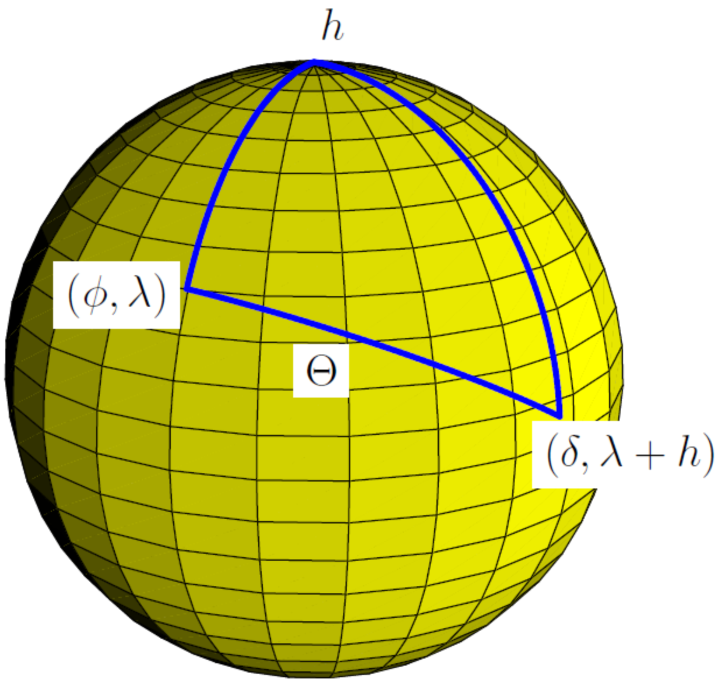
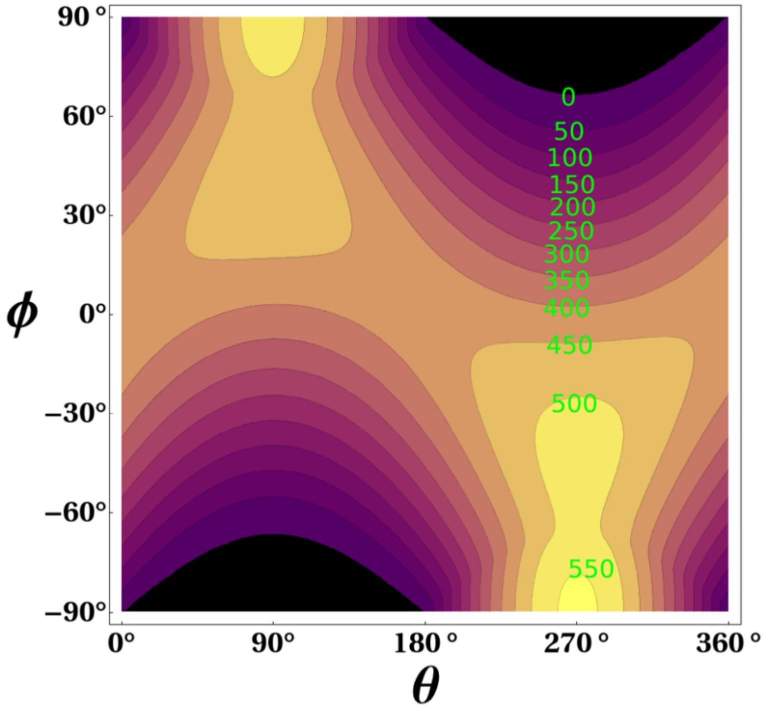
Distribution of insolation at the top of the atmosphere

The theory for the **distribution of solar radiation at the top of the atmosphere** concerns how the solar irradiance (the power of solar radiation per unit area) at the top of the atmosphere is determined by the sphericity and orbital parameters of Earth. The theory could be applied to any monodirectional beam of radiation incident onto a rotating sphere, but is most usually applied to sunlight, and in particular for application in numerical weather prediction, and theory for the seasons and the ice ages. The last application is known as Milankovitch cycles.



