably stratified atmosphere, this work extends the analysis of Held and Hou to large thermal Rossby numbers (slowly rotating atmospheres). For parametric values appropriate to the Venus atmosphere, heat flux by the Hadley circulation leads to a temperature distribution nearly uniform with latitude, close to the asymptotic limit for a nonrotating atmosphere, while the zonal wind shows a strong polar jet but does not superrotate at the equator, contrary to what is observed on Venus.

For the Venus atmosphere to superrotate above the cloud base, where most of the solar radiation is deposited, momentum sources and sinks must be provided by asymmetric motions to offset transports by the mean meridional circulation. Diagnostically, the eddy momentum source/sink pattern required by a given meridional cell depends upon both the sign of the vertical zonal wind shear and the direction of the meridional flow. For a realistic vertical zonal wind profile, the implication is that the observed superrotation on Venus can be supported by alternating layers of sources and sinks.

1. Introduction

The purpose of this work is to develop a simple mechanistic model of a zonally averaged circulation forced by heat and momentum sources, and to apply the model to the Venus atmosphere in the light of recent data.

We shall focus on two important dynamic issues concerning the Venus troposphere (below 75 km): the mean meridional transport, to which the observed small pole-to-equator thermal contrast is often attributed, and the maintenance of the planet-wide zonal superrotation. Data relevant to the Venus circulation can be found in a recent review by Schubert (1983). To summarize, at the equator the zonal wind increases from less than 1 m s^{-1} at the surface up to 120 m s^{-1} at the cloud tops (all measured velocities are retrograde, in the same direction as the planetary rotation, which we take to be positive). At times the cloud tops ($\sim$65 km) appear close to a solid body rotation; at others a jet structure has been observed at midlatitudes. Thus, the bulk of the atmosphere superrotates relative to the surface, which has a maximum rotation speed of roughly 2 m s^{-1}. The atmospheric angular momentum per unit mass increases by a factor of 60 from the surface to the cloud tops, and its density-weighted profile has a local maximum at 20 km above the surface. The solar heating is deposited principally between 45 and 75 km (Tomasko, 1983), a region to which we shall pay special attention. Velocity measurements show that meridional motions at the cloud tops are poleward in both hemispheres, suggesting the presence of Hadley circulations in this directly heated region. The pole-to-equator temperature difference at the cloud levels is roughly 10–20 K (Seiff, 1983). The relevant numerical data together with brief comments upon their origins are given in the Appendix.

To account for the weak thermal gradient at the Venus cloud tops first observed in telescopic images of the planet, by simple scaling arguments, Goody and Robinson (1966), Gierasch *et al.* (1970), and Stone (1974, 1975) have shown that on a nonrotating planet Hadley circulations forced by the differential solar heating could lead to small horizontal temperature gradients; and Stone (1968) estimated that the effect of planetary rotation is small on Venus. Kalnay (1973, 1975) also calculated axisymmetric flows using assumed solar heating profiles. The basic mechanics of such thermally-induced, nearly-inviscid symmetric circulations have been analyzed by Held and Hou (1980, hereafter HH) for a rapidly rotating Boussinesq atmosphere. In this work we will extend the HH model to a wide range of rotation rates and examine its properties for parameters appropriate for the Venus atmosphere.

According to Hide's theorem (Hide, 1969), an axisymmetric atmosphere cannot superrotate at the equator if small-scale mixing of angular momentum is down-gradient. The strong superrotation observed on Venus therefore implies up-gradient transport of angular momentum by asymmetric (eddy) motions, even though the actual physical mechanism is not yet known. Gierasch (1975) analyzed the global transport requirements for maintaining a stratified solid-body rotation in the presence of an axisymmetric Hadley cell. In this study, we will use the extended HH model to characterize the Eliassen-Palm flux divergence pattern for given zonal wind and solar heating distributions. A limitation of this simple model is that the height of the meridional circulation and the static stability must be treated as external parameters. In the case of Venus it is especially difficult to assign a depth to the meridional circulation because the height of the cloud tops (65 km), the depth of the region heated by the sun (30 km), and the scale height (4–16 km) differ so greatly; in addition, there have been suggestions that multiple direct and indirect cells may exist in the lower atmosphere (Schubert, 1983). The use of a finite-difference model can overcome these difficulties, but it lacks the flexibility and clarity of the simple model. We will, therefore, in this paper restrict consideration to the diagnostic maintenance requirement for an individual meridional cell of a given depth. The working assumption is that the depth of the meridional cell is determined locally by the vertical scale of the thermal or momentum forcing, whichever is dominant, as will be shown by finite-difference model calculations in a subsequent paper (Hou and Goody, 1985). However, the analysis of this paper provides the basis for understanding our numerical model results.

The structure of this paper is as follows. In Section 2 we discuss basic equations for a steady-state axisymmetric circulation. In Section 3, based on the work of HH we derive a general model for circulations forced by heat and momentum sources. In Section 4 we examine the parametric dependence of a nearly inviscid Hadley circulation in the absence of eddy forcing. Here we extend the analysis of HH to a wide range of thermal Rossby numbers. In section 5 we consider the effect of diffusion and find it to be small for the Venus cloud region. In Section 6 we determine the zonally averaged eddy sources and sinks required to support the zonal superrotation on Venus. In Section 7 we examine model approximations, and in Section 8 we summarize model results and discuss possible implications for the Venus atmosphere.

2. Basic equations

We follow HH except for the addition of a zonal momentum source term $G(y, z)$ and a heat source term $J(y, z)$, representing zonally averaged contributions by asymmetric (eddy) motions. We will, however, neglect eddy contributions in the meridional momentum equation as we will be concerned with flows that are in approximate gradient wind balance. The primitive equation system governing steady, nondivergent axisymmetric flows on a sphere consists of the zonal momentum equation:

$$\nabla\cdot(\mathbf{V}u) - 2\Omega yv - \frac{uv}{a}\frac{y}{(1-y^2)^{1/2}} = F(u) + G(y, z), \tag{1}$$

the meridional momentum equation:

$$\nabla\cdot(\mathbf{V}v) + 2\Omega yu + \frac{u^2}{a}\frac{y}{(1-y^2)^{1/2}} = -\frac{(1-y^2)^{1/2}}{a}\frac{\partial\phi}{\partial y} + F(v), \tag{2}$$

the hydrostatic relation:

$$\frac{\partial\phi}{\partial z} = L(\theta), \tag{3}$$

the thermodynamic equation:

$$\nabla\cdot(\mathbf{V}\theta) = \frac{\theta_E - \theta}{\tau_R} + F(\theta) + J(y, z), \tag{4}$$

and the continuity equation:

$$\nabla\cdot\mathbf{V} = 0, \tag{5}$$

where u is the zonal velocity, $\mathbf{V} = (v, w)$, v the meridional velocity, w the vertical velocity, y the sine of latitude, z the vertical coordinate, a the planetary radius, Ω the rotation rate, $\nabla(\ )$ the meridional gradient operator $\{a^{-1}\partial[(1-y^2)^{1/2}]/\partial y, \partial(\ )/\partial z\}$, θ the potential temperature, θ_E the radiative-equilibrium potential temperature, τ_R the radiative relaxation time, and F the small-scale vertical diffusion of heat or momentum. Equations (1)–(5) as written are appropriate to either a Boussinesq atmosphere in height coordinates or a perfect-gas atmosphere in pressure coordinates. In this work we restrict the analysis to a Boussinesq system in order to focus on the basic mechanics of symmetric flows. The generalization to the perfect-gas case will be given in Hou and Goody (1985).

In the Boussinesq case, z is physical height, ϕ is pressure, $L(\theta) = g\theta/\theta_0$, where g is the gravitational acceleration, and θ_0 the global mean potential temperature. For simplicity, we assume a constant τ_R, leaving the generalization to the perfect-gas case. Following HH we adopt an idealized θ_E that is symmetric with respect to the equator,

$$\frac{\theta_E}{\theta_0} = 1 - \frac{2}{3}\Delta_h P_2(y) + \Delta_v\left(\frac{z}{H} - \frac{1}{2}\right), \tag{6}$$

where H is the depth of the model atmosphere, P_2 the second Legendre polynomial $P_2(x) = (3x^2 - 1)/2$,

θ_0 the global mean, and Δ_h and Δ_v are, respectively, fractional horizontal and vertical differences of potential temperature; $\Delta_h > 0$ if the solar heating reaches a maximum in the tropics, and $\Delta_v > 0$ for a stable atmosphere.

Since the thermal forcing θ_E given by (6) is symmetric about the equator, we assume the same for G and J, so that the solution is symmetric in y. The model boundary conditions are: stress-free at the upper boundary, a linear drag at the lower surface and no heat or mass fluxes at the boundaries. For definiteness, we assume a unit Prandtl number and take $F(x) = \nu\partial^2 x/\partial z^2$, where ν is a turbulent mixing coefficient; thus,

$$\left.\begin{array}{lll} w = \dfrac{\partial \mathbf{V}_h}{\partial z} = \dfrac{\partial \theta}{\partial z} = 0 & \text{at} & z = H \\ w = \dfrac{\partial \theta}{\partial z} = 0, \quad \dfrac{\partial \mathbf{V}_h}{\partial z} = C\mathbf{V}_h & \text{at} & z = 0 \\ v = 0 & \text{at} & y = 0 \text{ and } 1 \end{array}\right\}, \tag{7}$$

where $\mathbf{V}_h = (u, v)$ and C is a linearized aerodynamic drag coefficient with dimensions of velocity.

As shown in HH, when $F = G = J = 0$, the solution to the exactly inviscid system (1) through (5) comprises $\theta = \theta_E$, $v = w = 0$, and u satisfies the thermal wind relation obtained from (2) and (3):

$$\frac{\partial}{\partial z}\left[2\Omega y u + \frac{u^2}{a}\frac{y}{(1-y^2)^{1/2}}\right] = -\frac{g(1-y^2)^{1/2}}{a}\frac{\partial}{\partial y}\left[\frac{\theta}{\theta_0}\right]. \tag{8}$$

Evaluating for radiative equilibrium conditions, HH give

$$\frac{u_E}{\Omega a} = \left[\left(1 + \frac{2Rz}{H}\right)^{1/2} - 1\right](1-y^2)^{1/2}, \tag{9}$$

where R is the external thermal Rossby number defined as

$$R = \frac{gH\Delta_h}{(\Omega a)^2}. \tag{10}$$

The point is that an exactly inviscid system does not support a meridional circulation and its solution is independent of both thermodynamic and angular momentum constraints, distinctly different from a model in which some dissipation, however small, is present. For $G = J = 0$, HH have shown that as $F \to 0$, the solution does not approach that for $F = 0$; i.e., viscosity acts as a singular perturbation on the inviscid system.

In the limit of $F \to 0$, the absolute angular momentum per unit mass

$$M = \Omega a^2(1-y^2) + ua(1-y^2)^{1/2} \tag{11}$$

is conserved along a streamline. A vortex ring rotating with an equatorial zonal velocity u_*, when displaced poleward, would then give rise to the momentum-conserving zonal wind profile

$$u_M = \frac{(u_* + \Omega a y^2)}{(1-y^2)^{1/2}}. \tag{12}$$

For a direct circulation forced by diabatic heating ($G = J = 0$, $u_* = 0$), HH have shown that u_M represents an upper bound on the zonal wind of the system, if F is down-gradient. Then, for $F \neq 0$, where u_E exceeds u_M a meridional circulation must exist in order to satisfy the angular momentum constraint. More generally, a necessary condition for the existence of meridional motions is, therefore, $(F + G) \neq 0$. Multiplying (1) by $a(1-y^2)^{1/2}$ we obtain an equation for the absolute angular momentum M,

$$\frac{1}{a}\frac{\partial}{\partial y}[vM(1-y^2)^{1/2}] + \frac{\partial}{\partial z}(wM) = (F(u) + G(y, z))a(1-y^2)^{1/2}. \tag{13}$$

Thus M is conserved in the limit $(F + G) \to 0$.

