Figure 3.6) of the particle that is now at point P therefore loops outward and forward again as the body passes by.

The streamlines and path lines of Figure 3.6 can be visualized in an experiment by suspending aluminum or other reflecting materials on the fluid surface, illuminated by a source of light. Suppose that the entire fluid is covered with such particles, and a *brief* time exposure is made. The photograph then shows short dashes, which indicate the instantaneous directions of particle movement. Smooth curves drawn through these dashes constitute the instantaneous streamlines. Now suppose that only a few particles are introduced, and that they are photographed with the shutter open for a *long* time. Then the photograph shows the paths of a few individual particles, that is, their path lines.

A *streak line* is another concept in flow visualization experiments. It is defined as the current location of all fluid particles that have passed through a fixed spatial point at a succession of previous times. It is determined by injecting dye or smoke at a fixed point for an interval of time. In steady flow the streamlines, path lines, and streak lines all coincide.

5. *Reference Frame and Streamline Pattern*

A flow that is steady in one reference frame is not necessarily so in another. Consider the flow past a ship moving at a steady velocity **U**, with the frame of reference (that is, the observer) attached to the river bank (Figure 3.7a). To this observer the local flow characteristics appear to change with time, and thus appear to be unsteady. If, on the other hand, the observer is standing on the ship, the flow pattern is steady (Figure 3.7b). The steady flow pattern can be obtained from the unsteady pattern of Figure 3.7a by superposing on the latter a velocity **U** to the right. This causes the ship to come to a halt and the river to move with velocity **U** at infinity. It follows that any velocity vector **u** in Figure 3.7b is obtained by adding the corresponding velocity vector **u**′ of Figure 3.7a and the free stream velocity vector **U**.

6. *Linear Strain Rate*

A study of the dynamics of fluid flows involves determination of the forces on an element, which depend on the amount and nature of its deformation, or strain. The deformation of a fluid is similar to that of a solid, where one defines normal strain as

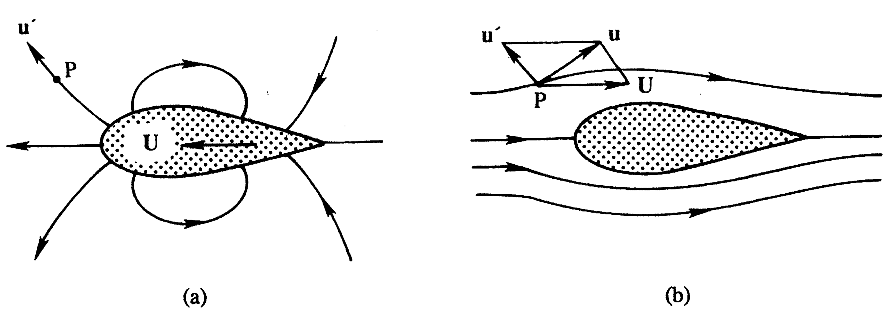

Figure 3.7 Flow past a ship with respect to two observers: (a) observer on river bank; (b) observer on ship.

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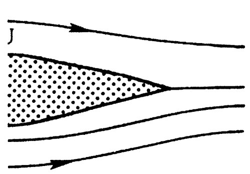

(b)

er bank; (b) observer on ship.

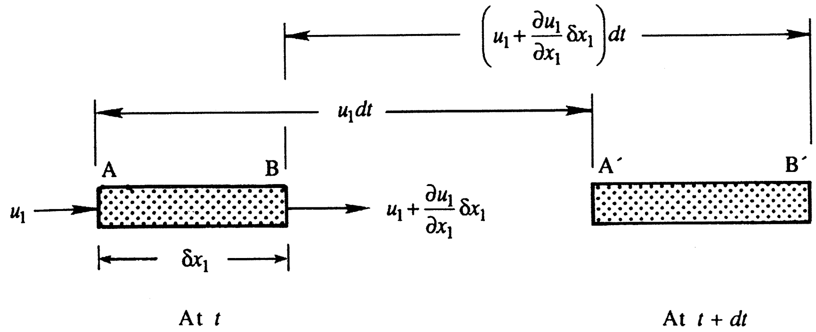

Figure 3.8 Linear strain rate. Here, $A'B' = AB + BB' - AA'$.

the change in length per unit length of a linear element, and shear strain as change of a 90° angle. Analogous quantities are defined in a fluid flow, the basic difference being that one defines strain *rates* in a fluid because it *continues* to deform.

