EPS 231

HW # 8 (Glacial 1)

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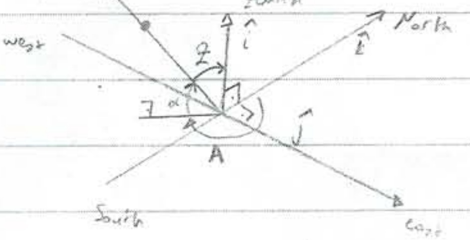
1. The energy flux ~~at~~ at a point on the surface of the Earth will be given by: $\tilde{W} = S_0 \left(\frac{a}{r}\right)^2 \cos(z)$, where a is the semi major axis, r is the Earth - Sun distance, S_0 is the flux at $r = a$, z is the solar zenith angle (see figure)



We know from the two-body problem that r is given by:

$$r = \frac{a(1-e^2)}{1 + e\cos(\lambda - \bar{\omega})}$$

, where e is the eccentricity defined as $e = \frac{\sqrt{a^2 - b^2}}{a}$, λ is the longitude of the Earth orbit around the sun, and $\bar{\omega}$ is the precession (It is basically an integration constant for that case). So W becomes:

$$\tilde{W} = S_0 \frac{(1 + e\cos(\lambda - \bar{\omega}))^2}{(1-e^2)^2} \cos(z)$$

Now, from the figure above we can see that the Sun's position according to an observer in Earth (in the observers system of coordinates) is give in terms of z and the solar azimuth angle A: $\bar{S} = \cos z\, \hat{i} + \sin z \cdot \sin A\, \hat{j} + \sin z \cos A\, \hat{k}$

Now, the Sun's position in a system of reference that has the origin at the center of the Earth is shown in the next figure

$$= \frac{S_0 \left(1+e\cos(\lambda-\bar{\omega})\right)^2}{\pi (1-e^2)^2} \left(H_0 \sin\delta \sin\phi + \cos\delta \cos\phi \sin H_0 \right) \quad ①$$

We have to note that in order to reach this equation we have assumed that there is sunset and sunrise for the given latitude ϕ ($|\cos H_0| = |\tan\phi \tan\delta| \leq 1$).

- If $\tan\delta \tan\phi > 1$, then there is no sunset: This yields that for $|\phi| \geq \frac{\pi}{2} - |\delta|$ $\left\{ \begin{array}{l} \phi>0, \text{ if } \delta>0 \\ \phi<0, \text{ if } \delta<0 \end{array} \right\}$ there is no sunset

and $W = \frac{S_0 \left(1+e\cos(\lambda-\bar{\omega})\right)^2}{\pi(1-e^2)^2} \left(2\pi \cdot \sin\delta \sin\phi \right) = \frac{S_0 \left(1+e\cos(\lambda-\bar{\omega})\right)^2 \sin\delta \sin\phi}{\pi (1-e^2)^2}$

- If $\tan\delta \tan\phi < -1$, then there is no sunrise. This yields that for $|\phi| \geq \frac{\pi}{2} - \delta$ $\left\{ \begin{array}{l} \phi<0, \text{ if } \delta>0 \\ \phi>0, \text{ if } \delta<0 \end{array} \right\}$ there is no sunrise and $W=0$

a) The annual average of W is: $\bar{W} = \frac{1}{T}\int_0^T W\,dt$, where $T = 365d$

In order to integrate we have to express $\lambda = \lambda(t)$. We know from the two-body problem that the angular momentum of the system is conserved: $L = r^2\dot{\theta} = r^2\dot{\lambda}$. In this case it has the constant value: $L = \frac{2\pi}{T} a^2 \sqrt{1-e^2}$

So: $\frac{d\lambda}{dt} = \frac{L}{r^2} = \frac{2\pi}{T}\sqrt{1-e^2}\,\frac{a^2}{r^2} = \frac{2\pi}{T}\sqrt{1-e^2}\,\frac{\left(1+e\cos(\lambda-\bar{\omega})\right)^2}{(1-e^2)^2}$ ②

and the angular velocity varies with λ with maximum velocity at the perihelion and minimum at the aphelion.

$$\bar{W} = \frac{1}{T}\int_0^T W\,dt = \frac{1}{T}\int_0^{2\pi} \frac{1}{\frac{d\lambda}{dt}} W\,d\lambda = \frac{1}{2\pi}\int_0^{2\pi} \frac{(1-e^2)^2}{\sqrt{1-e^2}} \frac{W}{(1+e\cos(\lambda-\bar{\omega}))^2}\,d\lambda =$$

$$= \frac{S_0}{2\pi^2\sqrt{1-e^2}} \int_0^{2\pi} \left(H_0 \sin\delta \sin\varphi + \cos\delta\cos\varphi \sin H_0 \right) d\lambda$$

