$$\frac{\partial h}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \underline{\epsilon_o h} = 0 \tag{7.26c}$$?

where $F_0 = \tau_0 L/(c_o^2 \rho H)$ is the dimensionless amplitude of the zonal wind stress and $\epsilon_o = a_m L/c_o$ is the dimensionless linear damping coefficient. In a finite basin on the equatorial β-plane, the boundary conditions are

$$x = 0, 1 \quad : \quad u = 0 \tag{7.27a}$$ $x \in [0,1]$

$$y \to \pm\infty \quad : \quad u, v, h \to 0 \tag{7.27b}$$

With $\mathbf{F} = (\tau^x, 0, 0)$ and $\mathbf{u} = (u, v, h)$, this system of equations can be written as ?

$$\mathcal{M}\frac{\partial \mathbf{u}}{\partial t} + \mathcal{L}\mathbf{u} = \mathbf{F} \tag{7.28a}$$

$$\mathcal{L} = \begin{pmatrix} \epsilon_o & -y & \frac{\partial}{\partial x} \\ y & \zeta_o \epsilon_o & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \epsilon_o \end{pmatrix} \quad ; \quad \mathcal{M} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \zeta_o^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{7.28b}$$

Applying Fourier transformation in x, according to

$$\hat{\mathbf{u}}(k, y, t) = \int_{-\infty}^{\infty} \mathbf{u}(x, y, t) e^{-ikx}\, dx \tag{7.29a}$$ $x \in [-\infty, \infty]$

$$\hat{\mathbf{F}}(k, y, t) = \int_{-\infty}^{\infty} \mathbf{F}(x, y, t) e^{-ikx}\, dx \tag{7.29b}$$

then all x-derivatives in $\mathcal{L}$ will transform to ik in $\hat{\mathcal{L}}$ and $\hat{\mathcal{M}} = \mathcal{M}$. All free wave solutions of the previous section, say written as $\hat{\mathbf{U}}$, are solutions of the eigenvalue problem (for $\epsilon_o = 0$)

$$\hat{\mathcal{L}}\hat{\mathbf{U}} = i\sigma\hat{\mathcal{M}}\hat{\mathbf{U}} \tag{7.30}$$

where σ is given through the dispersion relation (7.17).

In the limit $\zeta_o \to 0$, only the long (small k), low frequency modes (small σ) Rossby waves remain, having a dispersion relation and eigenfunctions for j = 1,2,...,

$$\sigma_j = \frac{-k}{2j+1} \tag{7.31a}$$

$$\hat{u}_j(y) = \frac{1}{2\sqrt{2}}\left(\frac{\psi_{j+1}(y)}{\sqrt{j+1}} - \frac{\psi_{j-1}(y)}{\sqrt{j}}\right) \tag{7.31b}$$

$$\hat{h}_j(y) = \frac{1}{2\sqrt{2}}\left(\frac{\psi_{j+1}(y)}{\sqrt{j+1}} + \frac{\psi_{j-1}(y)}{\sqrt{j}}\right) \tag{7.31c}$$

$$\hat{v}_j(y) = \psi_j(y) \tag{7.31d}$$