3. An idealized model of forced circulations

In this section we extend the HH model of nearly inviscid Hadley circulations on a rapidly rotating planet to include friction, arbitrary rotation rates, and eddy sources. Our treatment is qualitatively similar to that of Schneider (1983). Model results for different types of heat and momentum forcings will be presented in Sections 4–6, and the self-consistency of the various model assumptions will be discussed in Section 7.

We assume that in response to heat or momentum forcing there exists a steady meridional circulation of depth H corresponding to the scale of the forcing, and that the thermal wind relation (8) is valid. Averaging (8) vertically gives

$$\frac{2\Omega a y[u(y, H) - u(y, 0)]}{(1-y^2)^{1/2}} + \frac{y[u(y, H)^2 - u(y, 0)^2]}{1-y^2} = -gH\frac{d}{dy}\left[\frac{\bar{\theta}}{\theta_0}\right], \tag{14}$$

where

$$\overline{(\ )} = \frac{1}{H}\int_0^H (\ )dz. \tag{15}$$

Averaging (4) and (13) vertically and applying the boundary condition, (7) we obtain the averaged heat equation:

$$\frac{1}{a}\frac{d}{dy}[\overline{v\theta}(1-y^2)^{1/2}] = \frac{\bar{\theta}_E - \bar{\theta}}{\tau_R} + \bar{J}, \tag{16}$$

and the averaged angular momentum equation:

$$\frac{1}{a}\frac{d}{dy}[\overline{vM}(1-y^2)^{1/2}] = (\bar{F}+\bar{G})a(1-y^2)^{1/2}, \quad (17)$$

where $\bar{F} = -u(y, 0)/\tau_c$, and $\tau_c = H/C$ is the time scale associated with frictional drag at the lower boundary.

Following HH we assume an idealized circulation in which meridional flows are confined to two horizontal layers, each of thickness δ, at $z = H$ and $z = 0$, and define the following averages:

$$(\ )_u = \frac{1}{H}\int_{H-\delta}^{H}(\ )dz, \quad (\ )_l = \frac{1}{H}\int_0^{\delta}(\ )dz, \quad (18)$$

where subscripts u and l denote the upper and lower branches, respectively. Using (5) and (7) we obtain

$$\bar{v}(y) \approx v_u(y) + v_l(y) = 0. \quad (19)$$

It thus follows that horizontal fluxes of heat and momentum are restricted to the upper and lower layers and vertical fluxes to the interior.

Held and Hou also assume that $\delta/H \ll 1$; then according to (19) the averaged meridional flux of any quantity X that is not restricted to thin-layers may be approximated by

$$\begin{aligned}\overline{Xv}(1-y^2)^{1/2} &= [v_uX(y,H) + v_lX(y,0)](1-y^2)^{1/2}\\ &= v_u(1-y^2)^{1/2}[X(y,H) - X(y,0)]\\ &= V_b(y)[X(y,H) - X(y,0)], \quad (20)\end{aligned}$$

where $V_b \equiv v_u(1-y^2)^{1/2}$, proportional to the mass flux in the upper branch. For convenience, here we adopt this thin layer assumption, which provides a simple framework for discussing the circulation without loss of generality. By interpreting V_b as an effective meridional flux, one can obtain expressions similar to (20) for broad meridional flows by defining appropriate averaging functions. In Section 7 we will consider such an example and demonstrate that certain important model conclusions do not depend on the thin-layer assumption.

By (20) the averaged heat equation (16) may be written as

$$\frac{1}{a}\frac{d}{dy}(V_bS) = \frac{\bar{\theta}_E - \bar{\theta}}{\tau_R\theta_0} + \frac{\bar{J}}{\theta_0}, \quad (21)$$

where $S(y) = [\theta(y, H) - \theta(y, 0)]/\theta_0$, which is proportional to the static stability, S/H. In this analysis we will regard S as an external parameter, as determined from independent estimates or observations (see Section 7). Moreover, we assume S = constant > 0, since observations suggest that the static stability of the Venus troposphere does not vary greatly with latitude and Stone (1974) estimated that in the presence of a Hadley circulation driven by differential solar heating the static stability, however small, is always stable.

Note that $\bar{J}$ in (21) corresponds to the averaged horizontal convergence of eddy heat flux (see Holton, 1975); we may therefore write $\bar{J}/\theta_0 = -a^{-1}d\bar{h}/dy$, with $\bar{h}(y)$ = zonal average of $[\overline{v'\theta'}(1-y^2)^{1/2}]/\theta_0$ (where primes denote departures from the zonal mean). From (21) we then obtain an equation for a meridional circulation forced directly by the diabatic heating,

$$\frac{1}{a}\frac{dV_b^*(y)}{dy} = \frac{\bar{\theta}_E - \bar{\theta}}{S\tau_R\theta_0}, \quad (22a)$$

where,

$$V_b^*(y) = V_b(y) + \bar{h}(y)/S. \quad (22b)$$

This "transformed Eulerian-mean" velocity V_b^* (distinct from the Eulerian-mean motion V_b) defines a "residual" diabatic circulation similar to those of Boyd (1976), Andrews and McIntyre (1976, 1978) and Dunkerton (1978). The main difference is that here we transform the averaged meridional velocity rather than the velocity **V** but, according to (22a), the mass entrainment in the upper branch, $(H/a)dV_b^*/dy$, corresponds to the mean vertical residual velocity given by Dunkerton (1978).

From (17) and (20) we obtain the averaged momentum flux equation as

$$\frac{1}{a}\frac{d}{dy}[V_b(y)(u(y,H) - u(y,0))(1-y^2)^{1/2}] = (\bar{F}+\bar{G})(1-y^2)^{1/2}. \quad (23a)$$

By (22b) we write

$$\frac{1}{a}\frac{d}{dy}[V_b^*(y)(u(y,H) - u(y,0))(1-y^2)^{1/2}] = (\bar{F}+\bar{G}^*)(1-y^2)^{1/2}, \quad (23b)$$

where

$$\bar{G}^* = \bar{G} + \frac{1}{Sa}\frac{d}{dy}\{\bar{h}(y)[u(y,H) - u(y,0)](1-y^2)^{1/2}\}.$$

Equations (14), (22a), and (23b) contain five unknowns, namely, $\bar{\theta}(y)$, $u(y, H)$, $u(y, 0)$, $V_b^*(y)$, and $\bar{G}^*(y)$. In cases where zonal winds are known, this system can be solved as an inverse problem for the diabatic circulation and $\bar{G}^*$; but, the Eulerian-mean circulation and $\bar{G}$ cannot be uniquely determined unless the mean eddy heat flux is also known. Note also that even though (23b) states that a superrotating wind field can be supported by $\bar{h}$ with $\bar{G} = 0$, it does not violate Hide's theorem; it merely implies that locally $G \neq 0$ in the interior. We will return to this point in Section 6.

However, in cases where G and J vanish or are specified, the Eulerian-mean circulation itself can be obtained. With (14), (22), and (23a) for four unknowns: $\bar{\theta}$, $u(y, H)$, $u(y, 0)$, and V_b, one more equation is still required to close the system. This can be any one of three zonal momentum equations, of which two are associated with meridional flows in the upper and lower branches and one with vertical motions in the interior.

From (5) and (7) we see that $w = (H/a)dV_b/dy$ at $z = H - \delta$. It can then be shown from (13) that throughout the upper branch vertical advection of M is small compared with horizontal advection, so long as M varies smoothly and $\delta/H \ll 1$. Using (11), (18), and (13) we obtain an approximate angular momentum equation for the upper branch,

$$\frac{V_b}{a}\left\{\frac{d}{dy}[u(y, H)(1 - y^2)^{1/2}] - 2\Omega a y\right\} = (F_u(\nu, u) + G_u)(1 - y^2)^{1/2}. \quad (24)$$

Similarly, for the lower branch,

$$-\frac{V_b}{a}\left\{\frac{d}{dy}[u(y, 0)(1 - y^2)^{1/2}] - 2\Omega a y\right\} = (F_l(\nu, C, u) + G_l)(1 - y^2)^{1/2}. \quad (25)$$

Subtracting (24) and (25) from (23) gives

$$[u(y, H) - u(y, 0)](1 - y^2)^{1/2}\frac{1}{a}\frac{dV_b}{dy} = (F_i(\nu, C, u) + G_i)(1 - y^2)^{1/2}, \quad (26)$$

where $(\ )_i = \overline{(\ )} - (\ )_u - (\ )_l$. From (5), (7), and (20), it can be shown that

$$\overline{w\frac{\partial M}{\partial z}} = [u(y, H) - u(y, 0)](1 - y^2)^{1/2}\frac{dV_b(y)}{dy}, \quad (27)$$

so that $(F_i + G_i)$ in (26) are associated with the averaged vertical advection of M. In cases where G and J are not known, substituting (22b) into (24)–(26) yields the effective momentum sources associated with the residual circulation.

Once $\bar{\theta}$, $u(y, H)$, $u(y, 0)$, and V_b are known, additional properties of the Eulerian-mean circulation can be derived; these include the vertically averaged heat flux from (21), the averaged momentum flux from (23). Approximating $\partial M/\partial z$ by a constant zonal wind shear we obtain from (27) and (22) a rough estimate for the averaged vertical velocity,

$$\bar{w}(y) = \frac{H}{a}\frac{dV_b(y)}{dy} = \frac{H}{S\tau_R\theta_0}(\bar{\theta}_E - \bar{\theta}) + \frac{\bar{J}}{\theta_0}. \quad (28)$$

This model system contains four characteristic time constants, τ_R, $\tau_c = H/C$, $\tau_\nu = H^2/\nu$, and τ_D, the dynamic time scale associated with the Eulerian-mean circulation; of these, all except τ_D are external parameters. From (28) τ_D can be defined as,

$$\tau_D = \frac{H}{\bar{w}(0)} = \frac{S\tau_R\theta_0}{\bar{\theta}_E(0) - \bar{\theta}(0)}. \quad (29)$$

Of course, if V_b^* instead of V_b is known, we may obtain these diagnostic relations only for the residual circulation.

In the following sections we will use the model developed here to analyze a nearly inviscid thermal circulation (Section 4), the effect of vertical diffusion (Section 5), and the required eddy source pattern for maintaining the superrotation in the Venus cloud region (Section 6).

4. A nearly inviscid thermal circulation

When $G = J = 0$ and $F \rightarrow 0$, the conservation of angular momentum ensures that the flow forced by diabatic heating is nonlinear and nearly inviscid. Held and Hou analyzed this flow for rapidly rotating planets ($R \ll 1$); here we extend the analysis to a wide range of thermal Rossby numbers and present closed-form solutions to show the dependence of a nearly inviscid thermal circulation on the set of external parameters (Ω, a, g, H, θ_0, Δ_h, Δ_v, τ_R, and C).

Following HH, a nearly inviscid thermal circulation may be characterized as follows: (i) $\tau_D/\tau_\nu \ll 1$, which ensures that internal diffusion is weak so that the flow is essentially inviscid, and (ii) $\tau_c/\tau_D \ll 1$, which leads to the approximation $u(y, 0)/u(y, H) \ll 1$. Note that (i) and (ii) together ensure $\tau_c/\tau_\nu \ll 1$; i.e., frictional exchanges occur only at the lower boundary and the interior is effectively inviscid. The self-consistency of these requirements is discussed in Section 7.