Now since H_0, δ, φ do not depend on $\bar{\omega}$, $\bar{W}$ is independent of any changes in $\bar{\omega}$.

b) We have to calculate $\langle \bar{W} \rangle = \frac{1}{T\pi}\int_0^T \int_{-\pi/2}^{\pi/2} W\,d\varphi\,dt$

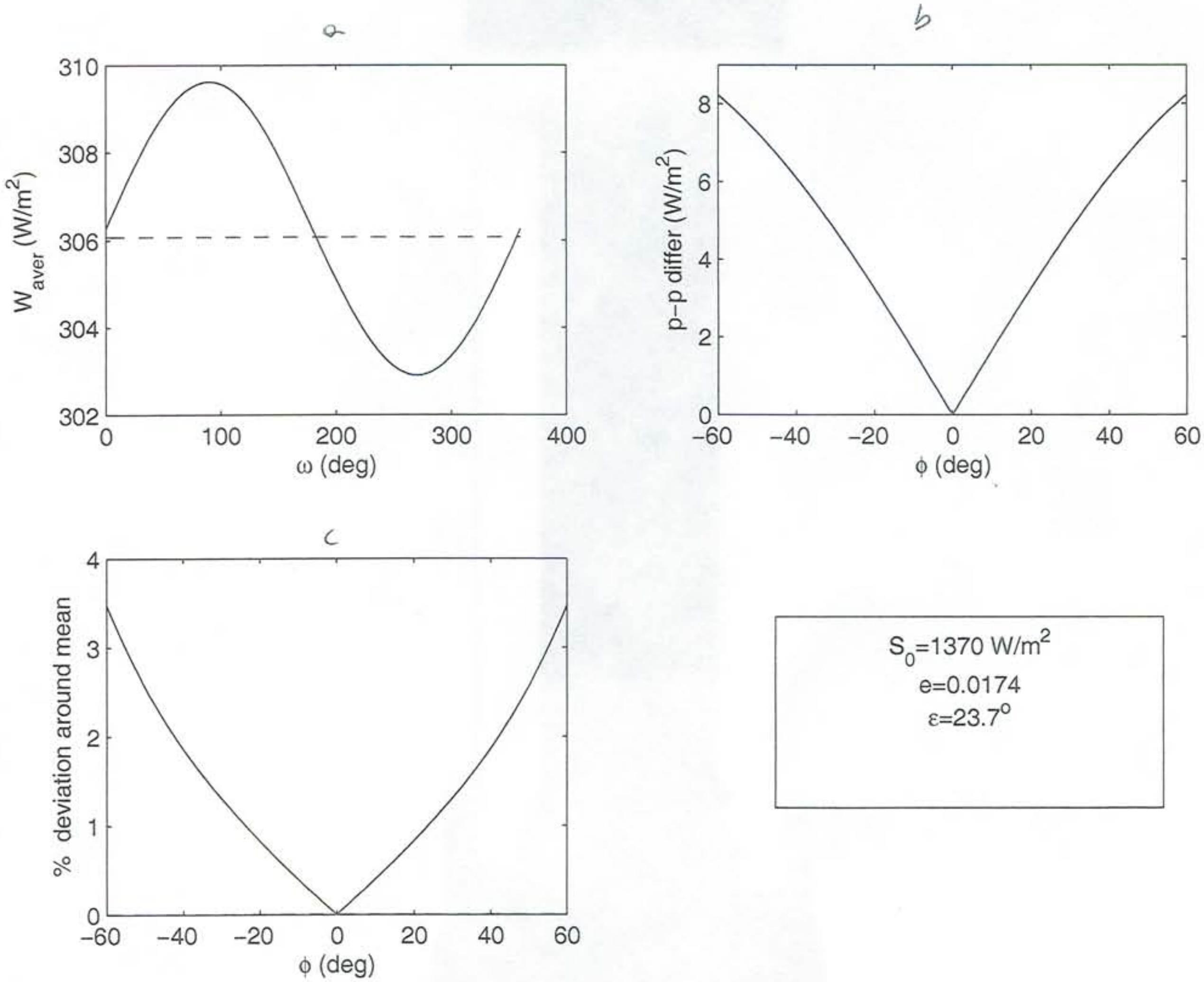
Since the constant velocity gave poor results for the previous case, we will use use equation ② to calculate $\lambda(t)$. The annual and global average as a function of ε is shown in Figure 2. We can see that it varies by $4\ W/m^2$ which corresponds to 1% deviation. The deviation is probably due to the fact that we neglected variation of λ during the one day integration.

