PALEOCEANOGRAPHY, VOL. 16, NO. 2, PAGES 190-198, APRIL 2001

Stochastic Resonance in the North Atlantic

R.B. Alley
Environment Institute and Department of Geosciences, Pennsylvania State University, University Park, Pennsylvania

S. Anandakrishnan
Department of Geology, University of Alabama, Tuscaloosa, Alabama

P. Jung
Department of Physics and Astronomy, Ohio University, Athens, Ohio

Abstract. Large, abrupt changes between warm and cold modes of North Atlantic climate exhibit spectral power at ~1500 years, yet some climatic changes were linked to outburst floods and other events that are unlikely to have been truly periodic. We hypothesize that a weak periodic forcing has combined with "noise" from ice sheet-related events to cause the observed mode switches. This stochastic resonance hypothesis predicts a recurrence pattern between warmings that is distinct from the predictions of simple periodic and stochastic models and at least some other models. The ice isotopic data from central Greenland ice cores are consistent with the stochastic resonance hypothesis but not with other models we have tested. We thus support arguments for the existence of a periodicity of ~1500 years in the North Atlantic climate system and for the importance of ice sheet events in forcing North Atlantic changes.

1. Introduction

Many climate records covering the last 110,000 years and beyond are dominated by large, abrupt, widespread changes with millennial spacing (Figure 1) [e.g., *Johnsen et al.*, 1992; *Alley et al.*, 1993; *Taylor et al.*, 1997; *Mayewski et al.*, 1997; *Severinghaus et al.*, 1998; *McManus et al.*, 1999]. These changes are typically believed to be linked to switches in mode of operation of oceanic circulation focused on the north Atlantic [e.g., *Stocker et al.*, 1992] forced by periodic [e.g., *Broecker et al.*, 1990; *Sakai and Peltier*, 1995] or stochastic [e.g., *Weaver and Hughes*, 1994] processes.

Numerous observational and modeling studies [e.g., *Bryan*, 1986; *Broecker et al.*, 1990; *Stocker et al.*, 1992; *Sarnthein et al.*, 1994; *Rahmstorf*, 1995] support the existence of two modes of North Atlantic behavior characterized by vigorous and reduced transport of warm waters to high latitudes, with most time spent in one of these modes and comparatively rapid transitions between modes, especially on warmings. This view is not universally accepted but is quite widespread and well documented, and we adopt it here. (Below we note that the proposed third or Heinrich mode [*Sarnthein et al.*, 1994; *Alley and Clark*, 1999], with even greater reduction in northward flow of warm waters in the Atlantic, is muted in the Greenland ice isotopic records we consider and so does not figure in our analyses.)

Some coolings occurred immediately after occasional outburst floods, which probably freshened North Atlantic surface waters, slowed or shifted sites of deepwater formation, and reduced northward flow of warm waters [e.g., *Broecker et al.*, 1988; *Barber et al.*, 1999]. This suggests stochastic forcing because outburst floods depend not only on climate affecting ice marginal positions [*Licciardi et al.*, 1999] but also on details of bedrock-sill locations that almost certainly include random elements. However, strong spectral power in relevant paleoclimatic records at ~1450-1500 years [*Denton and Karlen*, 1973; *Keigwin and Jones*, 1989; *Bond et al.*, 1997; *Grootes and Stuiver*, 1997; *Mayewski et al.*, 1997; *Yiou et al.*, 1997] argues for periodic forcing.

Many analyses, including ours reported here, show that this millennial periodicity is a feature of the climate system and not the result of aliasing of the annual cycle as a result of the sampling or interpolation scheme adopted, as was suggested by *Wunsch* [2000]. The possibility of aliasing certainly should be borne in mind. However, an extensive demonstration that this possibility does not explain results from Greenland ice cores is given by *L. D. Meeker et al.* (Comment on "On sharp Spectral lines in the climate record and the milleniel peak" by C. Wunsch, submitted to Paleoceanography, 2000), and our results support theirs. Briefly, for the data we analyze here the upper part of the Greenland Ice Sheet Project 2 (GISP2) ice core was sampled to obtain approximately biannual resolution, but lower parts were sampled on the basis of depth interval with a variable number of years per sample depending on layer thinning and changes in snow accumulation rate. The nearby Greenland Ice Core Project (GRIP) core was sampled on the basis of a different depth interval. We have conducted several interpolations with different intervals, including nonintegral numbers of years. As shown below, all interpolations produce similar results, with warmings preferentially spaced ~1500 years apart in both older and younger portions of both