For nearly inviscid Hadley circulations with rising motions at the equator, HH have shown that

$$u(y, H) \approx u_M = \frac{\Omega a y^2}{(1 - y^2)^{1/2}}, \quad (30)$$

where u_M is given by (12) with $u_* = 0$, which is consistent with the condition $\tau_c/\tau_D \ll 1$. Using (30) and the approximation $u(y, 0)/u(y, H) \ll 1$, we can integrate (14) to obtain

$$\bar{\theta}(y) = \bar{\theta}(0) - \frac{(\Omega a)^2}{gH}\frac{y^4}{1 - y^2}. \quad (31)$$

Assuming that the Hadley cell terminates before the pole at some latitude y_H and v vanishes at $y = 0$ and y_H, integrating (22) over the width of the circulation gives

$$\int_0^{y_H}(\bar{\theta}_E - \bar{\theta})dy = 0. \quad (32)$$

In the absence of internal diffusion and eddy motions, the atmosphere poleward of y_H may be assumed to be in radiative equilibrium, with u and θ given by (9) and (6), respectively. Since the continuity of temperature requires that $\bar{\theta}(y_H) = \bar{\theta}_E(y_H)$, (31) and (32) can be solved for y_H and $\bar{\theta}(0)$. This is equivalent to constructing a graphic solution by intersecting $\bar{\theta}$ with $\bar{\theta}_E$ in such a manner that the net heating (32) vanishes

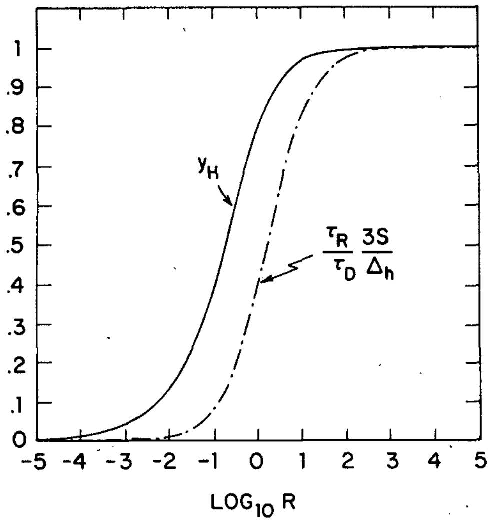

FIG. 1. The meridional width of the Hadley circulation y_H as a function of the external thermal Rossby number R (full line), obtained from (33). The broken line plots the ratio τ_R/τ_D given by (42), scaled by $\Delta_h/3S$.

(see HH, Fig. 1). Held and Hou give y_H as the solution to the equation

$$\frac{1}{3}(4R-1)y_H^3-\frac{y_H^5}{1-y_H^2}-y_H+\frac{1}{2}\ln\left[\frac{1+y_H}{1-y_H}\right]=0. \tag{33}$$

Hence, the width of the Hadley cell depends only on the external thermal Rossby number R. Figure 1 shows that as R increases (Ω decreases), the circulation expands poleward until it is confined by the geometric boundaries; an obvious effect of rotation is therefore the suppression of thermal convection. For limiting values of R we obtain from (33) the following asymptotic results for y_H:

$$y_H=\begin{cases}\left(\dfrac{5R}{3}\right)^{1/2}, & R\ll 1\\[2mm] 1-\dfrac{3}{8R}, & R\gg 1.\end{cases} \tag{34}$$

Thus, when $R \ll 1$, we recover the solution of HH.

Zonal winds at $z = H$ are shown in Fig. 2a for $R = 10^{-1}$, 1, 10, and 10^2. Over the width of the Hadley circulation $u(y, H)$ is given by (30), while poleward of y_H it is given by (9). The discontinuity at y_H manifests itself as a zonal jet which, through the thermal wind relation, implies a large horizontal thermal gradient near the poleward boundary of the Hadley cell.

For $0 \leqslant y \leqslant y_H$, the solution for $\bar\theta$ is

$$\frac{1}{\Delta_h}\left[\frac{\bar\theta(y)}{\theta_0}-1\right]=\frac{1}{3}-y_H^2+\frac{1}{2R}\left[\frac{y_H^4}{1-y_H^2}-\frac{y^4}{1-y^2}\right]. \tag{35}$$

For $y > y_H$, $\bar\theta$ is given by (6). Solutions for $\bar\theta$ are plotted in Fig. 2b for $R = 10^{-1}$, 1, 10, 10^2, and 10^4. The effectiveness of the Hadley circulation in smoothing out the horizontal temperature structure in a slowly rotating atmosphere is clearly demonstrated. In the limiting cases, (35) reduces to

$$\frac{1}{\Delta_h}\left[\frac{\bar\theta}{\theta_0}-1\right]=\begin{cases}\dfrac{1}{3}\left(1-\dfrac{5R}{6}-\dfrac{3y^4}{2R}\right), & R\ll 1\\[2mm] -\dfrac{1}{2R}\left(\dfrac{1}{2}+\dfrac{y^4}{1-y^2}\right), & R\gg 1.\end{cases} \tag{36}$$

The vertically averaged meridional heat flux can be obtained from (16), (35), (6), and (7). For $y \leqslant y_H$, with the aid of (33) it can be written as,

$$\overline{v\theta}(1-y^2)^{1/2}=\frac{a\Delta_h\theta_0}{\tau_R}\left\{\frac{1}{3}\left(1+\frac{1}{2R}\right)\frac{y}{y_H}\left[1-\left(\frac{y}{y_H}\right)^2\right]\times y_H^3+\frac{1}{4R}\left[\ln\left(\frac{1+y}{1-y}\right)-\frac{y}{y_H}\ln\left(\frac{1+y_H}{1-y_H}\right)\right]\right\}. \tag{37}$$

Equation (37) is plotted in Fig. 2c. The poleward heat flux increases as R increases (Ω decreases), ultimately asymptoting to the nonrotating limit, which can be shown from (37) to be $(a\Delta_h\theta_0/\tau_R)y(1-y^2)/3$.

Note that solutions for y_H, $\bar\theta$, $u(y, H)$, and the averaged heat flux are independent of the static stability S/H, the meridional velocity V_b, and the surface drag C. According to (20) the mean meridional heat flux is simply

$$\overline{v\theta}(1-y^2)^{1/2}=S\theta_0V_b(y), \tag{38}$$

so that (37) also yields $V_b(y)$.

Under the assumption $u(y, 0)/u(y, H) \ll 1$, (20) and (11) lead to

$$\overline{vM}(1-y^2)^{1/2}=V_b(y)u(y,H)a(1-y^2)^{1/2}.$$

By (30), (37), and (38), this can be expressed as

$$\overline{vM}(1-y^2)^{1/2}=\frac{\Omega a^3\Delta_h}{S\tau_R}f(y,R), \tag{39}$$

where $f(y, R)$ corresponds to the quantity in braces on the right-hand side of (37) multiplied by y^2. This is shown in Fig. 2d.

Surface winds can be determined from (17) and (39) as,

$$u(y,0)=-\frac{\Omega a\Delta_h\tau_c}{S\tau_R}\left\{\left(1+\frac{1}{2R}\right)(yy_H)^2\left[1-\frac{5}{3}\left(\frac{y}{y_H}\right)^2\right]+\frac{y}{2R}\left[\frac{y}{1-y^2}-\frac{3}{2}\left(\frac{y}{y_H}\right)\ln\left(\frac{1+y_H}{1-y_H}\right)+\ln\left(\frac{1+y}{1-y}\right)\right]\right\}\times(1-y^2)^{-1/2}. \tag{40}$$

The distribution of surface winds is shown in Fig. 2e.

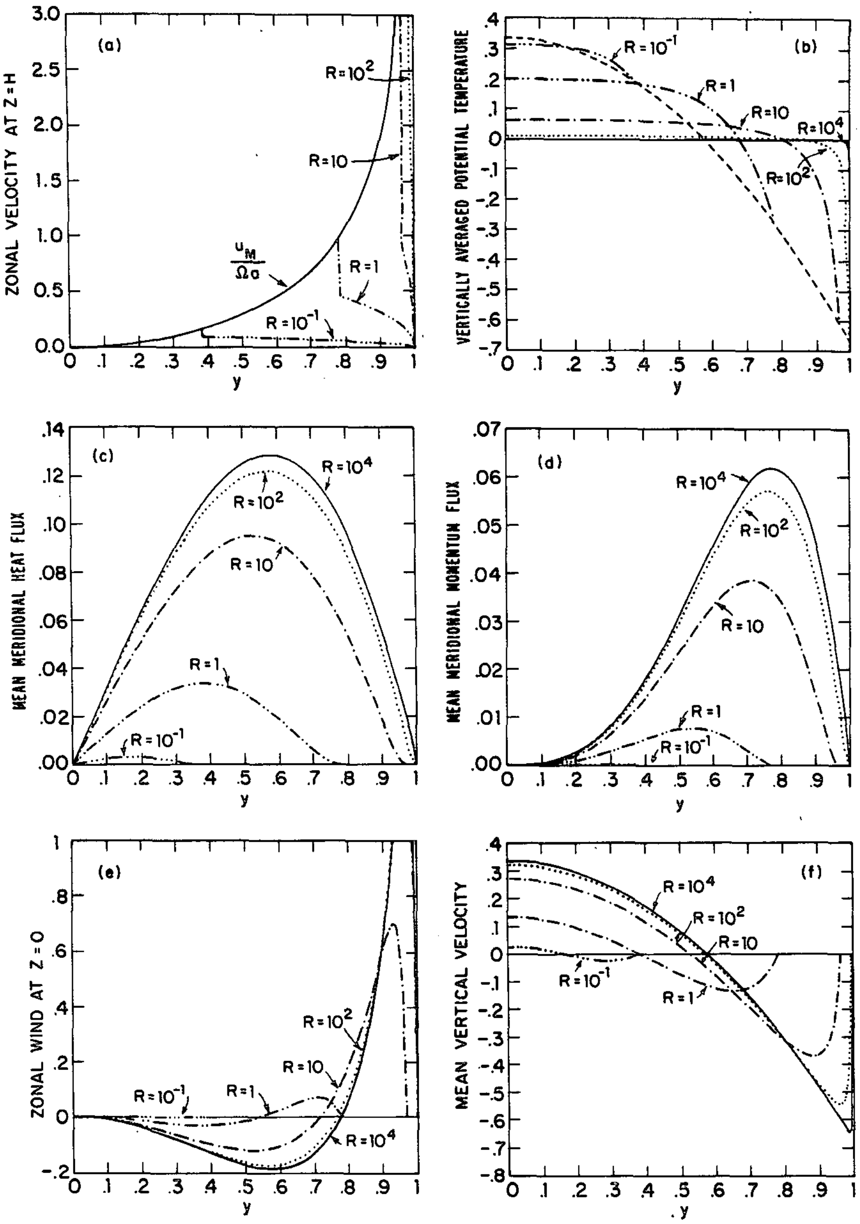

FIG. 2. Solutions for a nearly inviscid, thermally controlled Hadley circulation ($G = 0$, $\nu \to 0$). (a) Zonal wind at $z = H$ scaled by Ωa: for $0 \leqslant y \leqslant y_H(R)$, the solution is given by (30), for $y > y_H$, it coincides with u_E given by (9); (b) vertically averaged potential temperature $(\bar{\theta}(y)/\theta_0 - 1)$ from (35), scaled by Δ_h (full lines): the broken line gives the radiative equilibrium profile (6); (c) vertically averaged meridional heatflux $\overline{v\theta}(1 - y^2)^{1/2}$ from (37), scale by $a\theta_0\Delta_h/\tau_R$: according to (38), this also represents the meridional velocity V_b scaled by $a\Delta_h/S\tau_R$; (d) vertically averaged meridional flux of angular momentum $\overline{vM}(1 - y^2)^{1/2}$ from (39), scaled by $\Omega a^3\Delta_h/S\tau_R$; (e) zonal wind at $z = 0$ from (40), scaled by $\Omega a\Delta_h\tau_c/S\tau_R$; (f) the mean vertical velocity from (41), scaled by $H\Delta_h/S\tau_R$.

