Notes for climate dynamics course (EPS 231)

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Contents

1 Introduction

This is an evolving detailed syllabus of EPS 231, see course web page, all course materials are available under the downloads directory.

Homework assignments (quasi-weekly) are 50% of final grade, and a final course project constitutes the remaining 50%. There is an option to take this course as a pass/fail with approval of instructor.

2 Basics, energy balance, multiple climate equilibria

Downloads available here.

2.1 Multiple equilibria, climate stability, greenhouse

1. `energy_balance_0d.pdf` with the graphical solutions of the steady state solution to the equation $CT_t = (Q/4)(1-\alpha(T) - \varepsilon\sigma T^4$ obtained using energy_balance_0d.m, and then the quicktime animation of the bifurcation behavior.

2. Some nonlinear dynamics background: saddle node bifurcation ((p 45, Strogatz, 1994) or Applied Math 203 notes p 47), and then the energy balance model as two back to back saddle nodes and the resulting hysteresis as the insolation is varied;

3. Climate implications: (1) faint young sun paradox! (2) snowball (snowball obs from Ed Boyle's lecture);

4. 2-level greenhouse model (notes) and slide on actual mechanism (ppt).

2.2 Small ice cap instability

1. Introduction: The Budyko and Sellers 1d models Simple diffusive energy balance models produce an abrupt disappearance of polar ice as the global climate gradually warms, and a corresponding hysteresis. The SICI eliminates polar ice caps smaller than a critical size (18 deg from pole) determined by heat diffusion and radiative damping parameters. Below this size the ice cap is incapable of determining its own climate which then becomes dominated, instead, by heat transport from surrounding regions. (Above wording from Winton (2006), teaching notes below based on North et al. (1981)).

2. Derive the 1d energy balance model for the diffusive and Budyko versions (eqns 22 and 33): Assume the ice cap extends where the temperature is less than -10C, the ice-free areas have an absorption (one minus albedo) of $a(x) = a_f$, and in the ice-covered areas $a(x) = a_i$; approximate the latitudinal structure of the annual mean insolation as function of latitude using $S(x) = 1 + S_2 P_2(x)$, with $P_2(x) = (3x^2 - 1)/3$ being the second Legendre polynomial (HW); add the transport term represented by diffusion $(r^2 \cos\theta)^{-1}\partial/\partial\theta(D\cos\theta\partial T/\partial\theta)$, which, using $x = \sin\theta$, $1 - x^2 = \cos^2\theta$ and $d/dx = (1/\cos\theta)d/d\theta$ gives the result in the final steady state 1D equation (22),

$$-\frac{d}{dx}D(1-x^2)\frac{dT(x)}{dx} + A + BT(x) = QS(x)a(x,x_0).$$

as boundary condition, use the symmetry condition that $dT/dx = 0$ at the equator, leading to only the even Legendre polynomials.

3. (time permitting) Alternatively to the above, we could model the transport term a-la Budyko as $\gamma[T(x) - T_0]$ with T_0 being the temperature averaged over all latitudes, and the temperature then can be solved analytically (HW).

4. How steady state solution is calculated: eqns 22-29; then 15 and 37, the equation just before 37 and the 4 lines in the paragraph before these equations. Result is Fig. 8: plot of Q (solar intensity) as function of x_s (edge of ice cap). Analysis of results: see highlighted Fig. 8 in sources directory, unstable small ice cap (which cannot sustain its own climate against heat diffusion from mid-latitudes), unstable very-large ice cap (which is too efficient at creating its own cold global climate and grows to a snowball), and stable mid-size cap (where we are now).

5. Heuristic explanation of SICI: Compare Figs 6 and 8 in North et al. (1981), SICI appears only when diffusion is present. It is therefore due to the above mentioned mechanism: competition between diffusion and radiation. To find the scale of the small cap: it survives as long as the radiative effect dominates diffusion: $BT \sim DT/L^2$, implying that $L \sim \sqrt{D/B}$. Units: $[D]$ =watts/(degree K$\times$m^2) (page 96), $[B]$ =m^2/sec (p 93), so that $[L]$ is non dimensional. That's fine because it is in units of sine of latitude. Size comes out around 20 degrees from pole, roughly size of present-day sea ice!