Paper number 2000PA000518
0883-8305/01/2000PA000518$12.00

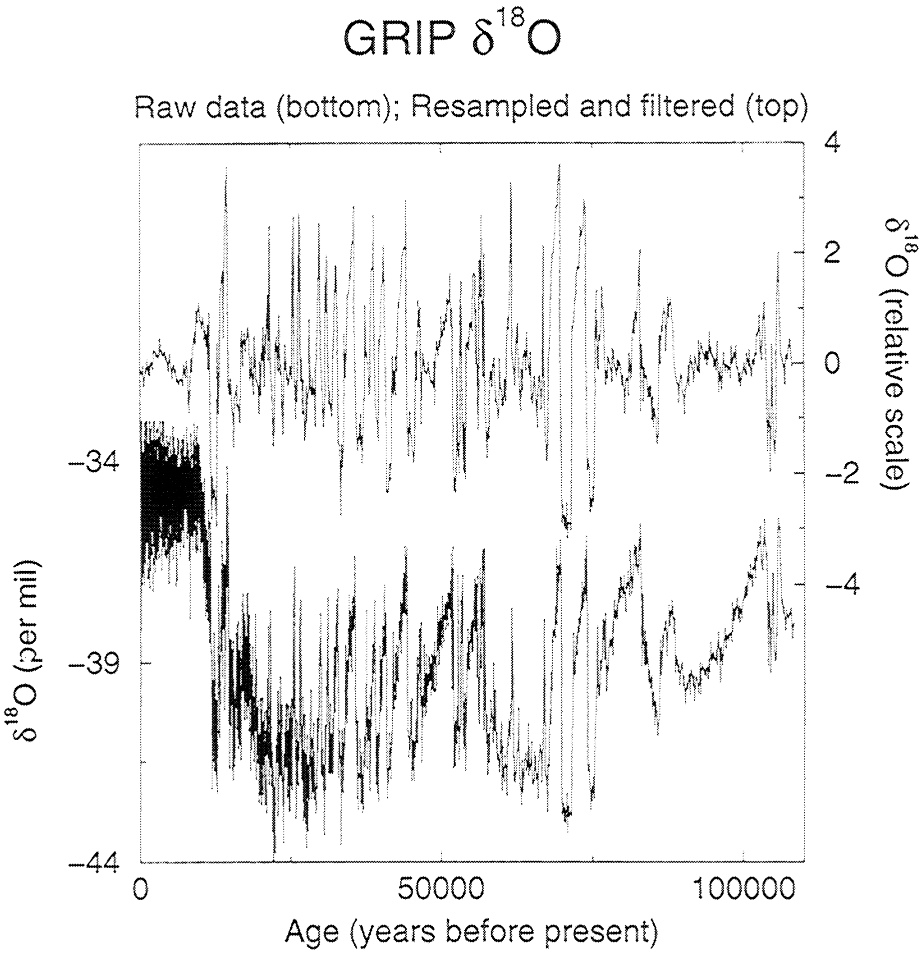

Figure 1. GRIP ice isotopic data (bottom) [*Johnsen et al.*, 1997], and the same data after resampling and high-pass filtering (top, where the isotopic values have been scaled to zero mean and extrema of +4 and -4). Data provided by the National Snow and Ice Data Center, University of Colorado at Boulder, and the World Data Center-A (WDC-A) for Paleoclimatology, National Geophysical Data Center, Boulder, Colorado, from The Greenland Summit Ice Cores CD-ROM (1997, see http://www.ngdc.noaa.gov/paleo/icecore/greenland/summit/).

GISP2 and GRIP data. Because significantly different interpolation schemes applied to data collected with significantly different sampling schemes all produce the same result, we reject the possibility that our signals have been produced by aliasing.

The clear evidence for periodic behavior in the one to two millennial or Dansgaard-Oeschger band of climate variability together with the evidence for meltwater outburst forcing of certain events argues either that there are different types of millennial events or, as we hypothesize, that both "signal" and "noise" have contributed to the observed climate change history. Systems in which a weak periodic signal combines with other signals (typically white noise or some other noise) to cause mode switches are often studied under the heading "stochastic resonance". The ideas of stochastic resonance were originally developed to model climate change [*Benzi et al.*, 1982] but have since found widespread application in diverse fields.

As reviewed by *Gammaitoni et al.* [1998], the simplest stochastically resonant system spends most of its time in one of two stable modes. A periodic forcing plus noise, which separately are too weak to cause frequent mode switches, typically combine to cause transitions when the phase of the periodic forcing is favorable. If a transition is missed, the next mode switch usually waits one or more periods T. Waiting times between successive transitions in a chosen direction can be measured, and a histogram can be formed showing how often different waiting times are observed. For a stochastically resonant system, such a recurrence histogram will exhibit peaks near T, $2T$, $3T$, ..., with the peak heights decreasing exponentially with increasing time between transitions and with few transitions near intermediate times $T/2$, $3T/2$, $5T/2$, ... (Figure 2). A truly periodic system will exhibit a single peak on such a plot at the period T, and a white noise system will exhibit an exponential decrease with increasing time but with no significant peaks.

We test the hypothesis that both signal and noise contributed to millennial oscillations in North Atlantic climate records using the recurrence histograms of different waiting times between mode switches.