According to (17) when $G = 0$, $u(y, 0)$ is proportional to the divergence of the averaged momentum flux carried by the Hadley cell (which is always poleward). As y increases, $u(y, 0)$ must change from easterlies to westerlies at some latitude. Since dynamic fluxes vanish at latitudinal boundaries, the net torque on the surface of the planet is guaranteed to be zero in a steady state.

Substituting (6) and (35) into (28), we get the averaged vertical velocity,

$$\bar{w}(y) = \frac{H\Delta_h}{S\tau_R}\left[y_H^2 - y^2 + \frac{1}{2R}\left(\frac{y_H^4}{1 - y_H^2} - \frac{y^4}{1 - y^2}\right)\right], \tag{41}$$

which is shown in Fig. 2f. Since according to (28), $\bar{w}$

is proportional to the mean radiative heating, which vanishes at y_H, the maximum sinking motion occurs at some distance from y_H.

Finally, by (29) and (41) we obtain an estimate for τ_D,

$$\frac{\tau_R}{\tau_D} = \frac{\Delta_h}{S}\left(y_H{}^2 - \frac{1}{2R}\frac{y_H{}^4}{1-y_H{}^2}\right). \quad (42)$$

The ratio τ_R/τ_D scaled by $\Delta_h/3S$ is shown in Fig. 1 to be a function of R only. For a slowly rotating atmosphere ($R \gg 1$), τ_D may be approximated by the nonrotating result $3S\tau_R/\Delta_h$.

The external thermal Rossby number emerges from this analysis as a key parameter governing the behavior of a nearly inviscid circulation. Figure 2 shows that the solutions, when suitably scaled, are universal functions of R. Moreover, the width of the Hadley cell, the vertically averaged temperature and heat flux are independent of the static stability parameter. As Ω decreases (R increases), both the width and the strength of the Hadley cell increase, so does the averaged poleward dynamic heat flux. Our analysis shows that the meridional thermal gradient is small not just in the nonrotating limit, but for $R \gtrsim 10$ the temperature structure is flat over most of the globe and the circulation effectively reaches the pole (Figs. 1 and 2b). For a model Venus atmosphere with parametric values appropriate for the directly-heated region (45–75 km, see Appendix), we obtain $R = 2.7 \times 10^4$ if we assume this region were not superrotating, or $R = 7.2$ if it as a whole rotates with a period of 4 days; both values are substantially greater than unity, suggesting that the small horizontal temperature gradient observed on Venus is largely a consequence of the slow atmospheric rotation rate. However, since superrotation implies eddy forcing, such a thermal circulation model is not directly applicable to the Venus atmosphere, as will be discussed in Section 6.

5. Effect of viscosity

In Section 4 we examined thermal flows for which the angular momentum is approximately conserved. Even in this nearly inviscid limit it is crucial that some mixing of momentum is present in order to prevent symmetric instability from developing, as discussed in HH. Moreover, if we assume $u(y, 0) \ll u(y, H)$, then the descending air cannot conserve angular momentum and viscosity must play a role in the sinking branch of the Hadley cell. However, HH and Hou (1981) have shown for $R \ll 1$ and $R \gg 1$ that the required viscosity is small so that nearly inviscid states are, in fact, realizable.

The effect of viscosity in a rapidly rotating atmosphere is to force the angular momentum equation to become linear, with the Coriolis force term dominating over nonlinear advections, as shown in HH. However, this is not necessarily true for a slowly rotating atmosphere, where Coriolis forces are negligible. In this section we investigate the model dependence on internal viscosity using the *ad hoc* diffusion model $F(u) = \nu\partial^2 u/\partial z^2$. Then by (18) and the stress-free boundary condition (7), we obtain $F_u(u) = -(\nu/H)(\partial u/\partial z)$, evaluated at $z = H - \delta$. For a zonal wind profile that increases with height over the depth of the circulation, we approximate $\partial u/\partial z$ by $[u(y, H) - u(y, 0)]/H$, thus,

$$F_u(\nu, u) \approx -\frac{u(y,H) - u(y,0)}{\tau_\nu} \approx -\frac{u(y,H)}{\tau_\nu}, \quad (43)$$

where $\tau_\nu = H^2/\nu$ is the characteristic diffusive time.

As in Section 4, we continue to assume $u(y, 0) \ll u(y, H)$; but, instead of approximating $u(y, H)$ by u_M, using (43) we can solve the coupled system (14), (22) and (24) for $u(y, H)$, $V_b(y)$ and $\bar{\theta}(y)$ with ν as a parameter. Unlike the nearly inviscid solutions of Section 4, these results depend on the actual value of the static stability parameter S. For $G = J = 0$ and parametric values for the Venus atmosphere listed in the Appendix, results have been obtained for two values of Ω, corresponding to the planetary rotation rate ($R = 2.7 \times 10^4$) and the 4-day rotation at the cloud top ($R = 7.2$); in each case calculations were performed for $\nu = 10^3$, 5×10^2, 10^2, and 50 $m^2\ s^{-1}$ and for $\nu \to 0$.

Numerical results are presented in Fig. 3. The scaling for each quantity is the same as for Fig. 2. In the slowly rotating ($R = 2.7 \times 10^4$) case the results show that diffusion leads to decreased zonal winds (Figs. 3a and 3e) and reduced mean angular momentum flux (Fig. 3d), but it has virtually no effect on the mean temperature or the mean meridional heat flux (Figs. 3b and 3c). This is because that $d\bar{\theta}/dy$ is proportional to $\Delta_h R^{-1}$ [as can be shown from (14)]; in the limit $R \to \infty$, $\bar{\theta}$ is effectively "decoupled" from u; the meridional flow is consequently insensitive to small changes in u. In the 4-day rotation ($R = 7.2$) case, the effect of $\nu = 10^2\ m^2\ s^{-1}$ is trivial, while that of $\nu = 10^3\ m^2\ s^{-1}$ is noticeable but not sufficient to alter the nonlinear character of the circulation (which will be evident as we inspect the angular momentum balance). Compared to the nearly inviscid case, the $\nu = 10^3\ m^2\ s^{-1}$ results show weaker zonal winds for most latitudes and a slightly stronger meridional flow (Figs. 3a, e, and f); the latter is reflected in increases in the departure of $\bar{\theta}$ from $\bar{\theta}_E$ (Fig. 3b). The net effect is to increase the mean meridional heat flux (Fig. 3c) and to decrease the mean angular momentum flux (Fig. 3d).

Figure 4 shows the angular momentum balance of the upper branch in terms of the ratio of the Coriolis force to the horizontal advection in (24) for the $R = 7.2$ case. For all values of ν this ratio is close to unity over most latitudes [which is equivalent to $u(y, H)$ approaching u_M]. Thus, the effect of viscosity is small and nonlinear transports are important, except in a narrow region near the poleward boundary of the circulation, where $d[u(1 - y^2)^{1/2}]/dy$ tends to zero

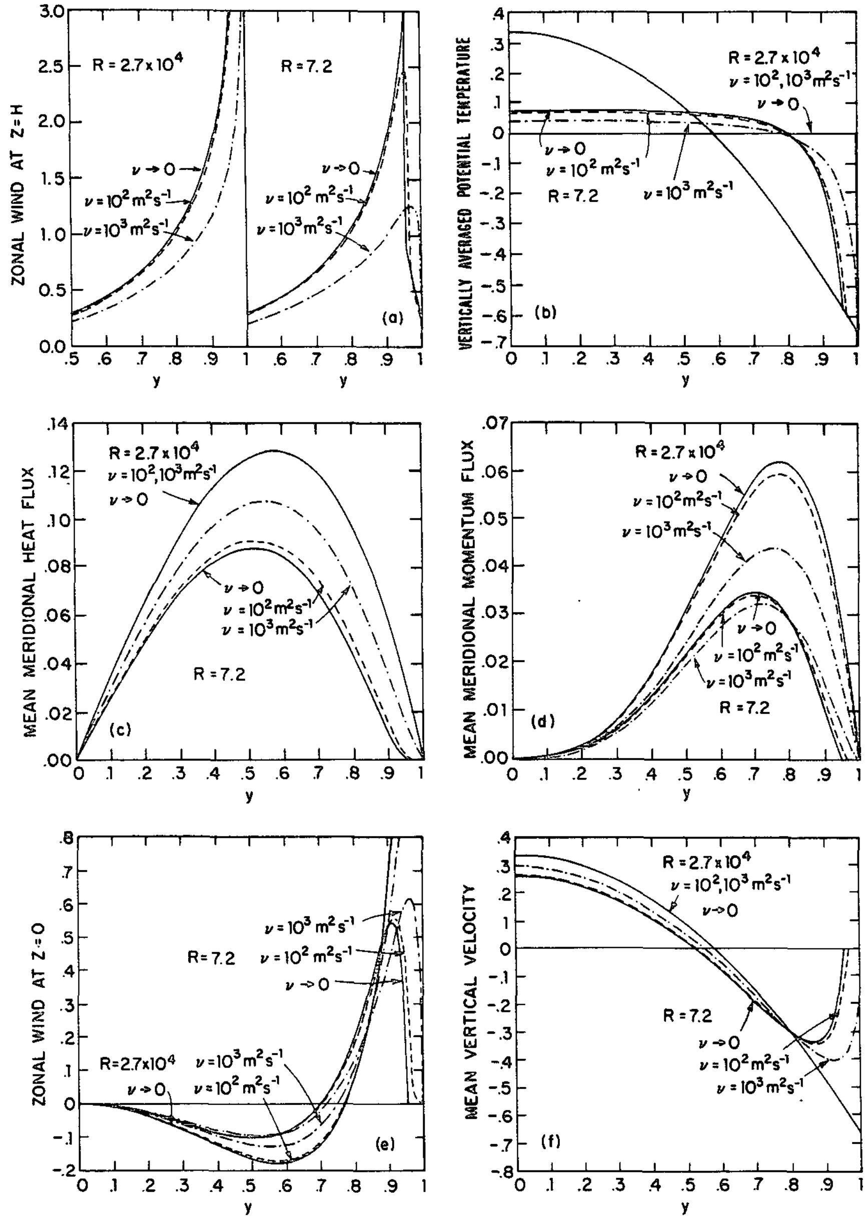

FIG. 3. Calculations of thermal circulations for $R = 2.7 \times 10^4$ and $R = 7.2$, and for three values of $\nu = 10^3$, 10^2 m^2 s^{-1}, and $\nu \rightarrow 0$: results presented as in Fig. 2.

and the angular momentum balance becomes linear and frictionally controlled. However, the width of this viscous boundary-layer region decreases as $\nu \rightarrow 0$. For $R = 2.7 \times 10^4$ we obtain similar results.

In Section 4 we assumed that the condition for diffusion to be negligible is $\tau_D/\tau_\nu \ll 1$, where $\tau_D = H/\bar{w}(0)$ is given by (42) for a nearly inviscid flow. From Fig. 1 and the Appendix we find that $\tau_D \approx 22$ days for $R = 2.7 \times 10^4$, and $\tau_D \approx 6.9$ days for $R = 7.2$; these values are insensitive to diffusion since vertical velocities are not significantly modified (Fig. 3f). The values $\nu = 10^3$, 10^2, and 50 m^2 s^{-1} then correspond to, respectively, $\tau_D/\tau_\nu = 0.52$, 0.052, and 0.026 in the slowly rotating case and 0.66, 0.066, and 0.033 in the 4-day rotation case. Hence, for $\nu \ll 10^3$ m^2 s^{-1}, the circulation is effectively inviscid. Since measurements on Venus suggest that $\nu \approx 10$ m^2 s^{-1} at cloud tops (see Appendix), diffusion is likely to be of no importance to the Hadley circulation at these levels.