6. The important lesson(!): p 95 in North et al. (1981), left column second paragraph, apologizing for the (correct...) prediction of a snowball state.

7. This is a complex PDE (infinite number of degrees of freedom), displaying a simple bifurcation structure. In such case we are guaranteed by the central manifold and normal form theorems that it can be transformed to the normal form of a saddle node near the appropriate place in parameter space. First, transformed to center manifold and get an equation independent of stable and unstable manifolds (first page of lecture_04_cntr_mnfld.pdf); next, transform to normal form within center manifold (lecture_03_bif1d2.pdf, p 89).

8. (time permitting) Numerically calculated hysteresis in 1D Budyko and Sellers models: figures ebm1d-budyko.jpg and ebm1d-sellers.jpg obtained using ebm_1dm.m).

9. (time permitting) One noteworthy difference between Budyko and Sellers is the transient behavior, with Budyko damping all scales at the same rate, and Sellers being scale-selective. (original references are Budyko, 1969; Sellers, 1969).

3 ENSO

Downloads available here.

3.1 ENSO background and delay oscillator models

Sources: Woods Hole (WH) notes (Cessi et al., 2001), lectures 0, 1, 2, here, plus the following: Gill's atmospheric model solution from Dijkstra (2000) technical box 7.2 p 347; recharge oscillator from Jin (1997) (section 2, possibly also section 3);

- The climatological background: easterlies, walker circulation, warm pool and cold tongue, thermocline slope (ppt, and lecture 1 from WH notes).

- Dynamical basics: equatorial Rossby and Kelvin waves, thermocline slope, SST dynamics, atmospheric heating and wind response to SST from Gill's model (rest of WH lecture 1). The coupled feedback,

- The heuristic delayed oscillator equation from section 2.1 in WH notes. One detail to note regarding how do we transition from $+\hat{b}h_{\text{off}-\text{eq}}(t-[\frac{1}{2}\tau_R+\tau_K])$ to $-\bar{b}\tau_{eq}(t-[\frac{1}{2}\tau_R+\tau_K])$ and then to $-bT(t-[\frac{1}{2}\tau_R+\tau_K])$: $h_{\text{off}-\text{eq}}$ depends on the Ekman pumping off the equator. In the northern hemisphere, if the wind curl is positive, the Ekman pumping is positive, upward ($w_E=\text{curl}(\vec{\tau}/f)/\rho$), and the induced thermocline depth anomaly is therefore negative (a shallowing signal). The wind curl may be approximated in terms of the equatorial wind only (larger than the off-equatorial wind), consider the northern hemisphere:

  $$h_{\text{off}-\text{eq}} \propto -w^{Ekman}_{\text{off}-\text{eq}} \propto -\text{curl}(\tau_{\text{off}-\text{eq}}) \approx \partial_y \tau^{(x)}_{\text{off}-\text{eq}} \approx (\tau^{(x)}_{\text{off}-\text{eq}} - \tau^{(x)}_{\text{eq}})/L \propto -\tau^{(x)}_{\text{eq}}.$$

  Finally, as the east Pacific temperature is increasing, the wind anomaly in the central Pacific is westerly (positive), leading to the minus sign in front of the T term,

  $$-\tau^{(x)}_{\text{eq}}(t-[\frac{1}{2}\tau_R+\tau_K])/L \propto -T(t-[\frac{1}{2}\tau_R+\tau_K]).$$

- Next, the linearized stability analysis of the Schopf-Suarez delayed oscillator from the WH notes section 2.1.1. Show numerical solution of this model for values on both sides of the first bifurcation point using delay_Schopf_Suarez_1989.m

- Self-sustained vs damped: nonlinear damping term and, more importantly, the proximity of ENSO to the first bifurcation point beyond steady state, discuss Hopf bifurcation from non linear dynamics notes pp 26-28 here or Strogatz (1994).

