EPS231, Climate dynamics
Dept of Earth and Planetary Sciences, Harvard University
Homeworks #6 & #7
Equable climate
Q1–3/HW6: Assigned April 3, 2025, due: Apr 15, 2025
Q4–6/HW7: Assigned April 15, 2025, due: April 24, 2025

1. **Back-of-the-envelope Arctic energy balance:** consider the energy balance for the annually averaged temperature over the Arctic,

$$H/A + (1-\alpha)S + \epsilon_a \sigma T_a^4 = \sigma T_s^4$$
$$\epsilon_a \sigma T_s^4 = 2\epsilon_a \sigma T_a^4.$$

Here H is the meridional heat flux toward the Arctic, A the area of the Arctic (say north of 60N), and S the annually averaged insolation there. Let the modern climate values be $H = 3PW$, the atmospheric emissivity $\epsilon_a = 0.6$, albedo $\alpha = 0.55$, $S = 250$ W/m^2. Calculate the annual averaged surface temperature T_s. Assume that the doubling of CO_2 by itself leads to a warming of 1.5 degrees, and with water vapor feedback to a warming of 3 degrees and remember that the dependence of the emissivity on CO_2 and moisture is logarithmic. Deep water temperature proxies suggest that the Arctic was never colder than 10–12 °C(!). By how much should each of the following parameters/variables change in order to warm the Arctic to an annual average temperature of 10°C: H, α, CO_2 with and without water vapor, and the emissivity due to clouds, $\Delta\epsilon_a^{clouds}$. You need to write the atmospheric emissivity as $\epsilon_a = \epsilon_a^0 + A\log_2(CO_2/280) + B\log_2(q^*(T_a)/q^*(T_a^0)) + \Delta\epsilon_a^{clouds}$, where zero superscripts correspond to preindustrial values. The saturation specific humidity $q^*(T,p)$ is calculated by an approximate form of the Clausius-Clapeyron relationship, given in the attached code, and the atmospheric level of the model is at 500 mbar. Assume $\Delta\epsilon_a^{clouds} = 0$ for the preindustrial climate. *Hint:* set B to zero and experiment numerically to find A that satisfies the given information. Then, given the calculated value of A, solve the above equations for different values of B until finding the right one. Similarly for the other parameters.

2. **Equator-to-pole Hadley cell:** (sections a–d here are directly from problem 11.2 of Vallis (2006), numbers refer to sections and equations there, see a copy here, and this problem is an excellent way to review your understanding of the Hadley cell dynamics which is the base of the Farrell (1990) equator-to-pole Hadley cell idea for equable climate).

   Suppose that, in the vertically-integrated Hadley cell Held and Hou (1980) model considered in section 11.2, the radiative equilibrium temperature Θ_E falls linearly from the equator to the pole, so that $\Theta_E = \Theta_{E0} - \Delta\Theta\,(y/a)$, rather the quadratic fall-off in (11.16). Obtain and discuss the solutions for the end-latitude of the Hadley cell using the following guidelines.

   (a) Derive in detail an expression for the vertically averaged temperature as a function of latitude, $\Theta(\theta)$, starting from the frictionless, zonally-averaged zonal momentum equation (11.4).

   (b) Write and justify a conservation equation for the temperature, with forcing based on restoring to the specified radiative-equilibrium temperature.

   (c) Derive, explain and apply the heat budget argument for the Hadley cell, to find an expression for the end-latitude Y_H of the Hadley cell. How does this situation differ from that of the quadratic Held and Hou radiative equilibrium temperature?

   (d) Plot $\Theta_E(y)$, and $\Theta(y)$ for your solution, and indicate on the plot the end latitude Y_H of the Hadley cell using the parameter values from Figure 11.6, and compare to the quadratic solution.

   (e) Analyze how adding a large friction force to the zonal momentum equation is expected to change this solution: assume that the zonal velocity is equal to $\eta = 0.5$ times what it is without friction. Calculate the extent of the Hadley cell and the profiles of zonal velocity and potential temperature as a function of latitudes, as well as the new edge of the Hadley cell. Discuss the significance of your results in terms of the Eocene equator-to-pole temperature difference.