However, the value of $\tau_R = 20$ days is appropriate only to the cloud levels. The situation in the lower atmosphere is quite different. Near the surface τ_R could be as long as 10^4 days (Gierasch *et al.*, 1970; Pollack and Young, 1975), and data indicate that 3% of the total solar flux actually reaches the ground. If this surface heating drives a shallow Hadley cell, as suggested by Schubert (1983), then for a nearly inviscid circulation extending one scale height (H = 15 km), Δ_h = 0.03 (an order smaller than the cloud-level value), and the surface rotation rate, we obtain $R \approx 10^3$ and, from (42), $\tau_D \approx 3 \times 10^4$ days. For such a flow, the nearly inviscid condition $\tau_D/\tau_\nu \ll 1$ would require that $\nu \ll H^2/\tau_D \approx 0.1$ m^2 s^{-1}, a value significantly less than what might be reasonably expected from turbulent mixing. Therefore, if a Hadley cell exists near the surface, it is likely to be viscous.

6. Circulation forced by heat and momentum sources: The inverse problem

In Sections 4 and 5 we analyzed symmetric flows forced by diabatic heating in the absence of eddy sources ($G = J = 0$). The resulting zonal winds do not exceed the momentum-conserving u_M given by (12) with $u_* = 0$ (Figs. 2a and 3a), in marked contrast to those observed in the Venus troposphere, which can be as large as 120 m s^{-1} at the cloud tops, i.e., $u_*/\Omega a \approx 60$. We are, therefore, compelled to invoke up-gradient transport of angular momentum by asymmetric motions ($G \neq 0$) to maintain this equatorial superrotation. For parametric values appropriate for the Venus atmosphere, we will attempt to characterize, within the axisymmetric framework, the eddy source pattern $G(y, z)$ by solving the inverse problem. We ask what mean meridional circulation and $G(y, z)$ are consistent with given solar heating and zonal wind distributions? Our main objective is to analyze the *diagnostic* eddy source requirement for a single meridional cell, whose depth is known. In a subsequent paper we will present primitive-model calculations for the zonal-mean circulation and eddy source pattern for the Venus troposphere.

In Section 3 we showed that the inversion for the eddy momentum source G is unique only if the eddy heat source is known. However, in the presence of significant diabatic heating, the effect of the mean eddy heat flux $\bar{h}$ is merely to modify the diabatic circulation and the physical interpretation of the zonal momentum source. For applications to the directly-heated region in the Venus troposphere, we will assume that the eddy heat source J is small compared with solar heating so that the Eulerian-mean velocity V_b may be approximated by the diabatic flow V_b^*. We will thus interpret the effective momentum source G^* as the eddy momentum flux convergence G, which, by Hide's theorem, is the key to equatorial superrotation.[1]

Given that the essential cause for superrotation lies in G rather than J, to determine the general pattern of G we shall in this analysis assume $J = 0$, with the understanding that if $J \neq 0$ the physical interpretation of G can be modified acccordingly. Then for specified $\theta_E(y, z)$ and $u(y, z)$, (14) and (22) can be solved for $\bar{\theta}(y)$ and $V_b(y)$. By (23), (24), (25) and (26) we obtain all components of $(F + G)$. The vertically averaged meridional heat and momentum fluxes can be evaluated from (21) and (23), respectively; the dynamic time scale from (29) and the mean vertical velocity from (28). Note that in this case the meridional flow is controlled by both heat and momentum sources; the resulting circulation is not necessarily thermally direct, as will be demonstrated.

For parametric values appropriate to the Venus atmosphere (see Appendix), we shall present solutions for two alternative zonal wind profiles:

$$\frac{u(y, z)}{\Omega a} = \left[U_0 + U\left(\frac{z}{H}\right)\right]Y(y), \tag{44a}$$

where Ω is the planetary rotation rate, U_0 controls the magnitude of $u(0, 0)$, U/H defines the vertical shear, and the latitudinal distribution $Y(y)$ is given by either

$$Y(y) = (1 - y^2)^{1/2} \text{ for all } y, \tag{44b}$$

or

$$Y(y) = \begin{cases} \dfrac{1}{(1 - y^2)^{1/2}}\left[1 + \left(\dfrac{y}{y_1}\right)^2 (k(1 - y_1^2)^{1/2} - 1)\right], & y \leq y_1 \\ k + [(1 - y_2^2)^{1/2} - k]\dfrac{y - y_1}{y_2 - y_1}, & y_1 < y \leq y_2 \\ (1 - y^2)^{1/2}, & y > y_2. \end{cases} \tag{44c}$$

The profiles given by (44a) and (44b) represent an atmosphere which rotates as a rigid spherical shell at each level (Fig. 5a). A similar profile was used in an important paper by Gierasch (1975), and it resembles the velocity distribution observed at the cloud tops in the early days of Pioneer Venus (Schubert, 1983, Fig. 8). Measurements from Mariner 10, however, suggest a midlatitude zonal jet, here modeled by (44c) with $k = 1.2$, $y_1 = \sin 60°$, and $y_2 = \sin 70°$ (Fig. 6a).

First, we consider the case of stratified solid-body rotation. If we define "superrotation" to be wherever $M/(\Omega a^2) > 1$; then at $z = H$ the profile given by

[1] Note that in the diagnostic mode, the eddy heat source term J can be absorbed into the definition of θ_E, then by Hide's theorem u cannot superrotate at the equator if $G = 0$. However, superrotation is possible if $\bar{G} = 0$, as suggested by (23b), which requires only that $\bar{G}^* \neq 0$. In this case, G cannot be identically zero everywhere, as can be shown by the following example: Assuming $F \neq 0$ but $C = 0$, by (17) we find $\bar{F} = 0$, then (23a) gives V_b (Eulerian) = 0 if $\bar{G} = 0$. From (24) we obtain $G_u = -F_u \neq 0$.

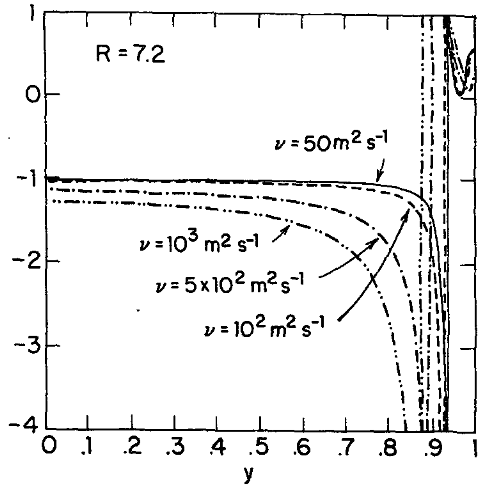

FIG. 4. Ratio of the Coriolis force to the meridional advection of $u(y, H)(1 - y^2)^{1/2}$ in the upper meridional branch for $R = 7.2$ [see the left-hand side of (24)].

(44a) and (44b) superrotates everywhere equatorward of $y = [(U_0 + U)/(1 + U_0 + U)]^{1/2}$ for $(U_0 + U) > 0$; the supply of angular momentum in the tropics is, therefore, a central issue in the maintenance problem.

Substituting (44a) and (44b) into (14) gives

$$U[U + 2(U_0 + 1)]y = -\frac{gH}{(\Omega a)^2}\frac{d}{dy}\left(\frac{\bar{\theta}}{\theta_0}\right). \quad (45)$$

Thus, in the special case of solid-body rotation, the vertically averaged temperature is independent of the latitudinal wind profile $Y(y)$. Integrating (45) over y with the identity $\theta_0 = \int_0^1 \bar{\theta} dy$ leads to

$$\frac{\bar{\theta}}{\theta_0} = 1 + \Delta_h(1 - \chi)\frac{1}{3}(1 - 3y^2), \quad (46)$$

where,

$$\chi = 1 - \frac{U}{2R}[U + 2(U_0 + 1)]. \quad (47)$$

Using (46) we can integrate (22) with the boundary condition $V_b(0) = 0$ to give

$$V_b(y) = \frac{a\Delta_h\chi}{S\tau_R}\frac{y}{3}(1 - y^2). \quad (48)$$

From (21), (22), and (48) we obtain

$$\overline{v\theta}(1 - y^2)^{1/2} = V_b S\theta_0 = \frac{a\theta_0\Delta_h\chi}{\tau_R}\frac{y}{3}(1 - y^2), \quad (49)$$

and from (17), (20), (44a), (44b), and (48),

$$\overline{vM}(1 - y^2)^{1/2} = \frac{\Omega a^3\Delta_h\chi U}{S\tau_R}\frac{y}{3}(1 - y^2)^2. \quad (50)$$

From (28) and (48),

$$\bar{w}(y) = \frac{H\Delta_h\chi}{S\tau_R}\frac{1}{3}(1 - 3y^2), \quad (51)$$

and from (29) and (51),

$$\tau_D = \frac{3S\tau_R}{\Delta_h|\chi|}. \quad (52)$$

Our primary aim is to determine the eddy momentum source pattern, which can be evaluated from (23) through (26) using (44a), (44b), and (48):

$$[\bar{G}(y) + \bar{F}(y)](1 - y^2)^{1/2} = \frac{\Omega a\Delta_h\chi U}{S\tau_R}\frac{1}{3}(1 - 5y^2)(1 - y^2), \quad (53)$$

$$[G(y) + F(y)]_u(1 - y^2)^{1/2} = \frac{\Omega a\Delta_h\chi}{S\tau_R}(U + U_0 + 1)\left(-\frac{2}{3}\right)y^2(1 - y^2), \quad (54)$$

$$[G(y) + F(y)]_l(1 - y^2)^{1/2} = \frac{\Omega a\Delta_h\chi}{S\tau_R}(U_0 + 1)\left(\frac{2}{3}\right)y^2(1 - y^2), \quad (55)$$

and

$$[G(y) + F(y)]_i(1 - y^2)^{1/2} = \frac{\Omega a\Delta_h\chi U}{S\tau_R}\frac{1}{3}(1 - 3y^2)(1 - y^2). \quad (56)$$

From Section 3 we see that $(\bar{G} + \bar{F})(1 - y^2)^{1/2}$ balances the vertically averaged divergence of M by the mean circulation; $(G + F)_u(1 - y^2)^{1/2}$ and $(G + F)_l(1 - y^2)^{1/2}$ are induced by horizontal motions in the upper and lower layers, respectively; and $(G + F)_i(1 - y^2)^{1/2}$ is associated with vertical motions in the interior. Note that $(G + F)_u$ and $(G + F)_l$ are similar in structure but opposite in sign, which give the appearance of a vertical dipole if the ratio of their magnitudes, $[1 + U/(U_0 + 1)]$, is positive.