- (time permitting) A more quantitative derivation of delay oscillator, starting from the shallow water equations and using Jin's two-strip approximation (WH notes, section 2.2).

3.2 ENSO's irregularity

3.2.1 Chaos

- Phase locking: flows on a circle, synchronization/ phase locking, fireflies.

- Circle map and quasi-periodicity route to chaos:

$$\theta_{n+1} = f(\theta_n) = \theta_n + \Omega - \frac{K}{2\pi}\sin 2\pi\theta_n, \quad \theta_n = mod(1)$$

  $K = 0$ and quasi-periodicity, winding number: $\lim_{n\to\infty}(f^n(\theta_0) - \theta_0)$, not taking θ_n as mod(1) for this calculation. $0 < K < 1$ and phase locking, Arnold tongues, Farey tree. $K = 1$ and the devil's staircase.

- Some generalities on identifying quasi-periodicity route to chaos in a complex system, including delay coordinate phase space reconstruction.

- References for phase locking: (Strogatz, 1994), for quasi-periodicity route to chaos: Schuster (1989). For both: course notes for applied math 203, pages 71,75,77-83 in lecture_bif1d2_eli.pdf. delay coordinate phase space reconstruction in lecture_bif2d3_eli.pdf.

- Slides on transition to chaos in CZ model.

3.2.2 Noise

- Non normal amplification from WH notes, plus: maximization of $\Psi(\tau)^T\Psi(\tau)$ instead of $d/dt\Psi(t = 0)$ described in the WH notes. Eigenvalue is the amplification factor from the initial conditions to the amplified state.

- WWBs in observations: seem stochastic, seen to precede each El Nino event, affect Pacific by forcing of equatorial Kelvin waves. Show Hovmoller diagram with WWBs and SST from Yu et al. (2003), in jpg file; wind stress sequence showing WWB evolution from Vecchi and Harrison (1997) (all Figs at end of this report); effects of wind bursts on SST and thermocline depth (heat content) from Mcphaden and Yu (1999) Figs 1,2,3 (last one is model results); ocean-only model response to a strong WWB: Zhang and Rothstein (1998), Figs. 4, 5, showing the response to a wind burst after 10 days and after several months;

- Stochastic optimals: (can also leave this to be discussed in the context of ENSO) the derivation from Tziperman and Ioannou (2002): consider a stochastically forced linear system:

$$\dot{P} = AP + f(t)$$

solution is

$$P(\tau) = e^{A\tau}P(0) + \int_0^\tau ds\, e^{A(\tau-s)} f(s) = B(\tau,0)P(0) + \int_0^\tau ds\, B(\tau,s) f(s)$$

variance of the solution is given by

$$\begin{aligned} var(\|P\|) &= \langle P_i(\tau)P_i(\tau)\rangle - \langle P_i(\tau)\rangle\langle P_i(\tau)\rangle \\ &= \left\langle \int_0^\tau ds \int_0^\tau dt B_{il}(\tau,s) f_l(s) B_{in}(\tau,t) f_n(t) \right\rangle \\ &= \int_0^\tau ds \int_0^\tau dt B_{il}(\tau,s) B_{in}(\tau,t) \langle f_l(s) f_n(t)\rangle \end{aligned}$$

Specifying the noise statistics as separable in space and time, with C_{ln} being the noise spatial correlation matrix and $D(t-s)$ the temporal correlation function (delta function for white noise),

$$\langle f_l(s) f_n(t)\rangle = C_{ln} D(t-s)$$

we have

$$\begin{aligned} var(\|P\|) &= \int_0^\tau ds \int_0^\tau dt B_{il}(\tau,s) B_{in}(\tau,t) C_{ln} D(t-s) \\ &= Tr(\int_0^\tau ds \int_0^\tau dt [B^T(\tau,s) B(\tau,t)] C) \\ &\equiv Tr(ZC) \end{aligned}$$

This implies that the most efficient way to excite the variance is to make the noise spatial structure be the first eigenvector of Z. To show this, show that eigenvectors of Z maximize $J = Tr(CZ) = Z_{ij}C_{ji}$; assuming that the spatial noise structure is f_i, we have $C_{ij} = f_i f_j$ and we need to maximize $Z_{ik} f_j f_k + \lambda(1 - f_k f_k)$; differentiating wrt f_i we get that f_i is an eigenvector of Z; show picture of optimal modes from WH notes; discuss their model dependence; model dependence of the optimals from Fig. 11, 12 and 17 in Moore and Kleeman (2001).

