EPS231, Climate dynamics
Dept of Earth and Planetary Sciences, Harvard University
Homework #4
DO, Heinrich events and Milankovitch forcing
Assigned March 6, 2025; due: 10 am March 25, 2025

1. **DO events:**

   (a) Plot the NGRIP GICC05 time series for δ^{18}O (column 3 vs column 1). Smooth the data using a 1–2–1 filter *twice* before plotting and using it for analysis.

   (b) Identify the time of onset of DO events by plotting the derivative of the smoothed time series and finding the 13 strongest peaks. Then plot all DO events superimposed such that their onsets are aligned.

   (c) **optional extra credit:** Plot on the unit circle the points corresponding $\theta_n = 2\pi t_j/\tau$ for $\tau = 1470$ (suspected period) and for $\tau = 1700$ (to see the sensitivity of the plot on τ).

   (d) **optional extra credit:** Calculate the Rayleigh measure $R(\tau) = (1/N)|\Sigma_j \cos(2\pi t_j/\tau) + i \sin(2\pi t_j/\tau)|$. Discuss (without doing any calculations) how this can be compared to a Monte-Carlo simulation of a Poisson process to figure out if this R value is significant.

2. **Heinrich events:**

   (a) Estimate the ice volume involved in Heinrich events: Suppose the thickness of a layer of sediment corresponding to a Heinrich event is 20 cm found in a core at (lon, lat)= $(-24, 50)$ and that this thickness extends to Hudson Bay from 40N to 60N. Estimate the volume of sediments involved in this Heinrich layer. Provide a couple of crude estimates for the expected volume of ice involved in carrying this amount of IRD, and present your result in terms of the sea level equivalent: one estimate based on the minimum ice volume that is needed to float that much IRD (assuming IRD density of 3000 kg/m^3), and a second estimate that assumes that the ice volume is 100 times larger than that minimum value.

   (b) Hydrofracturing: consider a 10 m deep wedge-like crack in an ice shelf, increasing in width upward at an angle of 5 degrees. To what height does it need to be filled with water for the water to act as a wedge extending the crack further down?

   (c) Summarize the proposed mechanisms for Heinrich events in 20 words each. Explain what sea level signatures are expected for each.

3. **Back-of-the-envelope Milankovitch:** Use the following steps to understand why the very small eccentricity of the Earth orbit has such a significant effect on the maximum summer insolation, and to understand the effects of obliquity.

   (a) Plot a thin blue line *ellipse* with major axis $a = 149,597,887$ km, a fairly large Earth orbit eccentricity of 0.03, and a minor axis b (which you should be able to calculate). Superimpose a thin red dash line *circle* with a radius of $(a + b)/2$.

   (b) In one short sentence: what are the configurations of the eccentricity, precession angle and obliquity that lead to a maximum June 21st insolation at 65N and to a minimum June 21st insolation there? Assume the obliquity range is 22.3–24.4° and the eccentricity range is 0.01–0.04. Calculate the June 21st summer peak daily ("noon") insolation at 65N at each of the two configurations, using a reference solar constant of 1360 W/m^2 that corresponds to a reference distance between the Earth and the Sun of 150,000,000 km.

   (c) How big is the difference in W/m^2 between the two configurations? As a percentage of the weaker insolation?

   (d) Calculate the above difference if the obliquity is fixed at 23.3°.

4. **Understanding Milankovitch forcing:** Let the date during the year be represented by the longitude λ of Earth's orbit around the sun, measured from the vernal equinox, March 21st (June 21st corresponds to $\lambda = 90°$). The averaged daily insolation, W, as a function of obliquity ε, precession angle $\tilde{\omega}$ and eccentricity e, may be written at latitudes where there is daily sunrise and sunset, to which we limit our analysis, as by combining equations 21,22,50 of Berger & Loutre,1994,

$$W = \frac{S_0}{\pi} \frac{[1 + e\cos(\lambda - \tilde{\omega})]^2}{(1 - e^2)^2} (h_0 \sin\phi \sin\delta + \cos\phi \cos\delta \sin h_0), \tag{1}$$

   where S_0 is the solar constant (1360 W/m^2), δ is the declination angle of the Sun (given by $\sin\delta = \sin\varepsilon \sin\lambda$), $e = c/a = s(a^2 - b^2)^{1/2}/a$ is the eccentricity, a and b are the semi-major and semi-minor axes, ϕ is the latitude, $\tilde{\omega}$ is the precession angle, and h_0 is the hour angle at sunrise and sunset, given by $\cos h_0 = -\tan\phi \tan\delta$. Refer to this reference to understand how each of these parameters is defined (including when precession is defined to be 0). Our objective is to understand Eqn. 1 using the following steps.

   (a) What is the precession angle when the Earth is closest to the sun during northern hemisphere summer? When it is furthest away during NH summer?

(b) The effect of precession on summer insolation: draw two line plots, of the averaged daily insolation as a function of latitude, for June 21st (more accurately, when $\sin\delta$ is at its maximum), for high and low precession angle values ($\pm 90°$), obliquity of 23° and $e = 0.04$. Limit your plot to latitudes that have sunrise and sunset all year.

(c) The effect of obliquity on summer insolation: draw two line plots of the averaged daily insolation as a function of latitude, for June 21st, for high and low obliquity $\epsilon = 22.3°$ and $\epsilon = 24.4°$, for $\tilde{\omega} = 90°$ and $e = 0.04$. Discuss the dependence of the difference between the two lines on latitude.

(d) Draw a line plot of summer insolation for high obliquity ($\epsilon = 24.4°$) and a precession angle of 90°, and for low obliquity ($\epsilon = 22.3°$) and precession angle of $-90°$. Discuss.

(e) Understanding the seasonal structure of the obliquity signal: draw two contour plots of the daily averaged insolation as a function of latitude and month for one year, for high and low obliquity (24.4°, 22.3°), for a low precession angle ($-90°$) and an eccentricity of 0.04. Discuss.

[What's the point of **optional extra credit challenge problems:** apart from the fun of doing them, they will count against other problems in the same HW assignment in which you may have missed an answer...]