c) $$\langle \bar{W} \rangle = \frac{1}{T\pi}\int_0^T \int_{-\pi/2}^{\pi/2} W\,d\varphi\,dt = \frac{1}{T\pi}\int_0^{2\pi}\int_{-\pi/2}^{\pi/2} \frac{W}{\frac{d\lambda}{dt}}\,d\varphi\,d\lambda =$$

$$= \frac{S_0}{2\pi^2\sqrt{1-e^2}} \int_0^{2\pi}\int_{-\pi/2}^{\pi/2} \frac{(1-e^2)^2}{(1+e\cos(\lambda-\bar{\omega}))^2}\, W\,d\varphi\,d\lambda =$$

$$= \frac{S_0}{2\pi^2\sqrt{1-e^2}} \int_0^{2\pi}\int_{-\pi/2}^{\pi/2} \left(H_0 \sin\delta\sin\varphi + \cos\delta\cos\varphi\sin H_0 \right) d\lambda\,d\varphi$$

Since H_0, δ, φ do not depend on e $\rightarrow$ $\langle \bar{W} \rangle \propto (1-e^2)^{-1/2}$



a
310
308
306
304
302
0
100
200
300
400
W_aver (W/m^2)
ω (deg)
b
8
6
4
2
0
−60
−40
−20
0
20
40
60
p−p differ (W/m^2)
φ (deg)
c
4
3
2
1
0
−60
−40
−20
0
20
40
60
% deviation around mean
φ (deg)
S_0=1370 W/m^2
e=0.0174
ε=23.7°