These results are plotted in Fig. 5, which shows that the scaled solutions are qualitatively similar to those in Fig. 2 for $R \gg 1$ (except the profiles of u and $\bar{\theta}$). The basic differences between the two cases lie in the scaling factors. The magnitudes of V_b, $\overline{v\theta}$, $\bar{w}$, and τ_D in the present case differ from those for $G = 0$ by, roughly, the factor χ, while $\overline{vM}$ differs by the factor χU. When $U = 0$, (47) yields $\chi = 1$, so that the present case reduces to that of a thermal circulation on a nonrotating planet of Section 4. To estimate the effect of superrotation, if (from the Appendix) we take $\Omega a \approx 2$ m s^{-1}, $R = 2.7 \times 10^4$ and a model circulation with $U = 120/2 \gg (U_0 + 1)$, from (47) we obtain $\chi = 1 - U^2/2R \approx 0.92$; alternatively, applying the model to the directly-heated region (45–75 km) by setting $U_0 = 50/2$, $U = (120 - 50)/2$, and $R = 2.7 \times 10^4$, we obtain $\chi \approx 0.94$. In both cases the effect of superrotation is to increase the bulk

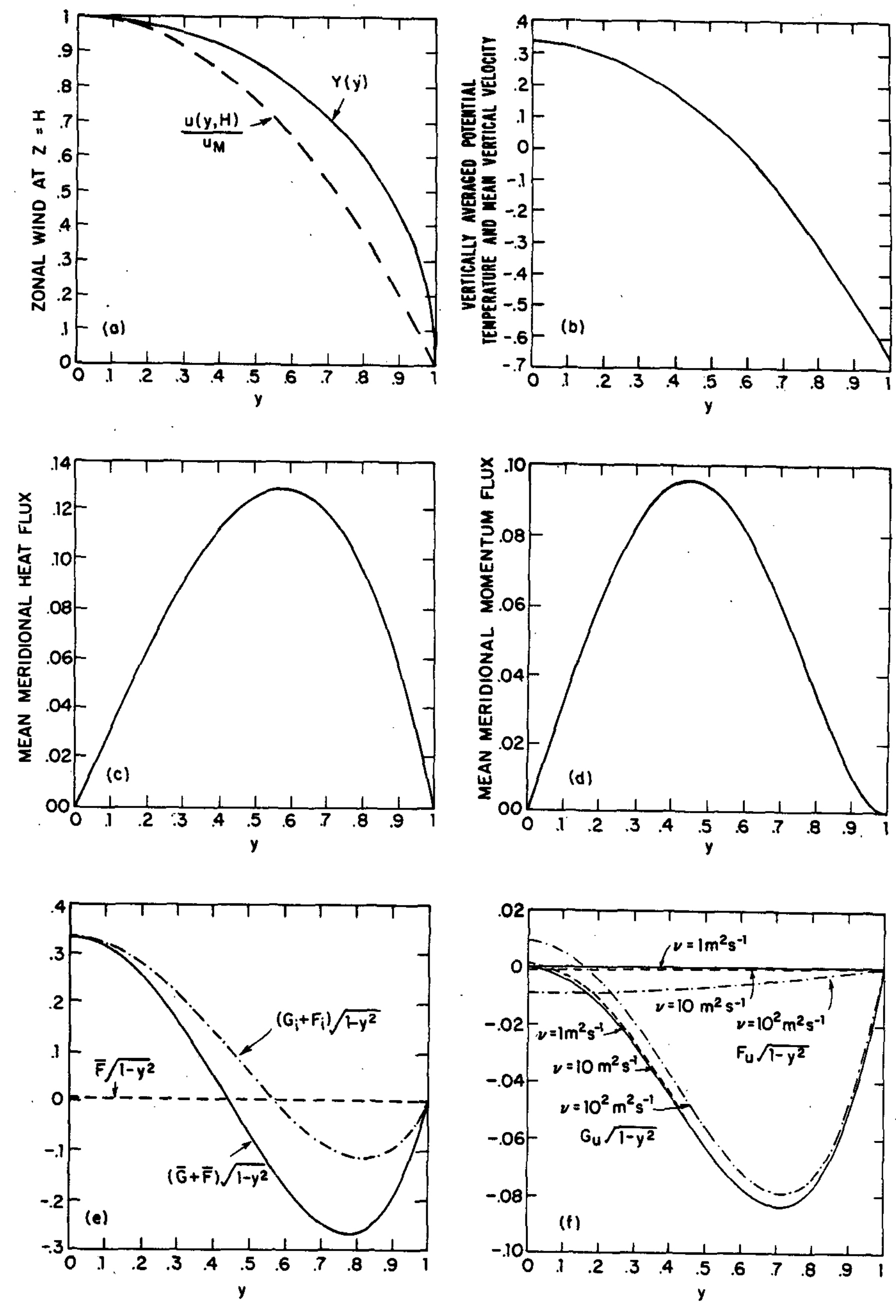

FIG. 5. Solutions to the inverse problem for the differentially rotating atmosphere given by (44a) and (44b). (a) Zonal wind at $z = H$ scaled by $u(0, H)$ (solid line) given by (44b). The broken line plots the ratio $u(y, H)/u_M$, where u_M is the momentum-conserving profile given by (12) for $u_* = u(0, H)$ and $U_0 + U = 60$; (b) vertically averaged potential temperature $(\bar{\theta}/\theta_0 - 1)$ from (46), scaled by $\Delta_h(1 - \chi)$. Coincidentally, this also represents the mean vertical velocity given by (51), scaled by $H\Delta_h\chi/S\tau_R$; (c) vertically averaged meridional heatflux $\overline{v\theta}(1 - y^2)^{1/2}$ from (49), scaled by $a\theta_0\Delta_h\chi/\tau_R$. According to (48), this also represents the meridional velocity V_b scaled by $a\Delta_h\chi/S\tau_R$; (d) vertically averaged meridional flux of angular momentum $\overline{vM}(1 - y^2)^{1/2}$ from (50), scaled by $\Omega a^3\Delta_h\chi U/S\tau_R$; (e) vertically averaged momentum sources from (53) and sources induced by vertical motions in the interior according to (56); each scaled by $\Omega a\Delta_h\chi U/S\tau_R$. The total inverted sources $(G + F)$ does not depend on friction; the estimated size of $\bar{F}$ is shown for $U_0 = 0.5$, $C = 0.1$ m s^{-1} and $\nu = 10^2$ m^2 s^{-1}; (f) momentum sources associated with meridional flows in the upper branch from (54), scaled by $\Omega a\Delta_h\chi(U + U_0 + 1)/S\tau_R$. According to (55), this corresponds to sources in the lower branch scaled by $-\Omega a\Delta_h\chi(U_0 + 1)/S\tau_R$. Also shown are the estimated sizes of F_u and G_u for $U_0 = 0$, $U = 60$, and $\nu = 10^2$, 10, and 1 m^2 s^{-1}.

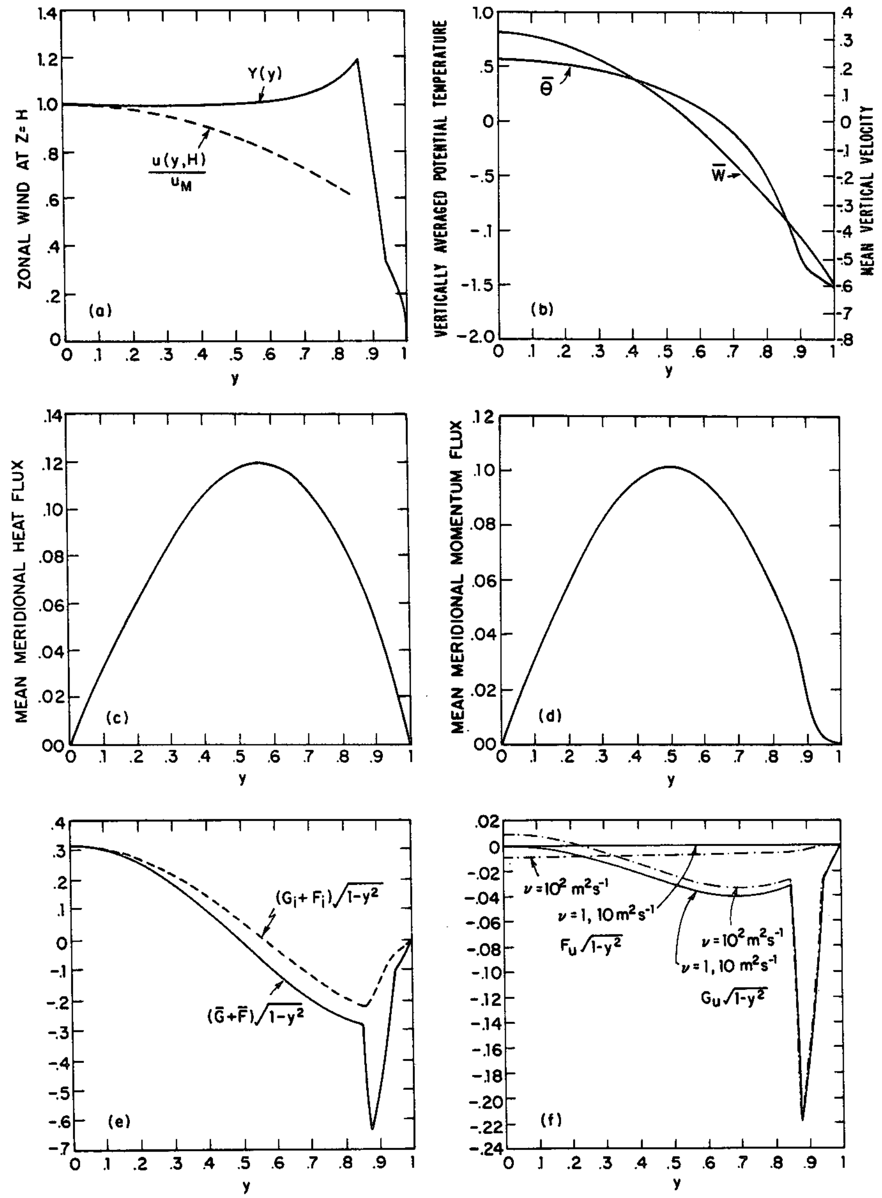

FIG. 6. Solutions to the inverse problem for a zonal jet at 60° latitude as given by (44a) and (44c): results presented as in Fig. 5.

atmospheric rotation rate, thereby weakening the poleward heat and mass fluxes, as anticipated in Sections 4 and 5. The major differences between Fig. 5 and Fig. 2 are the u, $\bar{\theta}$ and $\overline{vM}$ profiles, with the momentum transport in the present case being larger by a factor close to U. In Section 5 we have shown that for a thermal circulation the ratio $2\Omega ay$ to $d[u(y, H)(1 - y^2)^{1/2}]/dy$ in the upper branch is of the order unity (Fig. 4); in the current case this ratio is close to 0.016 for all latitudes. Thus, in the presence of strong superrotation the circulation is highly nonlinear, with the advection of angular momentum balancing $(G + F)$.

According to (48), (49), and (51), so long as $\chi > 0$ the circulation is thermally direct; i.e., both the mass flux in the upper branch and the mean meridional heat flux are poleward, and $\bar{w}$ is positive in equatorial regions and negative at high latitudes. However, (50) shows that the direction of the averaged meridional flux of angular momentum depends not only on the sign of χ but also that of U (the vertical wind shear); i.e., for $\chi > 0$, it is poleward if $U > 0$, or equatorward if $U < 0$. Moreover, for $-2(U_0 + 1) < U < 0$, the averaged temperature given by (45) is higher in polar regions, while the circulation remains direct according to (47). Recent *Pioneer Venus* data suggest that at altitudes above the cloud tops the temperature increases with latitude, with zonal winds decreasing with height (Schubert, 1983); it is possible that the mean circulation in this region may be thermally direct but transport angular momentum equatorward.

When $\chi = 0$ the meridional circulation is identically zero and the averaged diabatic heating vanishes; then $\bar{\theta} = \bar{\theta}_E$ and $u = u_E$, thereby reducing to the radiative equilibrium solution of Section 2.

When $\chi < 0$ the circulation is indirect; V_b is equatorward with sinking motions in the tropics. According to (47), this occurs if $U[U + 2(U_0 + 1)]/2R > 1$. It has been suggested that an indirect circulation may exist below the cloud base (Schubert, 1983); according to (47), this is possible if the heating below the clouds is small compared with the local "shear forcing." *Pioneer Venus* data show a region of relatively large shear just below the cloud base (~45 km), with the zonal wind gaining about 30 m s^{-1} over a depth of less than 5 km (Schubert, 1983). This effectively introduces a local vertical scale for the forcing function G, which would presumably force a local meridional cell if the solar heating at these levels is weak. Substituting $H = 5$ km, $g = 8.6$ m s^{-1}, $\Omega a \approx 2$ m s^{-1}, $U_0 = 50/2$, and $U = 30/2$ into (47) gives $\chi < 0$ if $\Delta_h < 1/20$. For an even more localized shear increase (a smaller H), an indirect cell may appear for a larger Δ_h. This will be demonstrated by primitive-model calculations in Hou and Goody (1985).