Consider first the *linear* or *normal strain rate* of a fluid element in the x_1 direction (Figure 3.8). The rate of change of length per unit length is

$$\frac{1}{\delta x_1}\frac{D}{Dt}(\delta x_1) = \frac{1}{dt}\frac{A'B' - AB}{AB}$$
$$= \frac{1}{dt}\frac{1}{\delta x_1}\left[\delta x_1 + \frac{\partial u_1}{\partial x_1}\delta x_1\, dt - \delta x_1\right] = \frac{\partial u_1}{\partial x_1}.$$

The material derivative symbol *D/Dt* has been used because we have implicitly *followed* a fluid particle. In general, the linear strain rate in the α direction is

$$\frac{\partial u_\alpha}{\partial x_\alpha}, \tag{3.8}$$

where *no summation* over the repeated index α is implied. Greek symbols such as α and β are commonly used when the summation convention is violated.

The sum of the linear strain rates in the three mutually orthogonal directions gives the rate of change of volume per unit volume, called the *volumetric strain rate* (also called the *bulk strain rate*). To see this, consider a fluid element of sides δx_1, δx_2, and δx_3. Defining $\delta\mathcal{V} \equiv \delta x_1\, \delta x_2\, \delta x_3$, the volumetric strain rate is

$$\frac{1}{\delta\mathcal{V}}\frac{D}{Dt}(\delta\mathcal{V}) = \frac{1}{\delta x_1\, \delta x_2\, \delta x_3}\frac{D}{Dt}(\delta x_1\, \delta x_2\, \delta x_3),$$
$$= \frac{1}{\delta x_1}\frac{D}{Dt}(\delta x_1) + \frac{1}{\delta x_2}\frac{D}{Dt}(\delta x_2) + \frac{1}{\delta x_3}\frac{D}{Dt}(\delta x_3),$$

that is,

$$\frac{1}{\delta\mathcal{V}}\frac{D}{Dt}(\delta\mathcal{V}) = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = \frac{\partial u_i}{\partial x_i}. \tag{3.9}$$

The quantity $\partial u_i/\partial x_i$ is the sum of the diagonal terms of the velocity gradient tensor $\partial u_i/\partial x_j$. As a scalar, it is invariant with respect to rotation of coordinates. Equation (3.9) will be used later in deriving the law of conservation of mass.

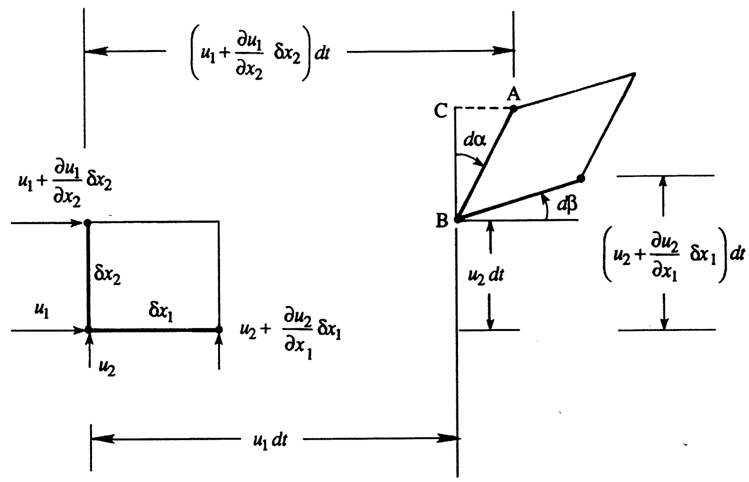

Figure 3.9 Deformation of a fluid element. Here, $d\alpha = CA/CB$; a similar expression represents $d\beta$.