Figure 1

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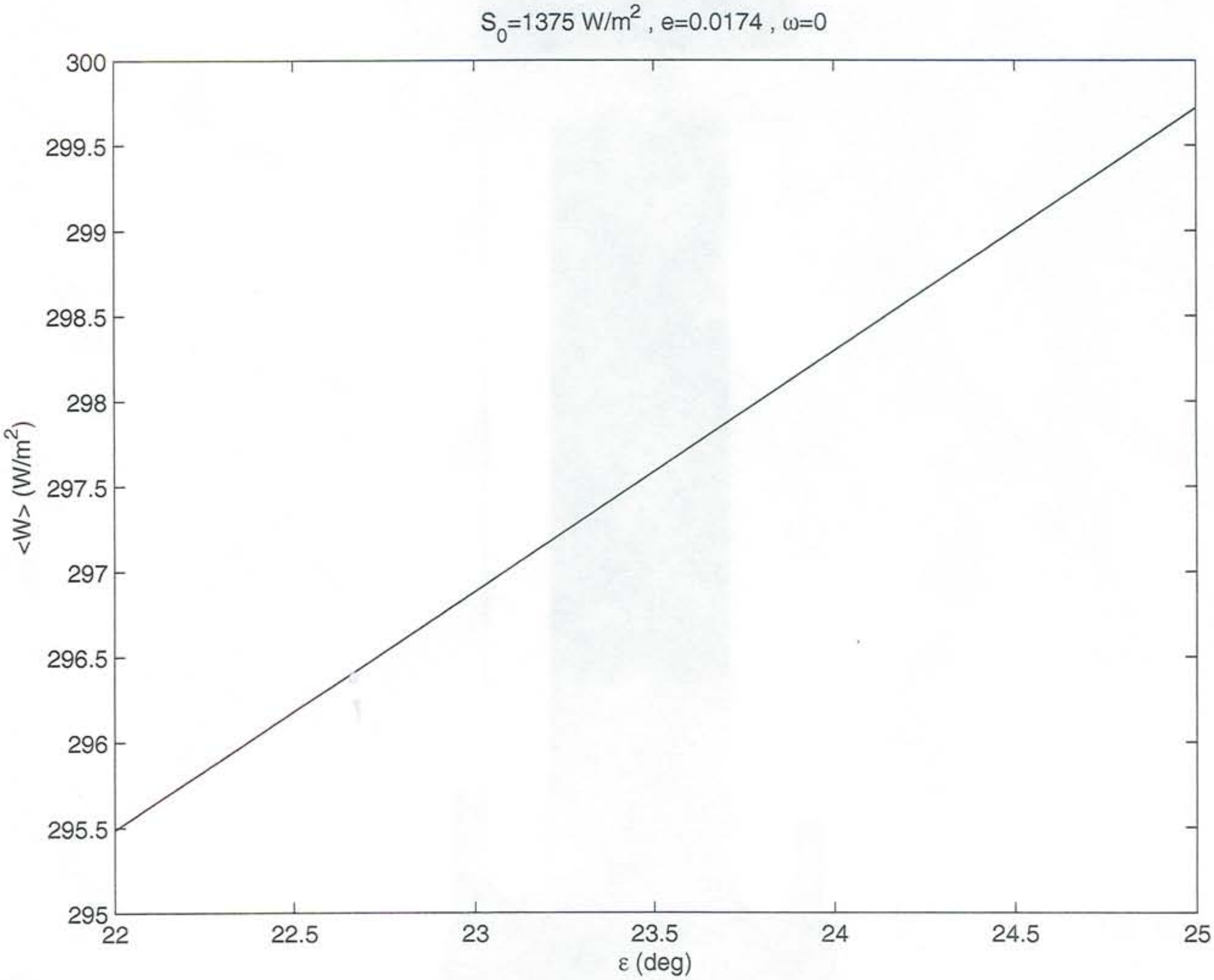
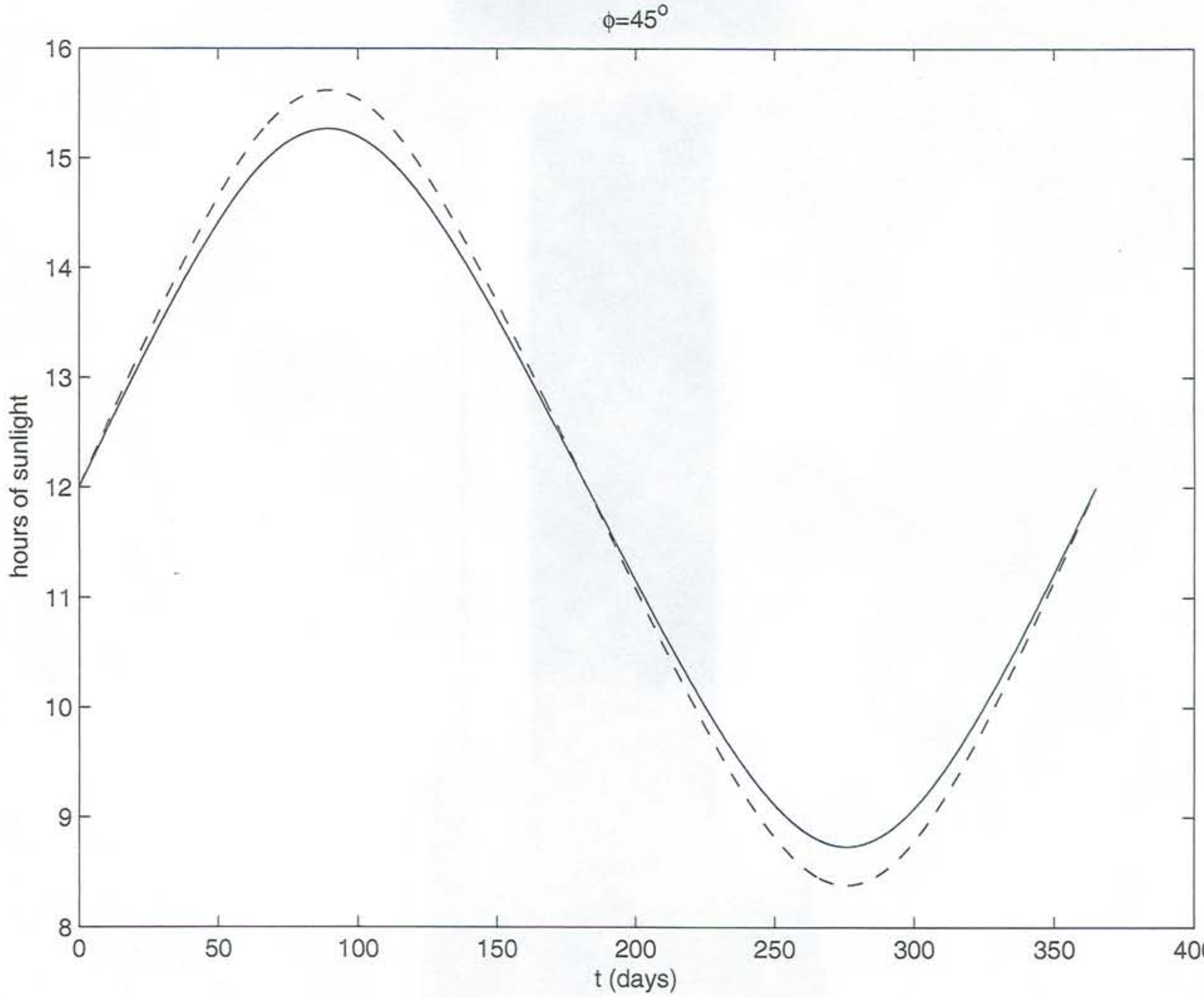


S_0=1375 W/m^2 , e=0.0174 , ω=0
<W> (W/m^2)
ε (deg)
300
299.5
299
298.5
298
297.5
297
296.5
296
295.5
295
22
22.5
23
23.5
24
24.5
25

Figure 2



ϕ=45°
16
15
14
13
12
11
10
9
8
hours of sunlight
0
50
100
150
200
250
300
350
400
t (days)

Figure 3