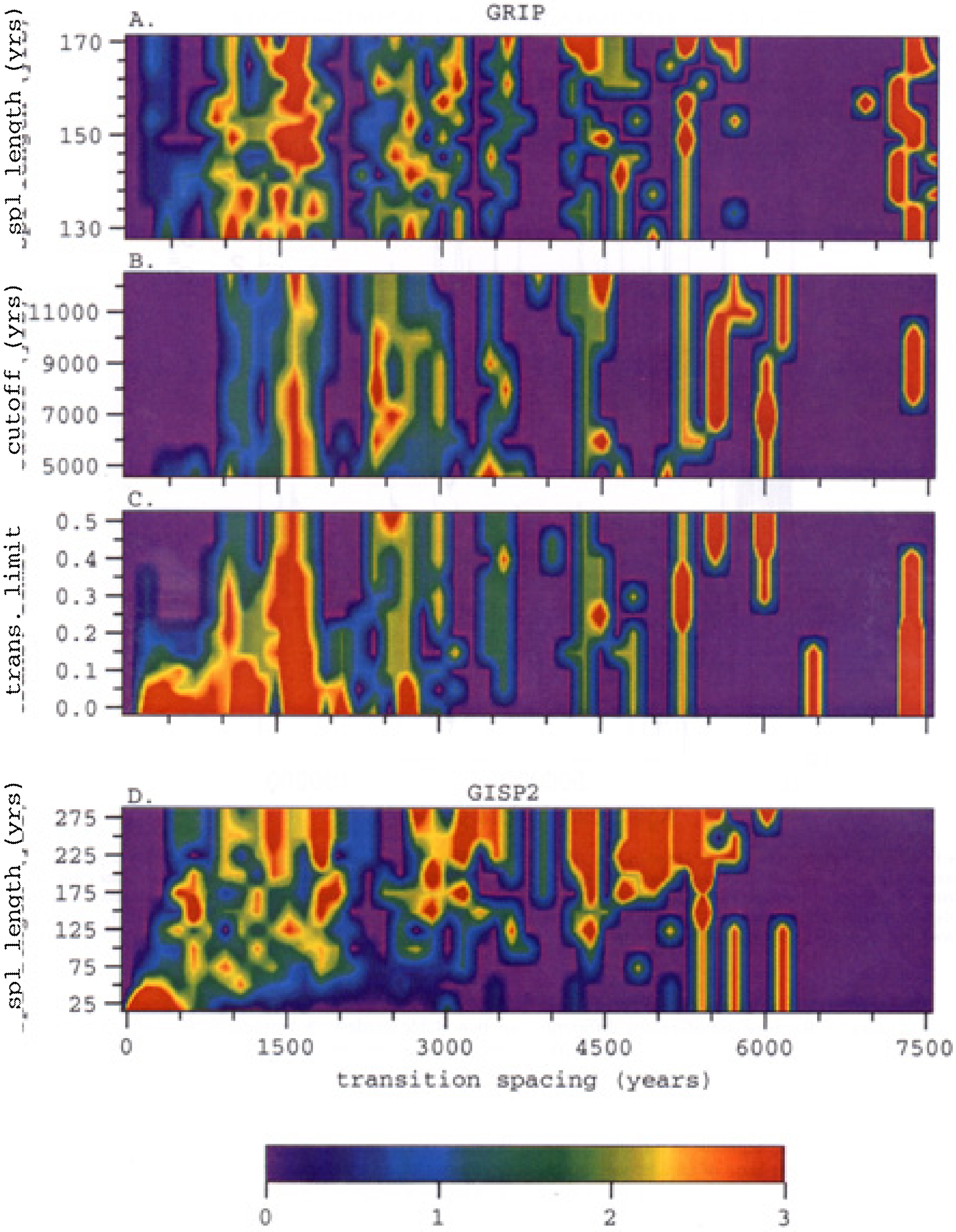

Plate 1. Sensitivity of transition times to variables in analysis, for (a-c) GRIP and (d) GISP2 [*Grootes and Stuiver*, 1997] ice isotopic data. Sample length (Plate 1a), high-pass filter cutoff (Plate 1b), and transition limit *d* (Plate 3c) for GRIP and sample length for GISP2 (Plate 1d) were varied around 150-year samples, high-pass cutoff of 7000 years, and *d*=0.25 or ~20% of the standard deviation of the resampled and high-pass-filtered data. For display, results were normalized by dividing by the expected number of occurrences from the exponentially decaying curve of a white noise process with a mean spacing between transitions of 3000 years, the observed time for GRIP data with a transition limit of between 0.35 and 0.4. "Hot" colors (red and orange) indicate favored transition spacings compared to "cool" colors (purple and blue). The normalization causes a stochastically resonant system with millennial-order mean waiting time to appear as vertical "hot" bands near times *T*, 2*T*, 3*T*, ..., separated by "cool" bands near *T/2, 3T/2, 5T/2*, ..., as observed. Data provided by the National Snow and Ice Data Center, University of Colorado at Boulder, and the WDC-A for Paleoclimatology, National Geophysical Data Center, Boulder, Colorado, from The Greenland Summit Ice Cores CD-ROM (1997, see http://www.ngdc.noaa.gov/paleo/icecore/greenland/summit/)

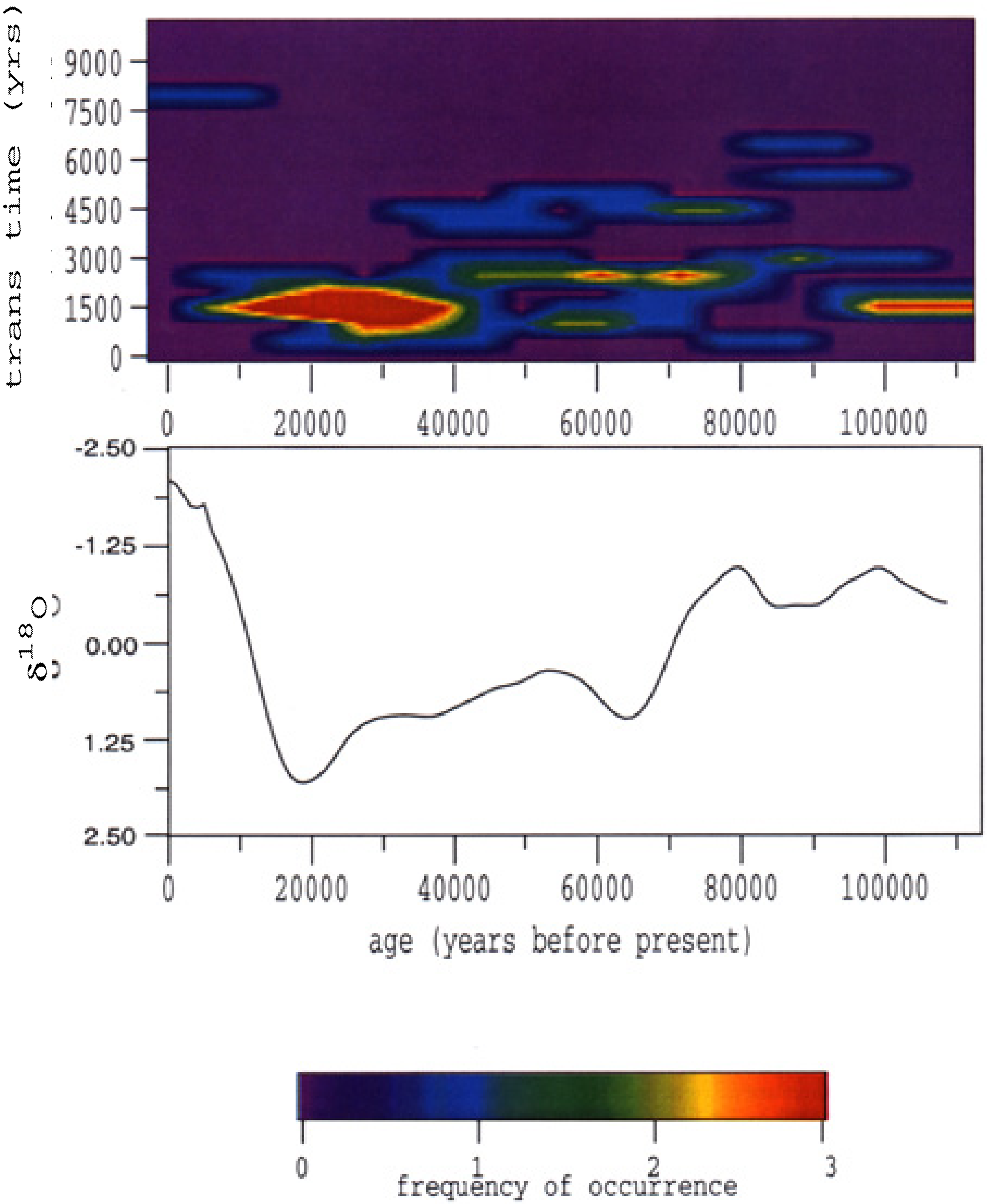

Plate 2. Evolution of the times between successive warmings in the GRIP ice-isotopic data with 150-year samples, *d*=0.25, and high-pass cutoff of 7000 years. Frequency of occurrence of a particular transition length in a 20,000-year-long window shifted 5000 years at a time is color-coded, with hot colors (red and orange) indicating frequent occurrence and cold colors (purple and blue) indicating rare occurrence in that interval. Also shown is the estimated ice volume on Earth based on the SPECMAP stack, in which lighter or more negative isotopic values of foraminifera shells indicate less ice and warmer conditions. Data provided from J. Imbrie, *et al.*, (1990), SPECMAP Archive 1, IGBP PAGES/World Data Center-A for Paleoclimatology Data Contribution Series 90-001, NOAA/NGDC Paleoclimatology Program, Boulder Colorado.