7. *Shear Strain Rate*

In addition to undergoing normal strain rates, a fluid element may also simply deform in *shape*. The shear strain rate of an element is defined as the rate of decrease of the angle formed by two mutually perpendicular lines on the element. The shear strain so calculated depends on the orientation of the line pair. Figure 3.9 shows the position of an element with sides parallel to the coordinate axes at time t, and its subsequent position at $t + dt$. The rate of shear strain is

$$\frac{d\alpha + d\beta}{dt} = \frac{1}{dt}\left\{\frac{1}{\delta x_2}\left(\frac{\partial u_1}{\partial x_2}\delta x_2\, dt\right) + \frac{1}{\delta x_1}\left(\frac{\partial u_2}{\partial x_1}\delta x_1\, dt\right)\right\}$$

$$= \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}. \qquad (3.10)$$

An examination of Eqs. (3.8) and (3.10) shows that we can describe the deformation of a fluid element in terms of the *strain rate tensor*

$$\boxed{e_{ij} \equiv \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right).} \qquad (3.11)$$

The diagonal terms of **e** are the normal strain rates given in (3.8), and the off-diagonal terms are *half* the shear strain rates given in (3.10). Obviously the strain rate tensor is symmetric as $e_{ij} = e_{ji}$.

8. *Vorticity and Circulation*

Fluid lines oriented along different directions rotate by different amounts. To define the rotation rate unambiguously, two mutually perpendicular lines are taken, and the

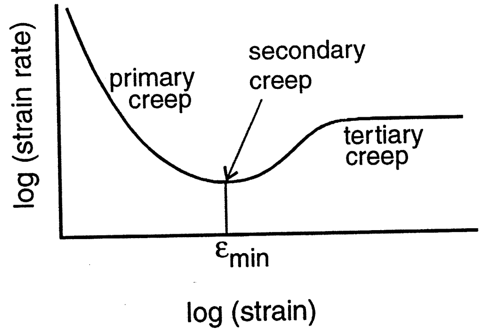

Figure 2.5. Typical creep curve for glacier ice.

2.3 CONSTITUTIVE RELATION

The experiments of Rigsby (1958) suggest that the deformation of ice is practically independent of the hydrostatic pressure, provided that the difference between the ice temperature and the pressure-melting temperature is kept constant. Then the constitutive relation, linking the rate of deformation to applied stress, must be independent of the hydrostatic pressure. To achieve this, deviatoric stresses, rather than full stresses, are used to describe the rheological properties of glacier ice.

Let σ_{ij} denote the components of the full stress tensor. The hydrostatic pressure is the sum of the three normal stresses ($P = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$). The full stress tensor is now split as follows

$$\begin{pmatrix} \sigma_{xx} & \sigma_{yx} & \sigma_{zx} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{zy} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{pmatrix} =$$

$$= \begin{pmatrix} (2\sigma_{xx} - \sigma_{yy} - \sigma_{zz})/3 & \sigma_{yx} & \sigma_{zx} \\ \sigma_{xy} & (2\sigma_{yy} - \sigma_{xx} - \sigma_{zz})/3 & \sigma_{zy} \\ \sigma_{xz} & \sigma_{yz} & (2\sigma_{zz} - \sigma_{xx} - \sigma_{yy})/3 \end{pmatrix} +$$

$$+ \begin{pmatrix} (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})/3 & 0 & 0 \\ 0 & (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})/3 & 0 \\ 0 & 0 & (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})/3 \end{pmatrix}. \quad (2.3.1)$$

The first tensor on the right-hand side is called the stress deviator. In short, its components are given by

$$\tau_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\,\delta_{ij}\,, \tag{2.3.2}$$

where δ_{ij} denotes the Kronecker delta ($\delta_{ij} = 1$ if $i = j$, and $\delta_{ij} = 0$ if $i \neq j$), and summation over repeat indices is implied. Thus, a deviatoric normal stress is defined as the full normal stress minus the hydrostatic pressure. The shear stresses are unaffected by this partitioning.

A few words on the notation commonly used in the glaciological literature may be helpful. In the literature, the components of stress are variably denoted by σ_{ij} or by τ_{ij}. Sometimes, σ_{ij} is reserved for normal stress ($i = j$) and τ_{ij} for shear stress ($i \neq j$). Deviatoric stresses are commonly denoted by a prime, e.g. σ'_{ij}. Here, in an effort to keep the notation simple and consistent, σ_{ij} refers to full stresses, and τ_{ij} to deviatoric stresses.