- Are WWBs actually stochastic, or are their statistics a strong function of the SST, making the stochastic element less relevant?

3.3 Teleconnections

- Motivation for ENSO teleconnection: show pdf copy of impacts page from El Nino theme page.

- Further motivation: results of barotropic model runs from Hoskins and Karoly (1981) Figures 3,4,6,8,9, showing global propagation of waves due to tropical disturbances.

- General ray tracing theory based on notes-ray-tracing.pdf. Note that this is a non-rigorous derivation, not using multiple-scale analysis.

- Qualitative discussion from Hoskins and Karoly (1981) after solution 5.23. Start from dispersion relation for stationary waves, $0=\omega=\bar{u}_M k-\frac{\beta_M k}{k^2+l^2}$, and define $K_s=(\beta_M/u_M)^{1/2}=k^2+l^2$. Based on these only, discuss trapping by jet. Note that if $k>K_s$ then l must be imaginary, which implies evanescent behavior and trapping/ reflection of the ray in latitude.

- (time permitting) For baroclinic atmospheric waves, $0=\omega=\bar{u}_M k-\frac{\beta_M k}{k^2+l^2+L_R^{-2}}$, so that $K_s=(\beta_M/u_M)^{1/2}-L_R^{-2}=k^2+l^2$, and they are more easily trapped, and are going to be trapped at the equator with a scale of the Rossby radius of deformation which is some 1000km or so (see discussion on page 1195 left column).

- Derivation of quantitative solution from the beginning of subsection 5b, equations 5.1-5.13.

- To get some idea of the amplitude of the propagating waves, use the WKB solution (Bender and Orszag (1978) section 10.1): start with equation 5.18. Substituting into 5.9 this leads to $d^2P/dy^2+l^2(\varepsilon y)P=0$ for $l^2(\varepsilon y)$ defined in 5.20; to transform to standard WKB form, define $Y=\varepsilon y$ so that $\varepsilon^2 d^2P/dY^2+l^2(Y)P=0$; try a WKB solution corresponding to a wave-like exponential with a rapidly varying phase plus a slower correction $P=\exp(S_0(Y)/\delta+S_1(Y))$ to find
$$\varepsilon^2[(S_0'/\delta+S_1')^2+(S_0''/\delta+S_1'')]P+l^2P=0;$$
let $\delta=\varepsilon$ and then $O(1)$ equation is ${S_0'}^2+l^2(Y)=0$ so that $S_0=i\int l(Y)\,dy$ (if l^2 is nearly constant, this simply reduces to the usual wave solution e^{ily}). Next, consider $O(\varepsilon)$ equation which, after using the $O(1)$ equation, is $2S_0'S_1'+S_0''=0$ and the solution is $S_1'=-S_0''/(2S_0')=-(dl/dY)/(2l)=-d/dY(\ln l^{1/2})$ so that $S_1=\ln l^{-1/2}$ which means that the wave amplitude is $l^{-1/2}$. This gives the solution in Hoskins and Karoly (1981) equation (5.21, 5.23), see further discussion there.

- Discuss the constant angular momentum flow solution (section 5c, page 1192, Fig. 12) and then the one using realistic zonal flows (Figs. 13, 14, 15, etc);

- Finally, mention that later works showed that stationary linear barotropic Rossby waves excite nonlinear eddy effects which may eventually dominate the teleconnection effects.

4 Thermohaline circulation

Downloads available here.

4.1 Phenomenology, Stommel box model

- Background, schematics of THC, animations of CFCs in ocean, sections and profiles of T, S from here, meridional mass and heat transport, climate relevance; RAPID measurements (Cunningham et al., 2007); anticipated response during global warming; MOC vs THC.