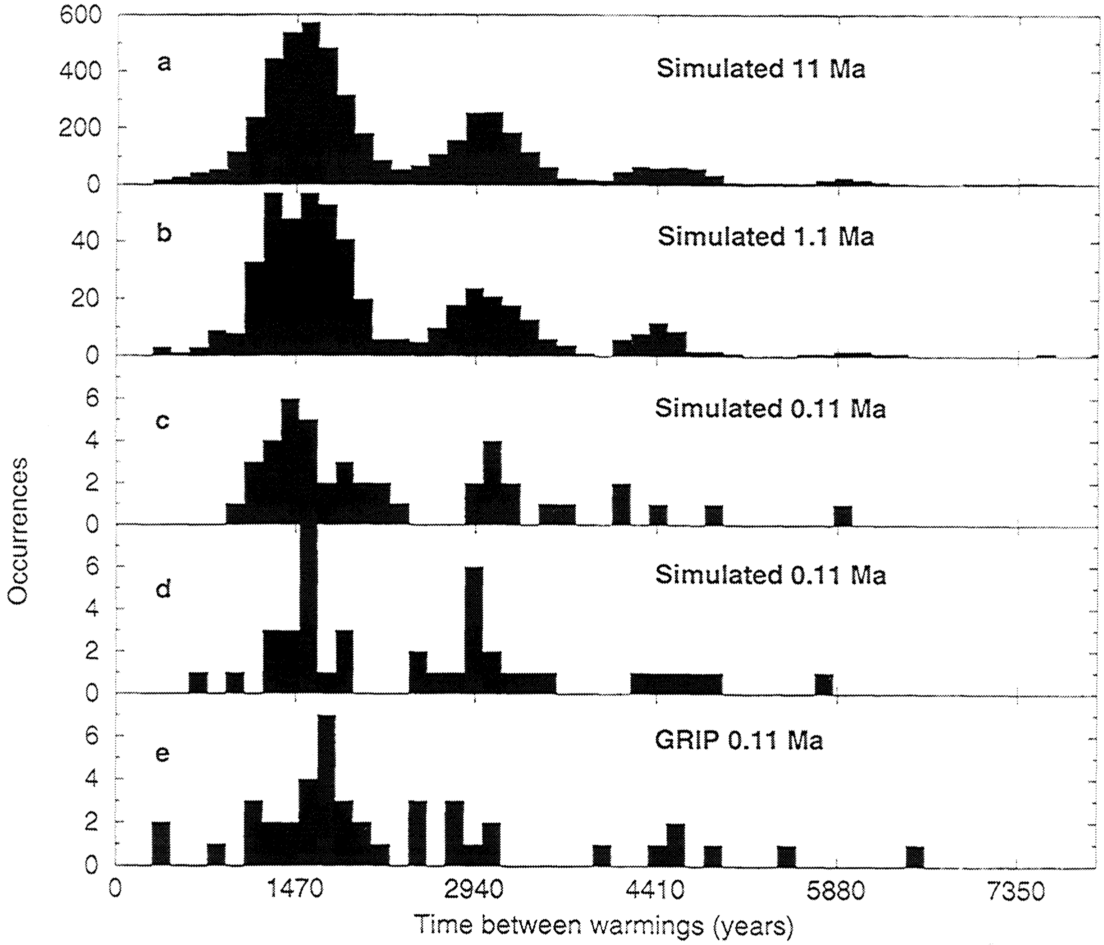

Figure 2. Number of occurrences of different waiting times between warmings for: (figure 2a) a simulated stochastically resonant time series 100 times longer than the GRIP data; (2b-d) subsets of that simulated time series that are 10x longer than GRIP (figure 2b) and of the same length as GRIP (figures 2c and 2d,); and (figure 2e) the observed GRIP ice-core data (150-year samples, high-pass cutoff of 7000 years, and d=0.25 or ~20% of the standard deviation of the resampled and high-pass-filtered data). The long-simulated series has the expected stochastically resonant pattern. The shorter subsets display deviations from this pattern related to the smaller sample size. The GRIP data are more similar to the long-simulated series than one of the two shorter subsets shown and than about one quarter of all GRIP-length subsets examined, based on chi-square testing.

2. Methods

Ice isotopic ratios are primary paleoclimatic indicators [e.g., *Cuffey et al.*, 1995; *Jouzel et al.*, 1997]. We analyzed the separately sampled and dated ice isotopic ratios from the GRIP [*Johnsen et al.*, 1997] (Figure 1a) and GISP2 [*Grootes and Stuiver*, 1997] central Greenland ice cores, using only the last 110,000 years undisturbed by ice flow [*Alley et al.*, 1995b; *Chappellaz et al.*, 1997]. We have analyzed other North Atlantic climate records from ocean sediments and ice\ and obtained broadly consistent results to those presented here. We focus in this paper on the GRIP and GISP2 ice isotopic records as having the most suitable combination of great length, accurate dating, sampling with high time resolution, and strong ties between proxy and climate. In subsequent work we hope to assess the spatial pattern of the results obtained here, but wish to establish the temporal pattern of variability for these "best" records first.

As noted above, layer thinning from ice flow causes older ice isotopic samples to span more years. We resampled the data in equal time increments at approximately the coarsest resolution present, by fitting a spline to the data, sampling the spline in subannual increments, and then combining those increments to the desired resolution. This resampling is sufficiently coarse that correction for diffusive smoothing of the paleoclimatic records is not needed [*Johnsen et al.*, 1997]. The resampling also serves as a low-pass filter to remove seasonal cycles, anomalous storms, and other situations in which a variable changed from "low" to "high" and back without recording a persistent mode switch of the North Atlantic climate system.