To arrive at the constitutive relation most often used in glaciology, the analysis of Nye (1953) is followed; a mathematically more rigorous approach can be found in Hutter (1983). The first assumption is that ice is incompressible and that it remains isotropic throughout the flow. In that case, the principal axes of the strain rate tensor must be parallel to those of the stress tensor, and the components of strain rate are proportional to the components of deviatoric stress. That is

$$\dot{\varepsilon}_{ij} = \chi\,\tau_{ij}\,, \tag{2.3.3}$$

where χ is a positive scalar function that may depend on the entire stress state, temperature of the ice, and perhaps other factors as well.

The next step is to determine the form of the function χ. Because the constitutive response of glacier ice must be independent of the particular coordinate system chosen, χ must be a function of the invariants of the strain rate and stress deviator tensors. Invariants of a tensor are scalar quantities whose values are not affected by a transformation of the coordinate system. Both the stress and strain rate tensors are of second order, and therefore have three invariants each. For the deviatoric stress tensor, the invariants are

$$I_1 = \tau_{xx} + \tau_{yy} + \tau_{zz}\,, \tag{2.3.4}$$

$$I_2 = \tau_{xx}^2 + \tau_{yy}^2 + \tau_{zz}^2 + 2\left(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2\right), \tag{2.3.5}$$

$$I_3 = \det\left(\tau_{ij}\right). \tag{2.3.6}$$

From the definition of deviatoric stresses (2.3.2) it follows immediately that the first invariant is zero. By adopting equation (2.3.3), it also follows that the first invariant of the strain-rate tensor is zero (which is true for any incompressible material). It is now postulated that the rheology of glacier ice is independent of the third invariants of the strain-rate and deviatoric stress tensor. Thus

$$\chi = \chi\left[I_2(\tau), I_2(\dot{\varepsilon})\right]. \tag{2.3.7}$$

Rather than using the second invariants, the quantities called effective strain rate, $\dot{\varepsilon}_e$, and effective stress, τ_e, are used. These are defined through

$$2\dot{\varepsilon}_e^2 = \dot{\varepsilon}_{xx}^2 + \dot{\varepsilon}_{yy}^2 + \dot{\varepsilon}_{zz}^2 + 2\left(\dot{\varepsilon}_{xy}^2 + \dot{\varepsilon}_{xz}^2 + \dot{\varepsilon}_{yz}^2\right), \qquad (2.3.8)$$

$$2\tau_e^2 = \tau_{xx}^2 + \tau_{yy}^2 + \tau_{zz}^2 + 2\left(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2\right). \qquad (2.3.9)$$

Based on the laboratory experiments of Glen (1952), Nye (1953) suggested the following relation between the effective stress and effective strain rate

$$\dot{\varepsilon}_e = A\tau_e^n , \qquad (2.3.10)$$

where A and n are the flow parameters. From (2.3.3) and the definitions of effective stress and strain rate, it follows that

$$\chi = A\tau_e^{n-1} , \qquad (2.3.11)$$

and hence

$$\dot{\varepsilon}_{ij} = A\tau_e^{n-1}\tau_{ij} . \qquad (2.3.12)$$

Equation (2.3.12) is called Nye's generalization of Glen's law, or Glen's law for short. Virtually all modelling of the flow of glaciers is based on this constitutive relation.

For some applications, the inverse formulation of equation (2.3.12) is needed, for example when estimating stresses from measured strain rates. Combining (2.3.12) and (2.3.10) gives

$$\dot{\varepsilon}_{ij} = A\left(\frac{\dot{\varepsilon}_e}{A}\right)^{1-1/n}\tau_{ij} , \qquad (2.3.13)$$

and

$$\tau_{ij} = B\dot{\varepsilon}_e^{1/n-1}\,\dot{\varepsilon}_{ij} , \qquad (2.3.14)$$

with

$$B = A^{-1/n} . \qquad (2.3.15)$$

The flow parameters, A and B, are dependent on many factors, most notably the temperature of the ice. This is discussed further in Section 2.4.

2.4 FLOW PARAMETERS FOR GLACIER MODELLING

The constitutive relation (2.3.12), or the inverse formulation (2.3.14), contains two parameters, namely the factor n in the exponent, and the rate factor A or B (the latter two are related through equation 2.3.15). Many laboratory and field measurements have been conducted to determine the values of these parameters.