Figures 5e, f give the dimensionless solutions for the total source $(G + F)$. From (53)–(56) we see that the sign of the source requirement depends on the signs of χ and U. The sources induced by vertical motions are important at all latitudes; in particular, in equatorial regions, where the absolute angular momentum has a maximum, they dominate over those induced by meridional flows. However, the latter are important at middle and high latitudes, with a maximum influence at $y = 1/\sqrt{2}$ (45° latitude).

The quantity $(G + F)$ determined by inversion does not depend upon boundary friction or internal diffusion. We can, however, make certain upper-bound estimates for their magnitudes. From (17) and (43) we obtain $\bar{F}(u) = -u(y, 0)/\tau_c = -U_0(1 - y^2)^{1/2}/\tau_c$, and $F_u(u) = -[u(y, H) - u(y, 0)]/\tau_\nu = -U(1 - y^2)^{1/2}/\tau_\nu$. For a model circulation with $U_0 = 1/2$, $U = 120/2$, and several values of ν and C; we obtain the results shown in Figs. 5e, f. For a cloud-level circulation with a diffusive lower boundary condition, we obtain comparable estimates. Even though these assumed values are much larger than what we anticipate for Venus, nevertheless, the calculated frictional terms are still small enough to be neglected. This conclusion is in agreement with that reached in Section 5 and is important for understanding the Venus circulation. With $F \ll G$, the required sources correspond to the convergence pattern of eddy fluxes, based on which it may be possible to set an upper bound for the scale of the eddies.

The preceding calculations were repeated for the zonal wind profile of (44a) and (44c) for $R = 2.7 \times 10^4$, $U_0 = 0$, and $U = 60$. The dimensionless results are plotted in Fig. 6. Even though in this case the averaged potential temperature is no longer independent of the latitudinal wind profile $Y(y)$, in most respects Figs. 5 and 6 are qualitatively similar. The main differences lie in $\bar{G}$, G_u and G_l; Figs. 6e, f show pronounced peaks between $y = 0.86$ and 0.94, corresponding to an abrupt loss/gain of angular momentum associated with meridional flows across the zonal jet. Since the profiles of G_u and G_l are similar, this appears as a vertical dipole feature at these latitudes if $U > -(U_0 + 1)$.

Based on the results of this section we conclude that so long as the absolute angular momentum of the atmosphere decreases with latitude, the general source pattern remains qualitatively similar to those shown in Figs. 5 and 6. For the scaling factors used in this analysis, $(\bar{G} + \bar{F})$ and $(G + F)_i$ are positive in the tropics and negative at high latitudes; $(G + F)_u$ is negative, and $(G + F)_l$ is positive.

7. Discussion of approximations

In Section 3 the two assumptions which made it possible to obtain a simple model of nonlinear axisymmetric circulations are the thin layer character of the flow and the fixed static stability. We will examine them here in some detail.

In general, there is, of course, no *a priori* reason for meridional flows to be confined to thin layers. This is merely a convenient device by which we obtain a simple model to demonstrate the basic mechanics of the flow. If instead of thin layers, we assume a broad meridional flow structure, $v(y, z) = -v(y, H)\cos(\pi z/H)$, with $\delta = H/2$, and solve the inverse problem for the differentially rotating zonal wind profile given by (44a) and (44b), we find that all vertically averaged results remain unchanged if in (20) $V_b(y)$ is defined to be $2\pi^{-2}v(y, H)(1 - y^2)^{1/2}$. In other words, the results (45) through (52) given in Section 6 are independent of the depth δ or details of the vertical distribution of v. This holds true so long as $V_b(y)$ is interpreted as an effective meridional flux, and $u(y, z)/u(y, H)$ and $\theta(y, z)/\theta(y, H)$ are roughly similar, which is the case for the Venus troposphere (Schubert, 1983).

Given that the vertically averaged solutions are invariant, the only difference between the two cases lies in the partition of sources associated with horizontal and vertical motions. By (24) we find that for $U = 60$, the $(G + F)_u$ of the present case is at most 40% larger than that given by (54). However, if we assume $\delta = H/2$, the "interior" sources due to vertical motions vanish, then, according to (26), so does dV_b/dy, which is inconsistent with (48). Clearly, $\delta = H/2$ represents an overestimate of source contributions of the meridional layers, which must in this case also include the influence of vertical motions since the averaged sources are correctly obtained; in other words, the "interior" has effectively merged with the two broad meridional layers; thus the actual difference between the two cases is, in fact, less than the upper-bound estimate of 40%. The main weakness of the thin layer assumption is, therefore, the idealization that regions of meridional flows are divorced from that of vertical motions. In a realistic situation δ is likely to be a function of y, with a maximum depth away from the lateral boundaries, where v vanishes.

In the case of nearly inviscid thermal flows, the thin-layer model has been shown to provide good approximations to solutions to the full primitive-equation system obtained by HH for $R \ll 1$ and by Hou (1981) for $R \gg 1$; thus, to the extent that the flow achieves a boundary-layer character, the thin layer assumption is a reasonable simplification.

A more limiting feature of this simple model is that, in order to close the system of equations, $S(y)$ must be specified externally. This is adequate for diagnostic purposes; but if we want to use the model for predictive purposes, an additional relation is still needed. For an earth-like atmosphere, HH assumed that the tropical static stability is controlled mainly by radiative–convective processes, so that $S \approx \Delta_v$ over most of the Hadley domain, which is consistent with the model result $\tau_R/\tau_D \ll 1$. For a slowly rotating atmosphere, (42) shows that $\tau_R/\tau_D \ll 1$ is equivalent to $3\Delta_v/\Delta_h \gg 1$, which is the condition for S to remain close to Δ_v. This agrees with Stone's (1974) analysis for $\Delta_v > 1 \gg (\tau_g/\tau_R)^{1/2}$, where $\tau_g = a/\sqrt{gH}$ is the time scale for external gravity waves.

For the Venus cloud region we obtain from Fig. 1 and the Appendix the nominal value $\tau_R/\tau_D = \Delta_h|\chi|/3S \approx 3$; even though its exact value is uncertain, with τ_R/τ_D being the order of unity, we can expect that S is at least partially controlled by the circulation. Primitive-equation model calculations of Hadley circulations in a nonrotating atmosphere show that when $3\Delta_v/\Delta_h \leq 1$, S does vary somewhat with latitude, but in the nearly inviscid limit S is of the order of $\Delta_h/6$ over most latitudes, i.e., $\tau_R/\tau_D \rightarrow 2$, as $\nu \rightarrow 0$ (Hou, 1981). This differs from Stone's (1974) estimate that S is independent of Δ_h if $\Delta_v < (\tau_g/\tau_R)^{1/2} \ll 1$. In cases where eddy sources are present the problem becomes even more complicated. Clearly, a better understanding of what controls the static stability is needed before the model can be used for general prognostic purposes.

Based on the results of Sections 4 through 6 we can check the self-consistency of our model assumptions. First, we examine the nearly inviscid assumption $\tau_D/\tau_\nu \ll 1$ of Section 4. According to the definition $\tau_\nu = H^2/\nu$, the condition that the interior be essentially inviscid is equivalent to $\nu \ll H^2/\tau_D$, where τ_D can be obtained from (42) and Fig. 1. Evaluating this for the Venus atmosphere using the Appendix, we find that this condition implies $\nu \ll 1.9 \times 10^3$ m^2 s^{-1} for $R = 2.7 \times 10^4$, or 1.5×10^3 m^2 s^{-1} for $R = 7.2$, which is in agreement with the results of Section 5.

Another assumption in Sections 4 and 5 is that $\tau_c/\tau_D \ll 1$, which is required for the simplification $u(y, 0)/u(y, H) \ll 1$. From (40), (42), Fig. 1, Fig. 2a and Fig. 2e, we see that away from the equator $u(y, 0)$ is of the order $\Omega a\tau_c/\tau_D$ and $u(y, H)$ is of the order Ωa; hence, $u(y, 0)/u(y, H) \approx \tau_c/\tau_D$. Note, however, that this assumption can be easily relaxed by retaining $u(y, 0)$ as a coupled unknown.

Finally, we check the basic assumption that the thermal wind relation (14) holds, i.e., $\nabla\cdot(\mathbf{V}v)$ and $F(v)$ are small compared with the Coriolis and centrifugal terms in (2). This would not be true in the immediate vicinity of the equator or near poleward boundary of the Hadley cell, but it is sufficient if the condition holds over most of the Hadley domain. For an order of magnitude estimate, we may approximate the divergence term by v^2/ay_H and $F(v) = \nu\partial^2 v/\partial z^2$ by $(2\nu v/H)/\delta = (2H/\delta)(v/\tau_\nu)$, where v is an average meridional velocity defined as (ay_H/τ_D), with τ_D given by (29). Ignoring the spherical geometry, we can estimate δ from (18) and (20) as $\delta/H \approx [V_b]/v$, where the brackets denote the latitudinal average. If we take $[V_b]$ to be roughly half the maximum value of V_b, we can estimate δ/H directly from the scaled results for V_b given in Figs. 2c, 3c, 5c, and 6c. Comparing these terms with the cyclostrophic terms then provides the condition for which they are negligible.

In the case of thermal circulations, we may approximate u by $u(y_H, H) = \Omega a y_H^2/(1 - y_H^2)^{1/2}$; assuming maximum values for the divergence and viscous terms, we find that the relative sizes of $2\Omega yu$:$yu^2/a(1 - y^2)^{1/2}$:$\nabla\cdot(\mathbf{V}v)$:$F(v)$ are, respectively, $2(\Omega y_H)^2$:$(\Omega y_H)^2 y_H^2(1 - y_H^2)^{-1}$:$\tau_D^{-2}$:$(\tau_D\tau_\nu)^{-1}(2H/\delta)$. From (20), (37) and (42), we obtain $(H/\delta) = (2y_H/3)$ multiplied by the scaled solution for τ_R/τ_D (as in Fig. 1) and divided by the scale solution for $(V_b)_{\max}$ (as in Fig. 2c). Evaluating for $R = 10^4$, 10, 1, and 0.1, we obtain $H/\delta = 10$, 11, 12, and 14, respectively. Therefore, so long as $\tau_D/\tau_\nu \leq 10$, (which is required by the nearly inviscid assumption), $F(v)$ is at most comparable to the divergence term. Thus, the thermal wind approximation is valid if $(\Omega y_H\tau_D)^2 \gg 1$. However, in the nonrotating limit, the divergence and viscous terms must ultimately become important, while (14) gives $d\bar{\theta}/dy = 0$ as $\Omega \rightarrow 0$ $(u \rightarrow 0)$. Yet, the contribution of the divergence term to (8) can at most be v^2/aH, and that of the frictional term is roughly $F(v)/H \approx (v/H\tau_\nu)(2H/\delta) \leq a/H\tau_D^2$ for a nearly inviscid flow; dividing through by g/a we find that $d\bar{\theta}/dy$ is of order $v^2/gH = (a^2/\tau_D^2 gH)$, which, by (42), may be estimated by $(a\Delta_h/3S\tau_R)^2(gH)^{-1}$; therefore, so long as the thermal velocity v is much less than the speed of external gravity waves $\sqrt{gH}$, $d\bar{\theta}/dy$ remains small.

Similarly, for the results of Section 6 we can show that the thermal wind approximation is valid for most latitudes if $[\Omega(U_0 + U)\tau_D]^2 \gg 1$ and $[\Omega(U_0$

$+ U)]^2\tau_D\tau_\nu/10 \gg 1$, and τ_D may be evaluated using (29) and Figs. 5b and 6b.