Spherical triangle for application of the spherical law of cosines for the calculation the solar zenith angle Θ for observer at latitude φ and longitude λ from knowledge of the hour angle h and solar declination δ. (δ is latitude of subsolar point, and h is relative longitude of subsolar point).

The derivation of distribution is based on a fundamental identity from spherical trigonometry, the spherical law of cosines:



$\overline{Q}^{\text{day}}$, the theoretical daily-average insolation at the top of the atmosphere. The calculation assumed conditions appropriate for 2000 A.D.: a solar constant of S_0 = 1367 W m^{-2}, obliquity of ε = 23.4398°, longitude of perihelion of ϖ = 282.895°, eccentricity e = 0.016704. Contour labels (green) are in units of W m^{-2}.

$$\cos(c) = \cos(a)\cos(b) + \sin(a)\sin(b)\cos(C)$$

where a, b and c are arc lengths, in radians, of the sides of a spherical triangle. C is the angle in the vertex opposite the side which has arc length c. Applied to the calculation of solar zenith angle Θ, we equate the following for use in the spherical law of cosines:

$$C = h$$

$$c = \Theta$$

$$a = \tfrac{1}{2}\pi - \phi$$

$$b = \tfrac{1}{2}\pi - \delta$$

$$\cos(\Theta) = \sin(\phi)\sin(\delta) + \cos(\phi)\cos(\delta)\cos(h)$$

The distance of Earth from the sun can be denoted R_E, and the mean distance can be denoted R_0, which is very close to 1 AU. The insolation onto a plane normal to the solar radiation, at a distance 1 AU from the sun, is the solar constant, denoted S_0. The solar flux density (insolation) onto a plane tangent to the sphere of the Earth, but above the bulk of the atmosphere (elevation 100 km or greater) is:

$$Q = S_o \frac{R_o^2}{R_E^2} \cos(\Theta) \text{ when } \cos(\Theta) > 0$$

and

$$Q = 0 \text{ when } \cos(\Theta) \le 0$$

The average of Q over a day is the average of Q over one rotation, or the hour angle progressing from $h = \pi$ to $h = -\pi$:

$$\overline{Q}^{\text{day}} = -\frac{1}{2\pi}\int_{\pi}^{-\pi} Q\,dh$$

Let h_0 be the hour angle when Q becomes positive. This could occur at sunrise when $\Theta = \frac{1}{2}\pi$, or for h_0 as a solution of

$$\sin(\phi)\sin(\delta) + \cos(\phi)\cos(\delta)\cos(h_o) = 0$$

or

$$\cos(h_o) = -\tan(\phi)\tan(\delta)$$

If tan(φ)tan(δ) > 1, then the sun does not set and the sun is already risen at $h = \pi$, so $h_o = \pi$. If tan(φ)tan(δ) < −1, the sun does not rise and $\overline{Q}^{\text{day}} = 0$.

$\frac{R_o^2}{R_E^2}$ is nearly constant over the course of a day, and can be taken outside the integral

$$\int_{\pi}^{-\pi} Q\,dh = \int_{h_o}^{-h_o} Q\,dh = S_o\frac{R_o^2}{R_E^2}\int_{h_o}^{-h_o}\cos(\Theta)\,dh$$

$$\int_{\pi}^{-\pi} Q\,dh = S_o\frac{R_o^2}{R_E^2}\left[h\sin(\phi)\sin(\delta) + \cos(\phi)\cos(\delta)\sin(h)\right]_{h=h_o}^{h=-h_o}$$

$$\int_{\pi}^{-\pi} Q\,dh = -2S_o\frac{R_o^2}{R_E^2}\left[h_o\sin(\phi)\sin(\delta) + \cos(\phi)\cos(\delta)\sin(h_o)\right]$$

$$\overline{Q}^{\text{day}} = \frac{S_o}{\pi}\frac{R_o^2}{R_E^2}\left[h_o\sin(\phi)\sin(\delta) + \cos(\phi)\cos(\delta)\sin(h_o)\right]$$