Now consider the situation when the surface of the slab is not parallel to its base (Nye, 1952b). Let α, β be the slopes, taken as positive and assumed small. Take x- and z-axes as in Fig. 11.2; AB and CD are perpendicular to the base and distance δx apart. Consider the equilibrium of an element $ABCD$ of unit thickness in the direction normal to the xz-plane. It is assumed that the normal pressure on AB is given, to a good approximation, by the hydrostatic head, which increases from zero at B to $\rho g h$ at A, where $AB = h$. The normal force on AB is thus $\frac{1}{2}\rho g h^2$. The normal force on CD is therefore $\frac{1}{2}\rho g h^2 + d/dx\,(\frac{1}{2}\rho g h^2)\,\delta x$. The difference between the two is $\rho g h (dh/dx)\,\delta x$, acting uphill. Other forces parallel to the base are the component of the weight $(\rho g h \sin\beta)\delta x$ downhill, and $\tau_b \delta x$ uphill, where τ_b is the basal shear stress. Thus for equilibrium

$$-\rho g h \frac{dh}{dx}\delta x + (\rho g h \sin\beta)\,\delta x - \tau_b\,\delta x = 0.$$

But $dh/dx = \beta - \alpha$ and $\sin\beta = \beta$ for small angles. Thus

$$\tau_b = \rho g h \alpha.$$

Therefore, provided that the slopes are small, τ_b is the same as if the slab had parallel sides of slope α.

Thus the shear stress at the bed is determined by the surface slope. Ice should flow in the direction of maximum surface slope even though the bed slopes in the opposite direction. This agrees with observation. Glacial valleys are often "over-deepened" at some distance from the terminus.

It follows that flow-lines can be determined from a contour map of the ice surface, provided that "slope" is interpreted as the average value over distances of several times the ice thickness. Small-scale features such as hummocks have no effect on the flow.

SURFACE PROFILES OF ICE SHEETS

Profile Equations

We consider a steady-state ice sheet on a horizontal bed. Figure 11.3 represents a cross-section and shows the coordinate system. The total width is $2L$, and the thickness is h in general and H on the centre-line. The ice sheet is pictured as a long ridge perpendicular to the plane of the diagram.

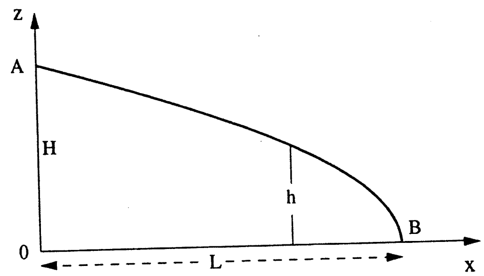

FIG. 11.3. Coordinate system for ice sheet.

Because the base is horizontal, the surface slope (taken to be positive) is $-dh/dx$ and, for small slopes, Eq. 1 shows that the shear stress at the base is

$$\tau_b = -\rho g h \frac{dh}{dx} . \tag{3}$$

Here ρ is density (assumed constant) and g is the acceleration due to gravity.

The simplest case is to treat ice as a perfectly-plastic material; the ice thickness adjusts itself so that at every point τ_b is equal to the yield stress τ_o. In this case, Eq. 3 can be integrated to give the equation of the surface profile

$$h^2 = \frac{2\tau_o}{\rho g}(L - x), \tag{4}$$

which is a parabola. This equation also applies to a perfectly-plastic ice sheet on a horizontal base with (1) a circular plan, if L is the radius and (2) a plan of irregular shape if $L - x$ is the distance from the edge measured along a flow-line (Nye, 1952a). The thickness at the centre is $H = (2\tau_o L/\rho g)^{1/2}$. As a first check, this formula, with $\tau_o = 100$ kPa was applied to central Greenland. It predicts that $H = 3150$ m compared with a true value of about 3200 m.

The best choice for τ_o is uncertain. Although basal shear stresses in alpine valley glaciers usually lie between 50 and 150 kPa, a mean value of 100 kPa is too high for most ice sheets. Reported values vary from 0 to 100 kPa with a mean of perhaps 50 kPa. With this value, Eq. 4 can be written

$$h = 3.4(L - x)^{1/2}, \tag{5}$$

with h, L, and x in metres. However, some of the southern lobes of the Laurentide Ice Sheet in North America had slopes appreciably smaller than those of present-day ice sheets (Mathews, 1974). They apparently had beds of deformable sediment with yield stresses significantly less than that of ice. (See Table 8.1.)