8. Summary and conclusions

In Sections 4 and 5, we analyzed the basic properties of an idealized model of an axisymmetric, thermally forced circulation in a stably stratified atmosphere. In the case of a nearly inviscid flow, we obtained closed-form solutions which, when suitably scaled, are strictly functions of the external thermal Rossby number R. For values appropriate to the Venus atmosphere, the solutions are close to the asymptotic limit of $R \rightarrow \infty$, characteristic of a nonrotating system. As R increases, both the meridional width and the strength of Hadley cell increase, so do the scaled meridional fluxes of heat and momentum. For $R = 10$ the Hadley cell terminates at about 74° latitude; for $R \gtrsim 10^2$ it effectively reaches the pole. In the slowly rotating limit, the nearly inviscid circulation maintains a temperature profile nearly constant with latitude, regardless of the value of the static stability. This agrees with the conclusion reached in earlier works by Goody and Robinson (1966), Gierasch *et al.* (1970), and others. It accounts for the weak latitudinal temperature gradient at the Venus cloud tops first observed in telescopic images of the planet.

In Section 5 we considered the effect of diffusion on thermal circulations for values of ν up to 10^3 m^2 s^{-1}. For the Venus data used, the effect is small for $\nu = 10^3$ m^2 s^{-1} and negligible for $\nu \leqslant 10^2$ m^2 s^{-1}. Recent measurements suggest that $\nu \approx 10$ m^2 s^{-1} at the cloud tops (see Appendix), the circulation is therefore likely to be fairly inviscid in this region. However, even in the nearly inviscid limit, diffusion must still play a secondary role near the poleward boundary of the Hadley cell, where an air parcel must lose its angular momentum along the sinking branch and some diffusion is required to prevent symmetric instability from developing, as discussed in HH.

Realistically, the solar radiation in the Venus atmosphere is not deposited uniformly at all levels, but principally over a 30 km region centered at the cloud tops. It is in this directly-heated region we expect to find a nearly inviscid Hadley circulation, and observations at these levels are consistent with the presence of a thermally direct cell (Schubert, 1983). However, this nearly inviscid circulation at the cloud levels cannot extend to the surface, for its strength is directly proportional to the differential solar heating and inversely proportional to the radiative relaxation time τ_R. While τ_R is of the order of 20 days in the cloud region, according to Pollack and Young (1975), it is 10^4 days at the surface (the Newtonian cooling approximation is invalid at these levels, but this suffices to illustrate the situation). For a circulation to be nearly inviscid near the Venus surface, the condition $\tau_D/\tau_\nu \ll 1$ would require $\nu \ll 0.1$ m^2 s^{-1}. Thus, unless there is evidence that ν is at least an order smaller than 0.1 m^2 s^{-1}, we expect diffusion to be important in the deep Venus atmosphere.

The results of Sections 4 and 5 also show that an axisymmetric circulation forced by thermal sources alone cannot by itself generate the planet-wide zonal superrotation found on Venus; we must therefore invoke eddy momentum sources to account for its presence. When the rapid rotation was first observed at the Venus cloud tops, discussion centered upon the problem of maintaining the high speed zonal winds against the inevitable downward diffusion of momentum by small-scale motions. The maintenance requirement was therefore for a mechanism capable of transferring momentum from the lower to the upper atmosphere to compensate for the diffusive loss. The result of Section 6 drastically changes this requirement in the directly-heated region, where diffusion is likely of minor importance in comparison with transport by the mean circulation; the required eddy source pattern in this region is consequently dictated by dynamic advection of zonal momentum rather than by diffusion.

For an atmosphere whose absolute angular momentum decreases with latitude, we have shown that the required source distribution depends on both the sense of the meridional circulation χ and the sign of the vertical zonal wind shear U. This is illustrated schematically in Fig. 7 for the case of a direct circulation ($\chi > 0$) and a positive shear ($U > 0$). In Fig. 7b the plus (+) sign indicates a region of source of momentum, and the minus (−) sign a sink region. At the equator as an air parcel rises into a region of higher values of M, it must gain angular momentum; hence the need for a local source of momentum. Given that M decreases with latitude, poleward flows in the upper branch must be associated with momentum sinks and return flows in the lower branch with sources. Similarly, downward motions in the presence of a positive wind shear must be maintained by momentum sinks. By the same argument, a latitudinal jet in the zonal wind profile corresponds to local intensifications of sources/sinks in meridional branches, implying a vertical eddy transport process that takes on the horizontal scale of the jet itself.

In the example shown in Fig. 7, the mean meridional circulation transports angular momentum upward and poleward, while the eddy processes do precisely the opposite; this agrees with Gierasch's (1975) analysis that a direct cell must transport momentum upward if the absolute angular momentum decreases with latitude. Gierasch's analysis is for an atmosphere in stratified solid-body rotation, whose vertical profile is supported by down-gradient vertical diffusion, but does not give the eddy source pattern. In this study we have analyzed the diagnostic relation between the required E-P flux divergence field and the specified zonal wind and solar heating distributions. Moreover, our estimates suggest that diffusion

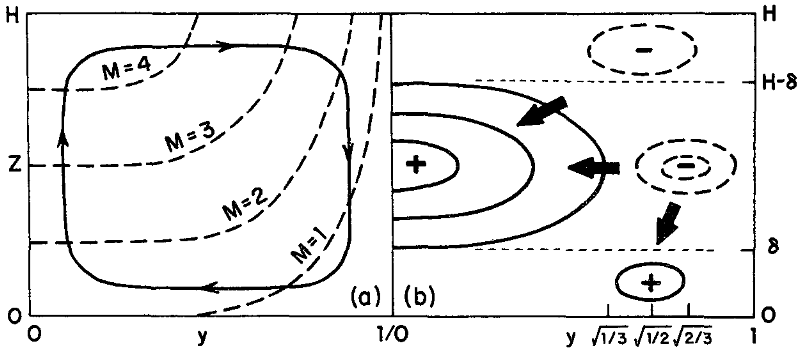

FIG. 7. Schematic representation of the momentum source pattern of Fig. 5. In panel (a) the solid line represents a meridional streamfunction contour and broken lines give constant absolute angular momentum contours (in arbitrary units). In panel (b) the depth δ corresponds to the depth associated with intense meridional flows. The symbol plus (+) means that momentum must be supplied to maintain the circulation; i.e., eddy fluxes in this region must be convergent in order to offset the local transport by the mean circulation. (In general, the actual signs depend upon the signs of U and χ.) The arrows indicate plausible momentum fluxes, though the choice is not unique. If a zonal jet is present as in Fig. 6, there would appear a set of concentrated sources and sinks in the upper and lower layers, which take on the meridional scale of the jet.

is probably too weak to support the observed wind shear in the cloud region. Rossow (1983) also found in his GCM calculations that vertical transports by the mean circulation cannot be compensated by diffusive transport.

We have, however, an additional clue suggesting that the Gierasch model may not be appropriate for the region above the cloud base. Recent observations show that over most latitudes in the direct-heated region, zonal winds generally increase with height in the cloud region, but decrease above the cloud tops (Schubert, 1983). Given that the directly-heated region is centered about the cloud tops, we might expect a situation similar to the one depicted in Fig. 8. Since the direct circulation still transports momentum upward, the required vertical eddy transport is downward, which is *down-gradient* only where $\partial u/\partial z > 0$, and is *up-gradient* where the wind decreases with height. The source pattern shown in Fig. 8b appears to be more complicated than what can be expected from a strictly up-or-down-the-gradient eddy transport mechanism. The implication is that asymmetric motions rather than small-scale diffusion are maintaining the observed circulation in this region.

For a more realistic zonal wind profile the analysis of Section 6 also suggests that an indirect meridional cell may appear below the cloud base, where there is a layer of strong vertical wind shear; this would result in additional layers of sources and sinks. Our conclusion is that the observed superrotation on Venus can be supported by alternating layers of eddy sources and sinks in the vertical. Fels and Lindzen (1974) have shown that thermally-excited gravity waves can produce alternating regions of mean-flow acceleration and deceleration. Our results suggest that in the presence of mean meridional motions, such layered sources and sinks can lead to changes of sign in the vertical wind shear rather than in the wind itself. A possible mechanism for providing the required eddy transport in the Venus atmosphere is the thermal

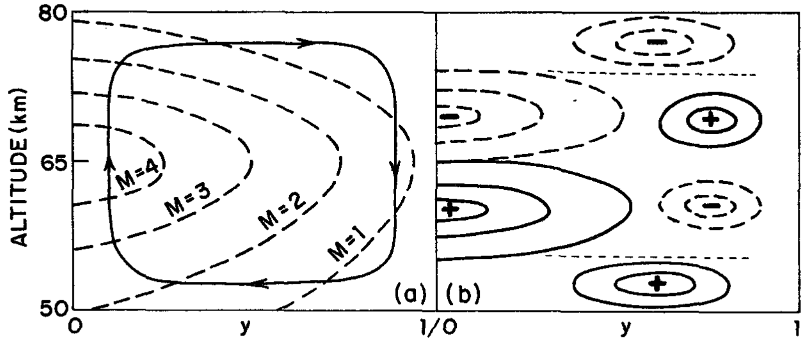

FIG. 8. Schematic representation of the situation in Venus' cloud-top region; symbols as for Fig. 7.

tide. We shall return to these questions in Hou and Goody (1985), in which we present numerical solutions to a perfect-gas, primitive-equation model atmosphere. Our numerical calculations fully confirm the simple model conclusions presented here, but they lack the ability to demonstrate parametric dependences. The simple model proves to be particularly revealing in that respect.

Acknowledgments. I am much indebted to R. M. Goody for revising an earlier version of this paper and for his encouragement. I wish to thank R. S. Lindzen for sponsoring a portion of this work as a part of my doctoral thesis. I also thank B. F. Farrell, I. M. Held, R. Kahn, E. S. Sarachik, and E. K. Schneider for helpful comments. This work was supported by NASA Grant NGL-22-007-228.

APPENDIX

Numerical Values Adopted for the Venus Atmosphere

Quantity	Symbol	Value	Note
Planetary radius	a	6.0×10^6 m	1
Rotation rate (planetary)	Ω	3.0×10^{-7} s^{-1}	1
Rotation rate (cloud level)	Ω	1.8×10^{-5} s^{-1}	3
Gravitational acceleration	g	8.6 m s^{-2}	1
Depth of the heated region	H	3.0×10^4 m	2
Cloud-top velocity (65 km)	u	120 m s^{-1}	3
Cloud-base velocity (45 km)	u	50 m s^{-1}	3
Fractional equator-to-pole difference in θ_E	Δ_h	⅓	4
Global-mean potential temperature	θ_0	10^3 K	4
Radiative relaxation time	τ_R	1.6×10^6 s	5
Static stability parameter	S	0.03	6
Surface drag coefficient	C	0.1 m s^{-1}	7
Vertical diffusion coefficient	ν	10 m^2 s^{-1}	8

Based on these values we obtain the following estimates for the external thermal Rossby number R: $R = 2.7 \times 10^4$ based on the planetary rotation rate and $R = 7.2$ based on the 4-day rotation at the cloud level.

Notes:
1. Colin (1983).
2. Most solar radiation is absorbed between 45 and 75 km (Tomasko, 1983). The cloud top is approximately 65 km above the surface.
3. Schubert (1983).
4. This is a nominal value for the Venus troposphere. According to grey radiative model calculations by the author, this parameter can be as large as $^7/_{10}$ in the cloud-top region. However, the thermal Rossby number based on these values of Δ_h corresponds to the nonrotating limit, so that the results are not sensitive to this parameter. It enters mainly as a scaling factor.
5. Calculated by Pollack and Young (1975) at 65 km.
6. For a mean potential temperature of 10^3 K between 45 and 75 km, this corresponds to 30 K difference, or 1 K km^{-1}. Observations suggest that this parameter can be as large as 0.15 in this region, but in Sections 4 and 6 this parameter enters only as a scaling factor.
7. According to Soviet measurements the drag coefficient at the surface of Venus is much smaller than 0.1 m s^{-1}; but this is only used as an upper-bound value for estimating the size of $\bar{F}$ in Fig. 5e.
8. This is a typical value required to explain chemical data between 60 and 80 km (von Zahn *et al.*, 1983).

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