Carbon-dioxide and land-ice changes associated with the tens-of-millennial Milankovitch orbital variations caused millennial oscillations to reach different levels at different times. We removed variations in the orbital band, while preserving the timing and size of millennial events, using the high-pass zero-phase Butterworth filter implemented in the software package MATLAB (Figure 1b). The few-millennial Bond cycles, comprising successively colder millennial Dansgaard-Oeschger oscillations culminating in a Heinrich event (reviewed by *Alley and Clark*, [1999]), are sufficiently subtle that they do not dominate our transition analysis of Greenland ice isotopic ratios; we obtain similar results whether the high-pass cutoff is longer or shorter than the typical Bond cycle length. We thus focus on the millennial Dansgaard-Oeschger oscillations.

Asymmetry in these Dansgaard-Oeschger oscillations, with cooling in a few steps and warming in one larger step, causes the times of coolings to depend on the threshold chosen to define a cooling, whereas times of warmings are less sensitive to the threshold chosen. Thus, we chose to analyze time intervals between warmings. Analyzing only coolings produces similar results, and analyzing all transitions produces results broadly consistent with the analysis of only warmings.

We defined a warming transition to have occurred when a paleoclimatic variable rose above its mean value by more than a specified threshold level d after falling more than d below the mean. We calculated waiting times between successive warmings, and formed histograms of how often different waiting times occurred. We then assessed sensitivity of these recurrence histograms to the three main parameters in the analysis: threshold d; resampling interval; and the high-pass level separating orbital from millennial changes. We varied one of these parameters at a time about a "standard" calculation with 150-year samples, high-pass cutoff of 7000 years, and d~20% of the standard deviation of the resampled and high-pass-filtered data (d=0.25, with the filtered data arbitrarily scaled to fall between +4 and -4). High sensitivity to small variations in these parameters would cast doubt on our results; insensitivity to variations in these parameters would suggest that the results are robust.

To assess significance of the results, we used chi-square tests to compare the observed recurrence histograms to results from a synthetic time series formed by adding random noise (using the **randn** function in MATLAB) to a sine wave with 1470-year period, sampled every 150 years to match the GRIP data. The threshold level d for the synthetic data was set so that the mean waiting time between warmings matches that observed from the ice cores. We calculated an 11-million-year synthetic time series, or 100 times as long as the ice core records, to approximate an ensemble average.

3. Results

Figure 2 compares the histograms of number of occurrences of various waiting times between warmings for the 0.11-million-year GRIP record, the 11-million-year synthetic time series, and one 1.1-million-year and two 0.11-million-year subsets of that synthetic time series. The 11-million-year synthetic data show the stochastic resonance pattern of histogram peaks at T, $2T$, $3T$, ..., with peak height decreasing exponentially with increasing waiting time between warmings. The subsets of this synthetic data set generally show the same pattern but with some mismatches owing to random fluctuations in the shorter time series. The GRIP data also exhibit this same basic pattern; few warmings are spaced less than a millennium apart or $3T/2$ or $5T/2$ apart, with T~1450-1500 years, but many warmings are spaced about T or $2T$ apart.

The chi-square test indicates that the recurrence histogram for the GRIP data is consistent with that for the synthetic stochastic resonance data. Some grouping of neighboring histogram cells is required to obtain enough occurrences in each cell for reliable statistics, and the results are somewhat sensitive to the groupings chosen. In general, we find that the GRIP data are more similar to the long synthetic stochastic resonance time series than are ~25% of GRIP-length subsets chosen from that synthetic time series.

The observed pattern for GRIP is statistically quite distinct from that of a simple sinusoid, of simple white noise, of the output of our analysis if the GRIP data are replaced by normally distributed random numbers with stationary variance, and of other models we have tested (such as a subharmonic response hypothesis, in which noisy climate responds nonlinearly to an oscillator of period T with generation of a 2:1 subharmonic; the power spectrum can have lines at T and $2T$, and the recurrence histogram can have exactly two peaks but not at T and $2T$, and does not contain the T, $2T$, $3T$ structure).

Plate 1 shows that our results are robust against small variations in important parameters in the analysis, for both GRIP and GISP2 data. The color code indicates the number of occurrences of a particular waiting time between warmings, normalized to improve display by dividing by the number of occurrences expected for a white noise process with a mean waiting time between warmings of 3000 years. On this plot the ensemble average of a stochastically resonant system with millennial mean spacing between warmings would produce vertical red-orange-yellow bands at waiting times T, $2T$, $3T$, ..., showing more-frequent occurrence than expected for a random system. These would be separated by purple-blue bands showing rare occurrence in comparison to a random system. Any finite realization of a stochastically resonant system would produce a noisy approximation of this ideal behavior. A white noise process would produce a single color gradient across the plate, with red on the left for mean waiting time less than the normalizing interval of 3000 years, red on the right for longer waiting times, and the plate entirely green for a mean waiting time of 3000 years. A sine wave would produce a single red band at the frequency, in a purple background.

The short length of the ice core time series typically allows only 40-50 total transitions, depending on the transition threshold chosen and other details of the analyses. Because of the roughly exponential decrease in number of occurrences with increasing waiting time in the recurrence histograms, the histogram peaks at $4T$ and $5T$ typically have either zero or one events, whereas the expected number of events for a white noise process falls between 0 and 1. This unavoidable granularity associated with the short length of the data set causes the data to appear "noisy" at long waiting times; we chose a normalizing time of 3000 years slightly longer than the mean waiting time in the observed time series to avoid further amplifying this noise at long waiting times.

Relative dating errors in the GISP2 data range from <1% in young samples to as much as 10% between older peaks [*Alley et al.*, 1997; *Meese et al.*, 1997], with similar uncertainty for GRIP [*Johnsen et al.*, 1992], introducing additional uncertainty to longer waiting times but having little effect on the peaks at short waiting times. This dating uncertainty prevents us from assessing whether the phase of the signal has been stationary over tens of thousands of years. Note, however, that although GRIP and GISP2 total ages may differ by more than a 1500-year cycle in older parts of the records, interval ages between successive warmings typically differ by only a few percent or less between the two timescales [*Meese et al.*, 1997; *Johnsen et al.*, 1997]. Hence, we would expect the independent records to yield similar recurrence histograms, as observed.