The next step is to replace the approximation of perfect plasticity by the flow relation for ice (Vialov, 1958). To derive the profile in this case we use the principle of mass conservation and an equation for the velocity averaged over the ice thickness derived in a subsequent section (Eq. 22). It is

$$\overline{u} = \frac{2A}{n+2} \tau_b{}^n h. \tag{6}$$

Here A and n are the flow parameters (Eq. 1 of Chapter 5), A being an average over the ice thickness. The ice is assumed to be frozen to the bed.

For a steady state, the mass of ice that accumulates on the surface between the crest and any point P must be equal to the mass flowing through a vertical section at P. For simplicity, assume that a uniform thickness of ice, c, is added to the surface in one year and that all the ablation is by calving at the edge. Then

$$cx = h\overline{u} = \frac{2A}{n+2} \left[-\rho g h \left(\frac{dh}{dx} \right) \right]^n h^2. \tag{7}$$

The solution of this differential equation gives the profile:

$$h^{2+2/n} = K \left(L^{1+1/n} - x^{1+1/n} \right), \tag{8}$$

with

$$K = \frac{2(n+2)^{1/n}}{\rho g} \left(\frac{c}{2A} \right)^{1/n}. \tag{9}$$

But $h = H$ when $x = 0$ so we can write the equation as

$$(h/H)^{2+2/n} + (x/L)^{1+1/n} = 1. \tag{10}$$

Perfect plasticity corresponds to $n \to \infty$, which reduces this equation to the parabola derived before.

Figure 11.4 shows that Eq. 10 provides a good fit to a profile surveyed in east Antarctica. However, agreement has to some extent been forced by using the known value of H, rather than estimating K from Eq. 9.

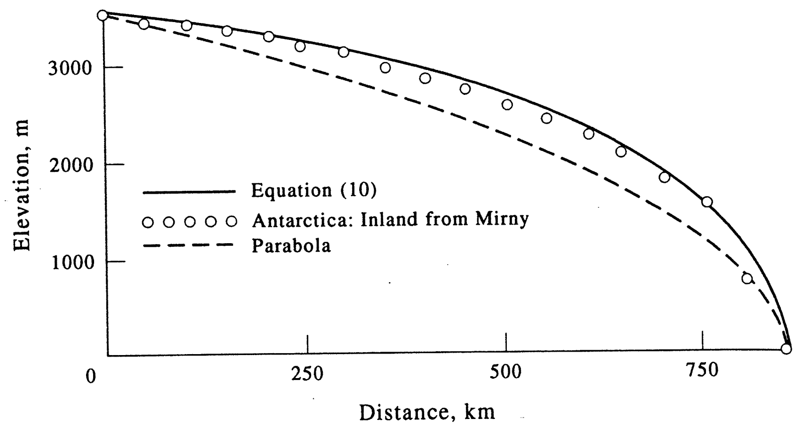

FIG. 11.4. Profile of Antarctic Ice Sheet inland from Mirny compared with theoretical profiles. Data from Vialov (1958).

Some important conclusions can be drawn from this analysis. Because $n \approx 3$, Eqs. 8 and 9 show that ice thickness is proportional to the eighth root of the accumulation rate. Thus the steady-state thickness of an ice sheet is insensitive to its mass balance. (With the approximation of perfect plasticity, the profile is independent of accumulation rate.) Equations 8 and 9 also show that h varies inversely as the eighth root of A, a parameter that depends on ice temperature. For the same accumulation rate, colder implies thicker ice. But the dependence is not very sensitive: a decrease in temperature from -10 to $-30°$C would increase h by about 35 per cent. This applies only if the base is below the melting point. Switching from a frozen to a melting base can cause a large change in the dimensions of a glacier or ice sheet. Note also that, if all the ablation is at the ice margin, Eq. 6 implies that τ_b increases steadily with increase of x.

These equations appear to describe the broad features of profiles along flow lines on ice masses with a wide range of sizes, ice temperatures, and accumulation and ablation rates. This supports the prediction that their shape is mainly determined by the plastic properties of ice. A single regularly-shaped dome is nevertheless a very crude picture of the ice sheets. The Greenland Ice Sheet is largely fringed by mountains that channel the flow into some twenty large outlet glaciers. In Antarctica, buried mountain ranges cause irregularities in the ice surface in the interior and, near the perimeter, much of the flow is channelled either by mountains or by deformable sediments beneath the ice.