Let θ be the conventional polar angle describing a planetary orbit. For convenience, let $\theta = 0$ at the vernal equinox. The declination δ as a function of orbital position is

$$\sin\delta = \sin\varepsilon\ \sin(\theta - \varpi)$$

where ε is the obliquity. The conventional longitude of perihelion ϖ is defined relative to the vernal equinox, so for the elliptical orbit:

$$R_E = \frac{R_o}{1 + e\cos(\theta - \varpi)}$$

or

$$\frac{R_o}{R_E} = 1 + e\cos(\theta - \varpi)$$

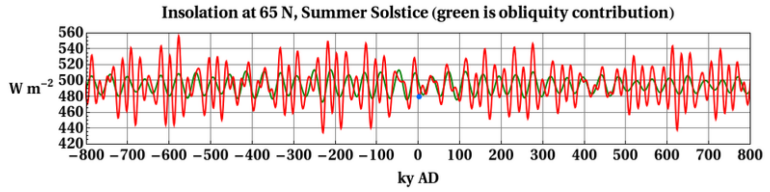
With knowledge of ϖ, ε and e from astrodynamical calculations [5] and S_o from a consensus of observations or theory, $\overline{Q}^{\text{day}}$ can be calculated for any latitude φ and θ. Note that because of the elliptical orbit, and as a simple consequence of Kepler's second law, θ does not progress exactly uniformly with time. Nevertheless, $\theta = 0°$ is exactly the time of the vernal equinox, $\theta = 90°$ is exactly the time of the summer solstice, $\theta = 180°$ is exactly the time of the autumnal equinox and $\theta = 270°$ is exactly the time of the winter solstice.

Application to Milankovitch cycles

Obtaining a time series for a $\overline{Q}^{\mathrm{day}}$ for a particular time of year, and particular latitude, is a useful application in the theory of Milankovitch cycles. For example, at the summer solstice, the declination δ is simply equal to the obliquity ε. The distance from the sun is

$$\frac{R_o}{R_E} = 1 + e\cos(\theta - \varpi) = 1 + e\cos(\tfrac{\pi}{2} - \varpi) = 1 + e\sin(\varpi)$$

For this summer solstice calculation, the role of the elliptical orbit is entirely contained within the important product $e\sin(\varpi)$, which is known as the **precession index**, the variation of which dominates the variations in insolation at 65 N when eccentricity is large. For the next 100,000 years, with variations in eccentricity being relatively small, variations in obliquity will be dominant.



Past and future of daily average insolation at top of the atmosphere on the day of the summer solstice, at 65 N latitude. The green curve is with eccentricity *e* hypothetically set to 0. The red curve uses the actual (predicted) value of *e*. Blue dot is current conditions, at 2 ky A.D.

Applications

In spacecraft design and planetology, it is the primary variable affecting equilibrium temperature.

In construction, insolation is an important consideration when designing a building for a particular climate. It is one of the most important climate variables for human comfort and building energy efficiency.[6]

The projection effect can be used in architecture to design buildings that are cool in summer and warm in winter, by providing large vertical windows on the equator-facing side of the building (the south face in the northern hemisphere, or the north face in the southern hemisphere): this maximizes insolation in the winter months when the Sun is low in the sky, and minimizes it in the summer when the noonday Sun is high in the sky. (The Sun's north/south path through the sky spans 47 degrees through the year).

Insolation figures are used as an input to worksheets to size solar power systems for the location where they will be installed.[7] This can be misleading since insolation figures assume the panels are parallel with the ground, when in fact they are almost always mounted at an angle[8] to face towards the sun. This gives inaccurately low estimates for winter.[9] The figures can be obtained from an insolation map or by city or region from insolation tables that were generated with historical data over the last 30–50 years. Photovoltaic panels are rated under standard conditions to determine the Wp rating (watts peak),[10] which can then be used with the insolation of a region to determine the expected output, along with other factors such as tilt, tracking and shading (which can be included to create the installed Wp rating).[11] Insolation values range from 800 to 950 kWh/(kWp·y) in Norway to up to 2,900 in Australia.

In the fields of civil engineering and hydrology, numerical models of snowmelt runoff use observations of insolation. This permits estimation of the rate at which water is released from a melting snowpack. Field measurement is accomplished using a pyranometer.