EPS231, Climate dynamics

Dept of Earth and Planetary Sciences, Harvard University

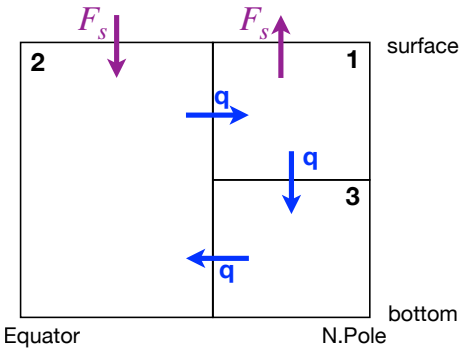
Homework #3

AMOC

Assigned Feb 25, 2025, due: 10 am March 6, 2025 via gradescope

1. Consider a three-box Stommel-like box model similar to that discussed in class (course notes),

$$\begin{aligned}
\frac{1}{2}V\frac{\partial S_1}{\partial t} &= |q|(S_2 - S_1) - F_s, \\
V\frac{\partial S_2}{\partial t} &= |q|(S_3 - S_2) + F_s, \\
\frac{1}{2}V\frac{\partial S_3}{\partial t} &= |q|(S_1 - S_3), \\
q &= K((\rho_1 + \rho_3)/2 - \rho_2).
\end{aligned}$$



assuming, as in class, that the temperatures are fixed.

(a) Solve for the steady states. Use the following parameter values: $\alpha = 0.2$ (kg/m^3)K^{-1}, $\beta = 0.8$ (kg/m^3)ppt^{-1}, $K = 10^9$ (m^3/s)(kg/m^3)$^{-1}$ and $T_1 = 2, T_3 = 1, T_2 = 15$ °C, and use the linear equation of state for density, found in the Stommel Box model notes. Assume a precipitation rate of 2 m/year, and use this to convert to F_S, following the notes and assuming a box depth of 4 km for box 2 and 2 km for boxes 1 and 3; the areas of boxes 1 and 2 are both 50000 km by 50000 km (ridiculous, but still). The reference salinity is $S_0 = 35$ ppt.

(b) Linearize the equations about the steady state with the larger salinity difference $\Delta S = (S_1 + S_3)/2 - S_2$, and write the linearized equations in matrix form. Calculate the eigenvectors and eigenvalues. *hint:* you should come up with something like:

$$\frac{\partial}{\partial t}\begin{bmatrix} S_1' \\ S_2' \\ S_3' \end{bmatrix} = \begin{bmatrix} -(q^* + K\beta\Delta S^*/2) & q^* + K\beta\Delta S^* & -K\beta\Delta S^*/2 \\ XX & XX & XX \\ XX & XX & XX \end{bmatrix} \begin{bmatrix} S_1' \\ S_2' \\ S_3' \end{bmatrix} \equiv \tilde{A}\begin{bmatrix} S_1' \\ S_2' \\ S_3' \end{bmatrix}$$

In which ΔS^* and q^* stand for the variables at steady state.

(c) For each eigenvector, calculate the terms in the salinity equation for each box and describe the contribution of each term [i.e., advection of high salinity water from the low latitudes to the high latitudes]. Your answer should be short sentences describing the balances for each box for each solution.

(d) Are you finding eigenvalues with imaginary parts? What do they mean in terms of variability?

2. **Convective variability:** consider the Welander flip-flop oscillator,

$$\frac{dT}{dt} = A(T_a - T) + k(\rho)(T_d - T)$$
$$\frac{dS}{dt} = B(S_a - S) + k(\rho)(S_d - S)$$
$$\rho = -\alpha T + \beta S$$
$$k(\rho) = \begin{cases} k_0 & \text{if } \rho > \epsilon \\ 0 & \text{else} \end{cases}$$

where,

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k0=5; epsilon=-0.01; % convection parameters:
alpha=0.2; beta=1;   % Eqn of state:
T_d=0; S_d=0;        % deep temp and salinity:
Sa=1; Ta=1;          % atmospheric values:
A=1; B=0.1;          % air-sea exchange coefficients for T, S:
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   (a) Calculate and plot the location of the convective and advective steady states on a $T-S$ plot. Why cannot these two steady states be realized at this parameter regime?

   (b) Find (analytically at first) the maximum convection parameter k that allows for oscillations to exist, keeping all other parameters unchanged. Explain your findings.

3. ***Climate modeling:***

   (a) Run CESM for 100 years after an abrupt increase to $4\times CO_2$ using the instructions provided. Plot a time series of GMST. Describe the functional form of its time evolution. What two timescales characterize the response? What two processes/ climate systems lead to these two time scales?

   (b) Plot the strength of AMOC as a function of time for the 100 years of your simulation. Additionally, plot the stream function as a contour in latitude and depth using the provided code at the beginning and end of your sinulation.

   (c) Contour the decadal average of the surface temperature for the $4\times CO_2$ simulation at the end of the run, that the beginning and the difference. Describe and discuss the spatial pattern of the surface temperature response.

   (d) Plot time series of the net incoming SW, outgoing LW and their difference at the TOA. Scatter-plot the net radiative imbalance versus the change in GMST (a Gregory plot) for the 100-year run. Is this simulation in steady state? Why?

   (e) **Optional extra challenge:** Perform a linear regression of TOA imbalance versus GMST for the last 50 years of the simulation to estimate the warming in equilibrium. What is the climate sensitivity (net warming per CO_2 doubling) of CESM2?

[What's the point of **optional extra credit challenge problems:** apart from the fun of doing them, they will count against other problems in the same HW assignment in which you may have missed an answer...]