3. **Polar stratospheric clouds:**

   (a) **Does the Brewer-Dobson circulation weaken in a warmer climate:** Analyze CMIP6 results at $\times 1CO_2$ and $\times 4CO_2$ (the SSP585 scenario) using provided model data accessed on Derecho (/glade/derecho/scratch/asalazar/CMIP6BD)(**A**) plot the northern hemisphere zonally-averaged Brewer-Dobson circulation, averaged over Northern Hemisphere winter time (DJF), as a quiver plot of V, W as a function of pressure and latitude for both a preindustrial control, for the greenhouse scenario, and the difference. Plot only in the stratosphere, above 100 hPa, to avoid having the plot overwhelmed by the larger tropospheric signal. Is the BD circulation stronger or weaker in the warmer climate? (**B**) plot the zonally averaged DJF temperature for both runs and the temperature diff between them. Describe how the meridional temperature gradient changes in the lower troposphere (1000–400 hPa) and in the upper troposphere (300–100 hPa). (**C**) Eddy motions: explain how you would calculate changes from control to RCP8.5 in planetary wave activity at $k = 1$, $k = 2$ from CMIP6 data, and how you would use that to test the stratospheric polar clouds idea. No need to calculate or plot anything here. (**D**) Briefly discuss the relevance of your results to the proposed polar stratospheric cloud mechanism for explaining equable climate.

   (b) **Vertical propagation of planetary waves into the stratosphere:** (section 13.3 of Vallis, 2017). Use the following steps to analyze this problem.

      i. ***Optional extra credit:** Show that $\overline{v'q'} = -\partial_y(\overline{u'v'}) + \partial_z(\frac{f_0}{N^2}\overline{v'b'})$.

      ii. Explain how and why vertically propagating planetary waves drive the Brewer-Dobson circulation. Write and explain the relevant equation for the mean stratospheric meridional flow.

      iii. Consider the potential vorticity equation in a stratified fluid,

$$\frac{\partial q}{\partial t} + J(\psi, q) = 0 \tag{1}$$

$$q = \nabla^2\psi + f_0 + \beta y + \frac{f_0^2}{\rho_R}\frac{\partial}{\partial z}\left(\frac{\rho_R}{N^2}\frac{\partial \psi}{\partial z}\right) \tag{2}$$

$$\nabla^2 = \partial_{xx} + \partial_{yy} + \partial_{zz} \tag{3}$$

$$\rho_R = \rho_0 e^{-z/H}, \tag{4}$$

$$H = RT(z = 0)/g \tag{5}$$

      where $J(a, b) \equiv a_x b_y - a_y b_x$; let $\psi = -\bar{u}(z)y + \psi'$ and linearize the above vorticity equation.

iv. Look for solutions of the form

$$\psi' = Re\tilde{\psi}(z)\sin(ly)e^{ik(x-ct)} \tag{6}$$

and derive an equation for $\tilde{\psi}(z)$.

v. Let $\Phi(z) = \tilde{\psi}(z)\left(\frac{\rho_R}{\rho_R(z=0)}\right)^{1/2}$, derive an equation of the form $\Phi''(z) + m^2\Phi = 0$, and write down the expression for m^2.

vi. Show that m must be positive for energy propagation to be upward.

vii. Show that vertically propagation with an upward energy flux occurs only for

$$0 < \overline{u} < \frac{\beta}{K^2 + \gamma^2}$$

with $\gamma \equiv f_0^2/(4N^2H^2)$.

viii. Discuss the implications for the seasonality of the Brewer-Dobson circulation and the Kirk-davidoff et al. (2002) feedback, building on the earlier work of Sloan et al. (1992), involving polar stratospheric clouds.

ix. ***Optional extra credit: Show that for $m^2 < 0$ the energy flux $\rho_R\overline{w'p'}$ vanishes where overbar denotes horizontal average (over x). This involves two steps: getting w from the thermodynamic equation and taking the real part of the vertical velocity and pressure anomalies before multiplying them and averaging.

4. **Hurricanes and ocean mixing:**

(a) *Hurricane strength as a function of SST:* Outline the derivation of the formula for the maximum hurricane wind velocity derived in class and explain each factor in the final result. Estimate the maximum possible hurricane wind velocity expected in the preindustrial climate. Compare this to a typical wind velocity in a category 4 hurricane. Consider a 5 °C warming of the tropics: Assume that the temperature at which air detrains from the top of the hurricane, T_0, is fixed and similar to that of the preindustrial period, and evaluate the expected change to the maximum hurricane speed.

(b) *Deep ocean mixing as a function of hurricane wind stress:* Hurricanes leave behind a trail of cold SST. Describe how this may be due to wind-induced mixing, in addition to the extraction of latent heat due to evaporation. Does this lead to mixing down to the depth that matters for the MOC? What is that deeper ocean mixing controlled by?