The observed purple bands in Plate 1 near 750, 2250, 3750 years and beyond, separated by red-orange-yellow bands near 1500, 3000, 4500 years and beyond, are fully consistent with the expected behavior from a system with characteristics of stochastic resonance and with T between 1450 and 1500 years, and show that the results are insensitive to small changes in the parameters used in our analyses. Insensitivity to the interpolation or sampling interval shows that the results are not caused by aliasing of an annual or other high-frequency signal [cf. *Wunsch*, 2000]. The shortest sampling intervals chosen for the GISP2 data (which were sampled more finely than the GRIP data available to us) and the lowest transition thresholds in the GRIP data (d near zero) do approach white noise behavior with short mean waiting time because these parameters include rapid and small changes that do not indicate persistent mode switches [*Ditlevsen*, 1999]. Except near these "noisy" limits, however, the data are consistent with the stochastic resonance hypothesis.

4. Discussion

Do these results prove that the north Atlantic is a stochastically resonant system? Of course not. No finite data sequence ever uniquely defines the process that produced it. This was illustrated by Roe and Allen [1999], who showed for ice-age cycles that available data cannot distinguish between the six distinct models they tested, which all fit the observations adequately. Although we have not identified another model that fits the ice-core data for Dansgaard-Oeschger cycles, we expect that other models are possible.

We note, however, that we were led to the signal plus noise, stochastic resonance hypothesis by the evidence that an ~1500-year periodicity exists in North Atlantic records, and by the evidence that events from the ice sheets around the north Atlantic contributed to important climate changes. This hypothesis makes a very specific prediction about the shape of the histogram of the number of occurrences of specific waiting times between warmings, and as discussed above, this prediction is distinct from the predictions of at least the simplest forms of previous hypotheses. We find that histograms for Greenland ice core data are fully consistent with the stochastic resonance hypothesis and not consistent with the simplest forms of the competing hypotheses. We thus argue that there is good support for a stochastically resonant North Atlantic and that this model merits further consideration.

With some variability, there is a tendency for the waiting times to have been long in the early part of the record and during the modern warm interval, with shorter waiting times during the colder parts of the ice age (Plate 2). This may indicate that a colder climate has a higher noise level, a lower transition threshold, or a stronger periodic oscillation.

Of these, we infer at least a higher noise level during times with more ice, on the basis of comparison to the estimated ice volume curve in Plate 2. Ice sheets store fresh water as ice and as ice-marginal or subglacial lakes dammed by ice. Ice and ice-dammed lakes typically grow slowly but shrink rapidly; Heinrich events in the North Atlantic and outburst floods from ice-dammed lakes are spectacular examples of this asymmetry. Ocean models indicate that rapid release of large volumes of fresh water could trigger reduction or collapse in North Atlantic circulation, producing cooling [e.g., *Broecker et al.*, 1990; *Stocker et al.*, 1992; *Weaver and Hughes*, 1994; *Sakai and Peltier*, 1995]. The ability of ice to supply fresh water must scale crudely with the ice volume; more ice can feed a bigger surge, and bigger ice can dam more rivers to make more and bigger ice marginal lakes. Thus, variability in freshwater supply was probably higher when ice sheets were bigger. Time changes in variability might cause deviation from a rigidly exponential fall-off of peak height with increasing waiting time on the recurrence histogram, but our data set is not long enough to detect any such trend reliably.

We have treated the ice sheet events as being entirely stochastic. This is probably not strictly correct; the timing of outburst floods will depend on locations of bedrock sills and other factors that are not synchronized with climate, but it also will depend on rates of ice marginal change, which may be related to climate [*Alley and Clark*, 1999; *Licciardi et al.*, 1999]. More-sophisticated modeling would be required to handle this additional complication.

That both "signal" and "noise" are significant is hardly a surprising result for almost any system. Nonetheless, if the stochastic resonance hypothesis is correct, it has several implications beyond providing a new "buzz word" for processes occurring in or around the North Atlantic. Our results suggest that 1) there is a 1450- to 1500-year climate cycle, as argued by many previous workers but 2) this cycle by itself is too weak to cause North Atlantic mode switches at most or all times and 3) the "noise" generated by ice sheet or other processes also is too weak to cause mode switches at most or all times and hence 4) both the weak periodicity and the ice sheet or other processes are required to explain North Atlantic behavior.

Again, we note that the presence of the ~1500-year preferred spacing between warmings in both younger and older parts of the GISP2 and GRIP records collected with different sampling schemes and robust against variation in the interpolation scheme (see Plates 1a and 1d, and the older and younger parts of Plate 2) demonstrates that we are not being misled by aliasing of the annual cycle or of any other higher-frequency signal [*Wunsch*, 2000]. However, the 1500-year peaks in our recurrence histograms are rather broad, perhaps more consistent with *Wunsch's* [2000] expectations about the climate system than are some other published records. Owing to the great differences in analysis between spectral techniques and our simple crossover analysis, we do not pursue this further here.

Existence of a periodic forcing raises the obvious question of its origin, which we cannot answer. The time-scale of the oscillator is consistent with deep-ocean processes [*Broecker et al.*, 1990], but solar [*Mayewski et al.*, 1990], tidal [*Keeling and Whorf*, 2000], and El Niño-Southern Oscillation (ENSO) processes perhaps related to orbital changes [*Cane and Clement*, 1999] have also been suggested. It is intriguing that the Little Ice Age may be related to the 1500-year cycle [e.g., *Bond et al.*, 1999], and although volcanism or other processes may have contributed, the Little Ice Age may have involved solar influence [e.g., *Crowley and Kim*, 1996]. Additionally, the ^{10}Be data presented by *Alley et al.* [1995a] indicate a decrease in Greenlandic atmospheric loading of this cosmogenic isotope between the beginning and end of the Younger Dryas cold event, which could have several explanations, including an increase in solar activity leading up to the termination of the event. If the solar model is applicable, we might expect to see ~1500-year power in Holocene records of cosmogenic isotopes in ice cores; some such power might also occur in radiocarbon records.