Discussion of Assumptions

The derivation of the profile equations rested on certain assumptions, which I now discuss.

1. Equation 1 is valid when the difference between surface and bed slopes is small and stresses vary only slowly with distance so that longitudinal stress gradients can be neglected. The difference between surface and bed slopes is often large near the ice margin. Flow in this region has received little study, except for an analysis by Nye (1967) for the case of perfect plasticity. Near the centre of an ice sheet, the longitudinal stress predominates because the surface slope and basal shear stress tend to zero. (The slope of a parabolic profile is not zero at the centre so this profile cannot apply there.) Flow at an ice divide is discussed later in this chapter.

2. The base was assumed to be horizontal. Weertman (1961a) considered the effect, on the surface profile, of depression of an originally horizontal bed by the weight of the ice. If local isostatic equilibrium is established, a thickness h of ice will depress the bedrock by $h(\rho/\rho')$, where ρ and ρ' are the densities of ice and rock. The appropriate value of ρ' is that of the upper mantle, where the isostatic adjustment occurs. It is $3300\,\mathrm{kg\,m^{-3}}$ and so $\rho' = 3.7\rho$. For perfect plasticity, with yield stress τ_o, the surface profile is

$$h_1{}^2 = (1 - \rho/\rho')\,\frac{2\tau_o}{\rho g}(L - x), \tag{11}$$

where h_1 is height above the original horizontal bed. The ice thickness is $h_1\rho'/(\rho' - \rho)$.

3. An accumulation rate uniform over the whole ice sheet was assumed. Weertman (1961c) and Paterson (1972a) derived equations analogous to Eq. 10 for an ice sheet with uniform accumulation rate c from $x = 0$ to $x = R$, and uniform ablation rate a from there to the edge. For steady state, $cR = a(L - R)$. The profile is

$$0 \le x \le R \qquad \left(\frac{h}{H}\right)^{2+2/n} + \left(1 + \frac{c}{a}\right)^{1/n}\left(\frac{x}{L}\right)^{1+1/n} = 1 \tag{12}$$

$$R \le x \le L \qquad \left(\frac{h}{H}\right)^{2} = \left(1 + \frac{a}{c}\right)^{1/n+1}\left(1 - \frac{x}{L}\right). \tag{13}$$

TABLE 11.2. Balance velocities in ice sheet of parabolic profile (Total width = 2000 km; net mass balance = 150 mm ice)

Km from centre	0	100	200	400	600	800	900	950
Velocity ($\mathrm{m\,a^{-1}}$)	0	1.5	3.5	8	15	29	45	68

VELOCITIES IN "LAMINAR FLOW"

We now return to the parallel-sided slab model of a glacier and calculate how velocity varies with depth. Take axes as in Fig. 11.1. Let u be the x-component of velocity. We assume that the slab deforms in simple shear that is, τ_{xz} is the only non-zero stress component. The flow-lines are therefore parallel to the surface. Glaciologists call this *laminar flow.* It follows that the z-component of velocity is zero and so $\dot{\varepsilon}_x = \frac{1}{2} du/dz$. By the flow relation

$$\frac{1}{2}\frac{du}{dz} = A{\tau_{xz}}^n \tag{18}$$

and, by the same argument as used to derive Eq. 1, the shear stress at depth $(h-z)$ is

$$\tau_{xz} = \rho g (h-z) \sin\alpha. \tag{19}$$

Integration of Eq. 18 with this value of τ_{xz} gives the velocities

$$u_s - u(z) = \frac{2A}{n+1}(\rho g \sin\alpha)^n (h-z)^{n+1} \tag{20}$$

$$u_s - u_b = \frac{2A}{n+1}(\rho g \sin\alpha)^n h^{n+1}. \tag{21}$$

Here u_s and u_b are the velocities at the surface and base and u is the velocity at depth $(h-z)$. The present theory deals only with deformation within the ice and gives no information about the sliding velocity u_b. Equation 20 shows that the velocity decreases continuously as depth increases and, because $n \approx 3$, most of the decrease takes place in the layers near the bed.

The flow parameter A was treated as a constant in this analysis. In fact, it depends on temperature. The only approximately isothermal glaciers are the so-called temperate glaciers, in which the ice is at melting