(c) *Ocean overturning as a function of vertical mixing:* develop scaling for the ocean meridional heat transport as a function of the diapycnal mixing coefficient. Assume the equations are,

$$
\begin{aligned}
w T_z &= \kappa T_{zz} && \text{temperature equation} \\
v_y + w_z &= 0 && \text{zonally-averaged continuity equation} \\
f v_z &= -\frac{g\alpha}{\rho_0}\frac{\partial T}{\partial x} && \text{thermal wind equation}
\end{aligned}
$$

Let the scales be $x \sim L_x$, $y \sim L_y$, $v \sim V$, $w \sim W$, $T \sim \Delta T$. What is the scale for the meridional overturning mass transport? For the meridional heat flux? (This is a classic scaling exercise and it follows Scott and Marotzke (2002), although note that the temperature equation assumes a balance that may not actually hold in the Atlantic, where heat transport by eddies seems to dominate diapycnal mixing.)

(d) *Back to the big picture:* using a literature search: what is the ocean meridional heat flux today at 60N? By what factor does it need to increase to contribute significantly to Arctic warming based on the back-of-the-envelope energy balance? By what factor would mixing need to increase? Discuss.

5. **Convective cloud feedback:** (based on Abbot and Tziperman (2009), available here.)

   (a) Write and explain the equations of the two-layer radiative-convective model discussed in class, for both convecting and non-convecting states.

   (b) Solve for the steady-state atmospheric and surface temperature for each of the two states as a function of base long-wave emissivity ϵ_0. You will need to solve for the convective states numerically, making use of the routines for calculating the specific humidity, available in the HW folder on the course web page. To solve the nonlinear equation for the convective state, you may use the Matlab function `fsolve` or similar.

   (c) Test your code and solutions by reproducing the four panels of Figure 1 from Abbot and Tziperman (2009). Make sure you plot each solution only in the regime in which it exists.

   (d) Interpret each of the four panels of your figure in terms of the model physics and equations. Discuss the possible effect of a gradual increase in atmospheric CO_2 (noting that $\epsilon_0 \approx \log(CO_2)$ and the existence of a climate equilibrium determined by this high-latitude convective cloud feedback) on the state of the Arctic as a function of time.

6. **Breakdown of subtropical stratocumulus clouds:**

   (a) Explain how stratocumulus clouds form via long wave cooling at the top of the cloud layer, leading to a convective coupling to the ocean. What is the role of this coupling?

   (b) Explain how an increased CO_2 might affect this process and eliminate the clouds.

   (c) Estimate the global area of subtropical stratocumulus clouds in the eastern margins of ocean basins (see Fig. 4 of Wood, 2012). Assume the clouds to be completely eliminated and estimate the effect on the SW surface heating. Assume 50% of this heat is transported to the Arctic north of 60N. Use the simple energy balance model for the Arctic discussed in class to estimate what annually-averaged temperature difference would be caused there.

   (d) Use the sample code for the toy stratocumulus box model (available under the Sources folder) to recreate the hysteresis in SST as a function of CO_2. At what CO_2 level does break-up occur?

   (e) Turn off the water vapor feedback by setting $B = 0$. How does this change the critical CO_2 at which break-up occurs? Explain the result.

   (f) Turn back on the water feedback and increase cloud-top entrainment from $w_e = 0.01$ m/s to $w_e = 0.013$ m/s. How does this change the critical CO_2 at which break-up occurs? Explain the result.

7. ***Climate modeling:*** analyzing an $8\times CO_2$ run [Available on Derecho at /glade/campaign/univ/uhar0024/asalazar/8xco2_spinup]. piControl data is in the PI_control directory.

   (a) Contour the surface temperature (TS) averaged over the last 100 years of the simulation, that for the preindustrial case, and the difference. Discuss the patterns and mechanisms of polar amplification.

   (b) Contour the zonally averaged pressure-latitude versions of the same quantities (T). Discuss the vertical structure of the warming.

   (c) Plot the PDFs of minimum daily temperatures in continental interiors in the high CO_2 case (TSMIN) and that for the preindustrial case.

   (d) **Optional extra challenge:** (i) Contour the convective precipitation (PRECC) during DJF averaged over a decade in the Arctic for end of run vs. that of PI. (ii) Analyze the stratocumulus response: Contour the low cloud (CLDLOW) fractions and their difference.

References

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[What's the point of **optional extra credit challenge problems:** problems: apart from the fun of doing them, they will count against other problems in the same HW assignment in which you may have missed an answer...]