Existence of a periodic forcing also invites speculation on the future. If we follow earlier workers [e.g., *Denton and Karlen*, 1973; *Keigwin and Jones*, 1989; *Bond et al.*, 1997; 1999] in taking the Little Ice Age as the most recent cold phase of the periodic oscillation and if we assume that the warming from the Little Ice Age began naturally sometime between 100 and 250 years ago, then the modern north Atlantic would be warming naturally. However, the Little Ice Age did not achieve a switch into a glacial ocean mode. If the periodic oscillation is a sine wave, then the climate would not yet have passed into the half of the cycle most stable against being forced into such a mode switch, opening the possibility that human-induced freshening of the north Atlantic could force a mode switch [e.g., *Stocker and Schmittner*, 1997]. These speculations probably do not greatly advance our predictive ability.

5. Conclusions

Much previous work has shown that the North Atlantic region and surroundings have experienced large, abrupt climate changes with ~1450-1500-year spacing but that some of these changes have been linked to events such as outburst floods that are unlikely to be truly periodic. Both periodic and event models would be correct if the North Atlantic is a stochastically resonant system, in which switches between warm and cold modes of the climate typically are triggered by more or less random events when the phase of the periodic forcing is favorable.

This stochastic resonance hypothesis predicts that in a history of North Atlantic climate, warmings most frequently will be separated by waiting time *T*, the period of the oscillation. Preferred waiting times between warmings also will be observed at 2*T*, 3*T*, ..., with exponentially decreasing number of observed occurrences with increasing waiting time and with few or no warmings separated by waiting times near *T*/2, 3*T*/2, 5*T*/2, ... Other hypotheses we have examined, including simple stochastic and simple periodic models, predict very different patterns for the recurrence histograms of waiting times between warmings.

The GRIP and GISP2 ice isotopic data from central Greenland for the last 110,000 years produce a recurrence histogram for waiting times between warmings that is fully consistent with the stochastic resonance model and inconsistent with other simple models we have tested. These observations do not prove that the North Atlantic is a stochastically resonant system, but we believe that they establish stochastic resonance as an important model to be tested in further research.

If the stochastic resonance model is correct, then study of periodic elements and of events linked to ice sheet behavior are both important. The cause of the periodic oscillation remains uncertain.

Acknowledgments. We thank the U.S. National Science Foundation for support, the GISP2 Science Management Office and chief scientist Paul Mayewski, the U.S. 109th Air National Guard, the Polar Ice Coring Office, the National Ice Core Lab, the NOAA NGDC Paleoclimatology Program for data archival, Eric Wolff, Minze Stuiver, Larry Wilen, Sigfus Johnsen, Kirk Maasch, other colleagues, and two anonymous reviewers.

References

Alley, R.B., and P.U. Clark, The deglaciation of the Northern Hemisphere: A global perspective, *Ann. Rev. Earth Planet. Sci.*, *27*, 149-182, 1999.

Alley, R.B., R.C. Finkel, K. Nishiizumi, S. Anandakrishnan, C.A. Shuman, G. Mershon, G.A. Zielinski, and P.A. Mayewski, Changes in continental and sea-salt atmospheric loadings in central Greenland during the most recent deglaciation: Model-based estimates, *J. Glaciol.*, *41*, 503-514, 1995a.

Alley, R.B., A.J. Gow, S.J. Johnsen, J. Kipfstuhl, D.A. Meese, and T. Thorsteinsson, Comparison of deep ice cores, *Nature*, *373*, 393-394, 1995b.

Alley, R.B., *et al.*, Visual-stratigraphic dating of the GISP2 ice core: Basis, reproducibility, and applications, *J. Geophys. Res.*, *102*, 26,367-26,381, 1997.

Alley, R.B., *et al.*, Abrupt increase in Greenland snow accumulation at the end of the Younger Dryas event, *Nature*, *362*, 527-529, 1993.

Barber, D.C., *et al.*, Forcing of the cold event of 8,200 years ago by catastrophic drainage of Laurentide lakes, *Nature*, *400*, 344-348, 1999.

Benzi, R., G. Parisi, A. Sutera, and A. Vulpiani, Stochastic resonance in climatic change, *Tellus*, *34*, 10-16, 1982.

Bond, G., W.J. Showers, M. Cheseby, R. Lotti, P. Almasi, P. deMenocal, P. Priore, H. Cullen, I. Hajdas, and G. Bonani, A pervasive millennial-scale cycle in North Atlantic Holocene and glacial climates, *Science*, *278*, 1257-1266, 1997.

Bond, G.C., W. Showers, M. Elliot, M. Evans, R. Lotti, I. Hajdas, G. Bonani and S. Johnsen, The North Atlantic's 1-2 kyr climate rhythm: relation to Heinrich events, Dansgaard/Oeschger cycles and the Little Ice Age, in *Mechanisms of Global Climate Change at Millennial Time Scales*, *Geophys. Monogr. Ser., vol. 112*, edited by P.U. Clark, R.S. Webb and L.D. Keigwin, pp. 35-58, AGU, Washington, D. C., 1999.

Broecker, W.S., M. Andree, W. Wolfli, H. Oeschger, G. Bonani, J. Kennett, and D. Peteet, The chronology of the last deglaciation: Implications to the cause of the Younger Dryas event, *Paleoceanography*, *3*, 1-19, 1988.

Broecker, W.S., G. Bond, M. Klas, G. Bonani, and W. Wolfli, A salt oscillator in the glacial Atlantic?, 1, The concept, *Paleoceanography*, *5*, 469-477, 1990.

Bryan, F., High-latitude salinity effects and interhemispheric thermohaline circulations, *Nature*, *323*, 301-304, 1986.

Cane, M., and A. Clement, A role for the tropical Pacific coupled ocean-atmosphere system on Milankovitch and millennial timescales. Part II: Global Impacts, in *Mechanisms of Global Climate Change at Millennial Time Scales*, *Geophys. Monogr. Ser., vol. 112*, edited by P.U. Clark, R.S. Webb and L.D. Keigwin, pp. 373-383, AGU, Washington, D. C., 1999.

Chappellaz, J., E. Brook, T. Blunier, and B. Malaize, CH_4 and $\delta^{18}O$ of O_2 records from Antarctic and Greenland ice: A clue for stratigraphic disturbances in the bottom part of the Greenland Ice Core Project and the Greenland Ice Sheet Project 2 ice cores, *J. Geophys. Res.*, *102*, 26,547-26,557, 1997.

