EPS231, Climate dynamics, spring 2023
Dept of Earth and Planetary Sciences, Harvard University
Homework #1
Energy balance, bifurcations, greenhouse, equatorial waves
Assigned: Thu Feb 4, 2025, due: Tue Feb 13, via Gradescope

1. **Simple Energy Balance**. Consider the simple 0-d energy balance model,

$$C\frac{\partial T}{\partial t} = \frac{S_0}{4}(1-\alpha(T)) - \epsilon\sigma T^4,$$
$$\alpha(T) = \alpha_1 - \alpha_2 \tanh((T-T_1)/\Delta T),$$

where $\alpha_1 = 0.6$, $\alpha_2 = 0.3$, $T_1 = 263K$, $\Delta T = 10K$, $S_0 = 1366\text{W/m}^2$, $\epsilon = 0.6$ and $\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4.

   (a) Solve graphically for the steady states $\bar{T}_i$ of this model by plotting dT/dt as function of T. Analyze the stability of each steady state based on this plot.

   (b) Transcritical bifurcation: consider $\dot{x} = \mu x - x^2$. Find the steady states (fixed points) for μ smaller than, equal to, and larger than 0. Draw each case and analyze the stability of each fixed point. Draw the bifurcation diagram (steady states as a function of μ), denoting stable with a solid line, unstable with a dashed line.

   (c) ***Optional extra credit:** Perform a linearized stability analysis of the three different steady-state solutions. To do this, let $T = \bar{T} + T'$, write the time-dependent linearized equation for T' and evaluate the coefficients for each of the three solutions. Which of the three solutions is stable? What are the time scales corresponding to the stable and unstable eigenvalues for each of the three solutions? Show that the results are qualitatively consistent with the results obtained graphically in class for this problem and explain what new information is obtained by the linearized analysis.

2. **Adding atmospheric absorption to two-layer greenhouse model:** change the two-layer model used in class to add 15% absorption of the downward SW flux by the atmosphere.

   (a) Write the modified equations.

   (b) Explain briefly what each term in each equation represents.

   (c) Solve using the same parameters as used in class, compare to the solution without SW absorption and discuss.

   (d) Calculate the change in height of the emission level needed to balance a 4 W/m^2 greenhouse forcing (increase in LW at the surface). Assume a lapse rate of 6.5 K/km.

3. The Recharge Oscillator mechanism for ENSO.

   (a) Explain each term on the RHS of the linear RO equations.

   (b) Solve analytically for the period and growth rate of the linear RO.

(c) Start with the following values for the RO parameters,

$$r = 0.38,\ \alpha b = 17.58,\ R = -1.13,\ \gamma = 0.19.$$

Adjust the parameter r to obtain a growth timescale of 10 years and a decay timescale of 10 years. Solve the linear equations numerically for both cases starting with $T = 1$ °C and $H = 10$ m. Plot the resulting time series for T, h for both cases for 50 years.

(d) **Optional extra credit:** the nonlinear RO. Put the RO in the parameter regime of exponential growth with a growth timescale of 10 years. Add a nonlinear damping term of the form $-cT^3$ to the temperature equation. Solve numerically and observe the development of a self-sustained oscillation. Plot the time series as above, and also as a phase space with coordinates (T, h). Start the oscillation with both a large amplitude and a small amplitude, describe what you see in each case.

(e) **Optional extra credit:** El Niño vs La Niña asymmetry: make γ a function of the thermocline depth as follows,

$$\gamma = \begin{cases} \gamma_0 & \text{if } h < 0 \\ 2\gamma_0 & \text{if } h > 0. \end{cases}$$

Solve in the self-sustained case and observe the amplitude of El Niño vs. La Niña events. Explain the physical reason for the observed asymmetry.

4. ***Climate modeling:***

(a) Run CESM for 20 years in a pre-industrial configuration using the B1850 compset.

(b) Calculate the yearly averaged, global-averaged net TOA radiative balance as a time series over the 20-year run. Is this simulation in a steady state? How can you tell?

(c) Calculate a time averaged, zonally averaged outgoing long wave radiation and incoming short wave radiation at the top of the atmosphere, at 10N and 50N. Are they in balance? Why not?

(d) Contour the time-mean TOA SW and LW energy fluxes.

(e) ***Optional extra credit:** Contour the RMS of the surface temperature from the *annually-averaged* data. Discuss areas where you see significant natural variability.

[What's the point of **Optional extra credit challenge problems:** apart from the fun of doing them, they will count against other problems in the same HW assignment in which you may have missed an answer...]