- Stommel model: mixed boundary conditions, Stommel-Taylor model notes; Qualitative discussion on proximity of present day THC to a stability threshold (Tziperman, 1997; Toggweiler et al., 1996). And again the surprising ability of simple models to predict/ explain GCM results (Fig. 2 from Rahmstorf (1995)). See also Dijkstra (2000, section 3.1.1, 3.1.2, 3.1.3)

4.2 Scaling and energetics

1. Scaling for the amplitude and depth of the THC from Vallis (2005) chapter 15, section 15.1, showing that the THC amplitude is a function of the vertical mixing which in turn is due to turbulence. Mention only briefly the issue of the "no turbulence theorem" and importance of mechanical forcing (details in the next section, not to be covered explicitly).

2. (Time permitting) Energetics, Sandstrom theorem stating that "heating must occur, on average, at a lower level than the cooling, in order that a steady circulation may be maintained against the regarding effects of friction" (eqn 15.23 Vallis, 2005). The "no turbulence theorem" in the absence of mechanical forcing by wind and tides (eqn 15.27) without mechanical mixing, and (15.30) with; hence the importance of mechanical energy/ mechanical forcing for the maintenance of turbulence and of the THC. Sections 15.2, 15.3 (much of this material is originally from Paparella and Young, 2002);

3. Tidal energy as a source for mixing energy (Munk and Wunsch, 1998, Figures 4,5).

4.3 Convective oscillations

- Advective feedback and convective feedback from sections 6.2.1, 6.2.2 in Dijkstra (2000) and from the original Lenderink and Haarsma (1994); Hysteresis from Fig. 8 in Lenderink and Haarsma (1994) and "potentially convective" regions in their GCM from Fig. 11.

- Flip-flop oscillations (Welander1982_flip_flop.m) and loop oscillations (Dijkstra, 2000, sections 6.2.3, 6.2.4); The Lenderink and Haarsma (1994) model, when put in the regime without any steady states, shows exactly the same flip-flop oscillations, yet without the artificial convection threshold necessary in Welander's model.

- Analysis of relaxation oscillations following Strogatz (1994) example 7.5.1 pages 212-213, or nonlinear dynamics course notes: slow phase and fast phases.
- Winton's deep decoupling oscillations will be covered below as part of DO events.

4.4 Additional variability mechanisms, stochastic excitation

A review of classes of THC oscillations: small amplitude/ large amplitude; linear and stochastically forced/ nonlinear self-sustained; loop oscillations due to advection around the THC path, or periodic switches between convective and non convective states; relaxation oscillations; noise induced switches between steady state, stochastic resonance;

Some types of THC oscillations not covered above:

- Linear Loop-oscillations due to advection around the circulation path: Stability regimes in a 4-box model: stable, stable oscillatory, [Hopf bifurcation], unstable oscillatory, unstable; Note changes from 2-box Stommel model: oscillatory behavior and change to the point of instability on the bifurcation diagram; Note the need of stochastic forcing to excite this type of variability. Next, the GCM study of Delworth et al. (1993); this paper also demonstrates the link between the variability of meridional density gradients and of the THC; Note the proposed role of changes to the gyre circulation in this paper, mention related mechanisms based on ocean mid-latitude Rossby wave propagation; then a box model fit to the GCM, showing that the horizontal gyre variability may not be critical and that the variability is due to the excitation of a damped oscillatory mode (Griffies and Tziperman, 1995); Useful and interesting analysis methods: composites (DMS Figs. 6,7), and regression analysis between scalar indices (Figs. 8,9) and between scalar indices and fields (Figs. 10, 11, 12).
- A complementary view of the above stochastic excitation of damped THC oscillatory mode: first, Hasselmann's model with a red spectrum

  $$\begin{aligned} \dot{x} + \gamma x &= \xi(t) \\ P(\omega) = |\hat{x}|^2 &= \xi_0^2/(\omega^2 + \gamma^2) \end{aligned}$$

  vs a damped oscillatory mode excited by noise that results in a spectral peak,

  $$\begin{aligned} \ddot{x} + \gamma \dot{x} + \Omega^2 x &= \xi(t) \\ P(\omega) = |\hat{x}|^2 &= \xi_0^2/((\Omega^2 - \omega^2) + \gamma^2) \end{aligned}$$

