EPS231, Climate dynamics
Dept of Earth and Planetary Sciences, Harvard University
Homework #2
ENSO
Assigned Feb 13, 2025, due: 10 am Feb 25, 2025 via gradescope

1. **Transient amplification:** Consider the system $d\mathbf{x}/dt = A\mathbf{x}$ with $\mathbf{x} = (x, y)^T$ and

```
A=[ -64.7965,  -22.9055;
    177.094,  62.5965];
```

   (a) Calculate the eigenvalues and eigenvectors of $A$, and estimate its normality in two different ways: **(A)** Using the `np.linalg.svd` command in python (using the fact that the norm of a matrix is given by its largest singular value): $\text{norm}(A^TA - AA^T)/\text{norm}(A)^2$, what are the maximum and minimum possible values of this measure and what do they tell us about the matrix and its eigenvectors? **(B)** By calculating the angle between the eigenvectors. Estimate the normality of $A + A^T$ using the same two approaches, and interpret your results.

   (b) Calculate the optimal initial conditions for $t = 0.75$, plot time series of $x, y, |\mathbf{x}|$.

   (c) Write the solution as a sum over eigenvectors/ exponents of eigenvalues, $\sum_i a_i \hat{\mathbf{e}}_i e^{\lambda_i t}$, plot the time series of the two terms and discuss.

2. **Optional extra credit challenge: The circle map:** iterate the circle map for $K = 0.9$ using the code on the course webpage. Identify a value of $\Omega$ that leads to a period 3 solution, one to a period 4 solution and a value close to the transition between the two. Plot the iterations after initial transients dissipated as a time series using 'x' symbols and no connecting line. Then solve and plot for $K > 1$ for an $\Omega$ value where you expect an overlap of the 3 and 4 resonances (likely close to the above-mentioned transition value). Do you see irregular behavior? Discuss the relevance to El Niño.

3. **Rossby ray tracing and atmospheric teleconnections** (Hoskins and Karoly (1981), available here). Consider a mean wind field given in spherical coordinates (lat, lon=$\phi, \lambda$) by (m/s)

$$\bar{U}(\phi) = \frac{\bar{\omega} a}{30} \cos^2 \phi$$

   where $\bar{\omega} = 2\pi/(24 \text{ hours})$ and $a$ is the Earth radius.

(a) Write the explicit expressions and plot the following quantities as a function of the Mercator coordinate $y$: $\bar{u}_M(y)$, $\beta_M(y)$ and $K_s(y)$. The Mercator projection is,

$$\begin{aligned} y &= a \ln[(1+\sin\phi)/\cos\phi] \\ \cos\phi &= \mathrm{sech}(y/a) = 1/\cosh(y/a) \\ \sin\phi &= \tanh(y/a) \end{aligned}$$

(b) Calculate and plot $l(y)$ for a few reasonable values of $k$ based on the stationary argument using $k^2 + l^2 = K_s^2$. How do you identify the turning latitude? Explain why Rossby wave reflection happens at the turning latitude and what happens to the wave then.

***The following sections of this problem are all optional extra credit:***

(c) The ray tracing equations (use class notes/slides to understand them) are,

$$dx/dt = c_g^{(x)} \tag{1}$$

$$dy/dt = c_g^{(y)} \tag{2}$$

$$\frac{d_g l}{dt} \equiv (\partial_t + \vec{c}_g \cdot \nabla) l = -\partial_y \Omega(y) \tag{3}$$

Write down explicitly the above equations (1–3) by expanding $c_g$, $\Omega$, etc for Barotropic Rossby waves using the dispersion relation used in class. What are the equations and expected solutions for $k$ and $\omega$? why?

(d) Use the ray tracing code provided to solve for $k = 1, 4, 9$, plot and discuss the calculated rays. Please explain each panel in the figure you get, and in particular explain the interplay between $k, K_s, l, dl/dt$ just before, during and after the point at which the ray turns back to lower latitudes.

(e) Plot $l$ as a function of latitude, and compare to that calculated from the stationary argument using $k^2 + l^2 = K_s^2$. Explain how reflection happens at the turning point when the ray tracing equations are used.

4. ***Climate modeling:***

(a) Calculate and plot the NINO3.4 index monthly time series from the preindustrial control run of the CESM model. It is available on Derecho at /glade/collections/cdg/data/CMIP6/CMIP/NCAR/CESM2/piControl/r1i1p1f1/.

(b) Calculate the power spectrum for the model NINO3.4 and for the provided observed NINO3.4. What is the dominant frequency of the ENSO-like oscillation in observations? In the model?

(c) Calculate and contour composite SST and precipitation for El Niño and La Niña over the Pacific basin from 140E to 285E, and from 30S to p30N, and the difference between them.

(d) **Optional extra credit:** perform PCA on the SST field over 30S to 30N, 140E to 285E. Plot the first two PCs and the corresponding time series. Calculate the percent variance explained by each.

[What's the point of **optional extra credit challenge problems:** apart from the fun of doing them, they will count against other problems in the same HW assignment in which you may have missed an answer...]

# References

Hoskins, B. J. and Karoly, D. J. (1981). The steady linear response of a spherical atmosphere to thermal and orographic forcing. *Journal of the Atmospheric Sciences*, 38(6):1179–1196.