EPS231, Climate dynamics
Dept of Earth and Planetary Sciences, Harvard University
Homework #5
Glacial cycles, Milankovitch forcing, CO_2
Assigned March 25, 2025; due: April 3, 2025

1. **Optional extra credit: More Milankovitch:** the following questions are meant to deepen your understanding of Milankovitch forcing. When needed, use the insolation formula (see slide "Geometry for calculating insolation"). For these calculations, you will need to include the effect due to the variations in the speed of Earth's motion around the sun along its trajectory around the ellipse. To include these effects, recall that the conservation of angular momentum for a body under the gravitational force is,

$$\mu r^2 \dot{\theta} = l,$$

where μ is the reduced mass of the two-body problem, r is the distance from the Earth to the sun, $\theta = \lambda - \tilde{\omega}$, and l is the magnitude of angular momentum, which is conserved along the Earth's orbit. It can be approximated as $l = M_e 2\pi a^2 (1-e^2)^{1/2}/T$ where M_e is the mass of the Earth and T is one year. (See the "two body problem" in the MathWorld Wolfram homepage, or any mechanics textbook, e.g., Thornton & Marion).

   (a) Show that the changes to the insolation at a given latitude due to precession variations vanish under an annual average: plot the June 1st 65N insolation as a function of precession. Then plot the annually averaged 65N radiation vs precession.

   (b) Use the insolation formula to show that the changes to the insolation due to obliquity variations vanish after being averaged globally and annually.

   (c) Show and explain why increasing obliquity results in an increase in the maximum summer insolation (June 21st) in high latitudes, and a decrease in winter insolation in the mid-latitudes. Plot the percentage change in summer insolation as a function of latitude as obliquity changes from 22.1° to 24.5°.

   (d) Show that the annual and global mean insolation is proportional to $(1-e^2)^{-1/2}$ where e is eccentricity. Explain why this is the case physically.

2. **Glacial phase locking:** Consider a highly simplified glacial model for the ice volume during glacial cycles, assuming linear volume growth due to a constant net accumulation rate Acc until a critical threshold nondimensional volume V_0 at which point the ice sheet collapses, plus a periodic forcing $F(t)$,

$$\frac{dV}{dt} = F(t) + \begin{cases} Acc & \text{if } V \le V_0 \\ -V/\tau & \text{else} \end{cases}$$

where, $V_0 = 500$, $\tau_{acc} = 90$ kyr , $Acc = V_0/\tau_{acc}$, $\tau = 10$ kyr. Use the provided python script as your beginning point.

(a) Apply perfectly periodic forcing. Change the initial conditions to five distinct values and then plot the resulting five time-series on the same axes. Discuss your findings.

(b) What happens if you significantly change the forcing frequency and/or amplitude? Use plots to support your answer.

(c) **optional extra credit:** Run the code forced with Milankovitch forcing representing insolation at 65N on June 21st (see the data file and corresponding readme file). Adjust your periodic forcing to have a zero mean, removing the average from the insolation given in the data file. Repeat the analysis in part (b). In what ways is this similar to or different from the results of part (b)?

(d) Discuss the model formulation and attempt to justify it based on physical mechanisms. Is this a useful glacial model? In what sense is it useful and/or not? What does it teach us and what problem(s) does it not solve?

3. **Glacial CO_2:**

(a) The ocean carbonate system: solve the approximate carbonate system discussed in class (section 4.2 of paleo_co2_tutorial.pdf in terms of specified CO_2(g) and Alkalinity. *Hint:* you will find a quadratic equation for, say, HCO_3^- which can be solved analytically. Calculate all carbonate system unknowns for $CO_2 = 280$ and for $CO_2 = 180$ ppm. Use the following constant values Alk $= 2350\,\mu$mol kg^{-1}, $K_H = 2.84 \times 10^{-2}$ mol kg^{-1}, $K_1 = 1.4 \times 10^{-6}$ mol kg^{-1}, $K_2 = 1.2 \times 10^{-9}$ mol kg^{-1}.

(b) **Optional extra credit:** Using the approximate set of equations derived in class: **(i)** Suppose bottom sediments of $CaCO_3$ dissolve, with atmospheric CO_2 fixed. Qualitatively, based on your solution, what is the effect on alkalinity and total carbon? What is the effect on pH? On CO_3^{2-}? **(ii)** Now suppose we add gaseous CO_2 to the ocean-atmosphere system such that C_T increases from $C_T = 2075$ mol kg^{-1} to 2076. Would other specified inputs change in this scenario? Calculate the new concentration of all ions involved, and rationalize the change you find in each of them.

(c) Consider Toggweiler's 3 box glacial CO_2 model described in section 6 of these notes, when the circulation is reversed. Calculate $C_{T,d} - C_{T,h}$ and $PO_{4,h}$ as function of the mixing f_{hd} and circulation T in this case. Discuss your results.

4. ***Climate modeling:*** Model Intercomparison/[P]MIP analysis.

(a) Download surface temperature (variable ts) and sea ice (variable sic) from all 8 models that ran an LGM simulation in PMIP3 [https://pmip3.lsce.ipsl.fr/] and for the preindustrial CMIP5 simulation from the same 8 models [same site].

(b) For each model, calculate a 100-year average of the surface temperature for PI and for the LGM, and their difference.

(c) Plot the multiple-model average of this difference, and the std across models. Add stripes over areas where the difference is smaller than the std. Discuss areas of model disagreement and what might lead to these.

Steps for downloading LGM and PI CMIP/PMIP data:

- Make an OpenID account: https://esgf-node.ipsl.upmc.fr/ (required to download data, only valid for 72 hours so download data right away)
- Save your OpenID somewhere
- Go to this website: https://aims2.llnl.gov/search/cmip6/
- Switch to CMIP5
- Click Only Globus Transferrable
- Under Identifiers: Experiment ID select either "lgm" or "piControl"
- Under Classifications: Variable ID select "ts" (surface temperature data)
- Under Classifications: Frequency select "mon" (monthly data)
- Download the wget script for each dataset and save in folder in your scratch directory (recommend different folder for each model/experiment)
- Execute the wget script to download data to your scratch directory using:
  $ bash filename -s
  You will be asked to input your OpenID and password

OR

- Go to this website: https://aims2.llnl.gov/search/cmip6/
- Switch to CMIP5
- Click Only Globus Transferrable
- Under Identifiers: Experiment ID select either "lgm" or "piControl"
- Under Classifications: Variable ID select "ts" (surface temperature data)
- Under Classifications: Frequency select "mon" (monthly data)
- Using the dropdown arrow for each model, select the .nc file for "ts" and save locally
- Upload the netcdf file to Derecho

[What's the point of **optional extra credit challenge problems:** apart from the fun of doing them, they will count against other problems in the same HW assignment in which you may have missed an answer...]