Crowley, T.J., and K.-Y. Kim, Comparison of proxy records of climate change and solar forcing, *Geophys. Res. Lett.*, *23*, 359-362, 1996.

Cuffey, K.M., G.D. Clow, R.B. Alley, M. Stuiver, E.D. Waddington, and R.W. Saltus, Large Arctic temperature change at the glacial-Holocene transition, *Science*, *270*, 455-458, 1995.

Denton, G.H., and W. Karlen, Holocene climatic variations—Their pattern and possible cause, *Quat. Res.*, *3*, 155-205, 1973.

Ditlevsen, P.D., Observation of α-stable noise induced millennial climate changes from an ice-core record, *Geophys. Res. Lett.*, *26*, 1441-1444, 1999.

Gammaitoni, L., P. Hanggi, P. Jung, and F. Marchesoni, Stochastic resonance, *Rev. Modern Phys.*, *70*, 223-287, 1998.

Grootes, P.M., and M. Stuiver, Oxygen 18/16 variability in Greenland snow and ice with 10^{-3}- to 10^5-year time resolution, *J. Geophys. Res.*, *102*, 26,455-26,470, 1997.

Johnsen, S.J., H.B. Clausen, W. Dansgaard, K. Fuhrer, N. Gundestrup, C.U. Hammer, P. Iversen, J. Jouzel, B. Stauffer, and J.P. Steffensen, Irregular glacial interstadials recorded in a new Greenland ice core, *Nature*, *359*, 311-313, 1992.

Johnsen, S.J., *et al.*, The δ^{18}O record along the Greenland Ice Core Project deep ice core and the problem of possible Eemian climatic instability, *J. Geophys. Res.*, *102*, 26,397-26,410, 1997.

Jouzel, J., *et al.*, Validity of the temperature reconstruction from water isotopes in ice cores, *J. Geophys. Res.*, *102*, 29,471-29,487, 1997.

Keeling, C.D., and T.P. Whorf, The 1800-year oceanic tidal cycle: A possible cause of rapid climate change, *Proc. Natl. Acad. Sci.*, *97*, 3814-3819, 2000.

Keigwin, L.D., and G.A. Jones, Glacial-Holocene stratigraphy, chronology, and paleoceanographic observations on some North Atlantic sediment drifts, *Deep -Sea Res.*, *36*, 845-867, 1989.

Licciardi, J.M., J.T. Teller, and P.U. Clark, Freshwater routing by the Laurentide Ice Sheet during the last deglaciation, in *Mechanisms of Global Climate Change at Millenial Time Scales*, *Geophys. Monogr. Ser.*, *vol. 112*, edited by P.U. Clark, R.S. Webb and L.D. Keigwin, 177-200, AGU, Washington, D. C., 1999.

Mayewski, P.A., L.D. Meeker, M.S. Twickler, S. Whitlow, Q. Yang, W.B Lyons, and M. Prentice, Major features and forcing of high-latitude Northern Hemisphere atmospheric circulation using a 110,000-year-long glaciochemical series, *J. Geophys. Res.*, *102*, 26,345-26,366, 1997.

McManus, J.F., D.W. Oppo, and J. Cullen, A 0.5-million-year record of millennial-scale climate variability in the North Atlantic, *Science*, *283*, 971-975, 1999.

Meese, D.A., A.J. Gow, R.B. Alley, G.A. Zielinski, P.M. Grootes, M. Ram, K.C. Taylor, P.A. Mayewski, and J.F. Bolzan, The Greenland Ice Sheet Project 2 depth-age scale: methods and results, *J. Geophys. Res.*, *102*, 26411-26423, 1997.

Rahmstorf, S., Bifurcations of the Atlantic thermohaline circulation in response to changes in the hydrological cycle, *Nature*, *378*, 145-149, 1995.

Sakai, K., and W.R. Peltier, A simple model of the Atlantic thermohaline circulation: Internal and forced variability with paleoclimatological implications, *J. Geophys. Res*, *100*, 13,455-13,479, 1995.

Sarnthein, M., K. Winn, S.J.A. Jung, J.C. Duplessy, L. Labeyrie, H. Erlenkeuser, and G. Ganssen, Changes in east Atlantic deepwater circulation over the last 30,000 years: Eight time slice reconstructions, *Paleoceanography*, *9*, 209-267, 1994.

Severinghaus, J.P., T. Sowers, E.J. Brook, R.B. Alley, and M.L. Bender, Timing of abrupt climate change at the end of the Younger Dryas interval from thermally fractionated gases in polar ice, *Nature*, *391*, 141-146, 1998.

Stocker, T.F., and A. Schmittner, Influence of CO_2 emission rates on the stability of the thermohaline circulation, *Nature*, *388*, 862-865, 1997.

Stocker, T.F., D.G. Wright, and W.S. Broecker, The influence of high-latitude surface forcing on the global thermohaline circulation, *Paleoceanography*, *7*, 529-541, 1992.

Taylor, K.C., *et al.*, The Holocene/Younger Dryas transition recorded at Summit, Greenland, *Science*, *278*, 825-827, 1997.

Weaver, A.J., and T.M.C. Hughes, Rapid interglacial climate fluctuations driven by North Atlantic Ocean circulation, *Nature*, *367*, 447-450, 1994.

Wunsch, C., On sharp spectral lines in the climate record and the millennial peak, *Paleoceanography*, *15*, 417-424, 2000.

Yiou, P., K. Fuhrer, L.D. Meeker, J. Jouzel, S. Johnsen, and P.A. Mayewski, Paleoclimatic variability inferred from the spectral analysis of Greenland and Antarctic ice-core data, *J. Geophys. Res.*, *102*, 26, 441-26,454, 1997.

R. B. Alley, Environment Institute and Department of Geosciences, Pennsylvania State University, University Park, PA 16802 (ralley@essc.psu.edu)

S. Anandakrishnan, Department of Geology, University of Alabama, Tuscaloosa AL 35487

P Jung, Department of Physics and Astronomy, Ohio University, Athens OH 45701

(Received March 2, 2000;
revised November 2, 2000;
accepted November 3, 2000.)