- Stochastic variability due to noise induced transitions between steady states (double potential well) Cessi (1994). A GCM version of jumping between two equilibria under sufficiently strong stochastic forcing: Weaver and Hughes (1994).
  (time permitting:) Cessi: Stommel 2 box model from section 2 with model derivation and in particular getting to eqn 2.9 with temperature fixed and salinity difference satisfying an equation of a particle on a double potential surface; section 3 with deterministic perturbation;

- Stochastic resonance: periodic FW forcing plus noise. Matlab code `Stommel_stochastic_resonance.m` from APM115, and jpeg figures with results: SRa.jpg, SRb.jpg, SRc.jpg;

- THC oscillations due to "Thermal" Rossby waves analyzed by Te Raa and Dijkstra (2002), using equations 8-10 and Figure 7 of Zanna et al. (2011).

- Stochastic forcing, non normal THC dynamics, transient amplification; stochastic optimals if this wasn't discussed in the context of ENSO, see ENSO notes above (e.g., in the context of The 3-box model of Tziperman and Ioannou (2002), or the spatially resolved 2d model of Zanna and Tziperman (2005)). Figures for the amplification and mechanism, taken from a talk on this subject (file nonormal_THC.pdf). (Time permitting): A more general issue that comes up in this application of transient amplification is the treatment of singular norm kernel (appendix) and infinite amplification; show and explain the first mechanism of amplification (Figure 2); note how limited the amplification may actually be in this mechanism.

- (Time permitting) Use of eigenvectors for finding the instability mechanism: linearize, solve eigenvalue problem, substitute spatial structure of eigenvalue into equations and see which equations provide the positive/ negative feedbacks; results for THC problem (e.g., Tziperman et al., 1994, section 3): destabilizing role of $v'\nabla\bar{S}$ and stabilizing role of $\bar{v}\nabla S'$; difference in stability mechanism in upper ocean ($v'\nabla\bar{S}$) vs that of the deep ocean (where $\nabla\bar{S} = 0$ and $\bar{v}\nabla S'$ is dominant); for temperature, also ($v'\nabla\bar{T}$ is more important, but from the eigenvectors one can see that v' is dominated by salinity effects; GCM verification and the distance of present-day THC from stability threshold: Figs 4,5,6 from Tziperman et al. (1994); Fig 3 from Toggweiler et al. (1996); Figs 1, 2, 3 from Tziperman (1997).

4.5 More on stochastic variability (time permitting)

- As a preparation for the rest of this class: the derivation of diffusion equation for Brownian motion following Einstein's derivation from Gardiner (1983) section 1.2.1; next, justify the drift term heuristically; then, derivation Fokker-Plank equation from Rodean (1996), chapter 5; Note that equation 5.17 has a typo, where the LHS should be $\frac{\partial}{\partial t}T_\tau(y_3|y_1)$; Then, first passage time for homogeneous processes from Gardiner (1983) section 5.2.7 equations 5.2.139-5.2.150; 5.2.153-5.2.158; then the one absorbing boundary (section b) and explain the relation of this to the escape over the potential barrier, where the potential barrier is actually an absorbing boundary, with equations 5.2.162-5.2.165; Note that 5.2.165 from Gardiner (1983) is identical to equation 4.7 from Cessi (1994); Next, random telegraph processes are explained in Gardiner (1983) section 3.8.5, including the correlation function for such a process; Cessi (1994) takes the Fourier transform of these correlation functions to obtain the spectrum in the limit of large jumps, for which the double well potential problem is similar to the random telegraph problem.

  Next, back to Cessi (1994) section 4: equation 4.4 (Fokker-Planck), 4.6 and Fig. 6 (the stationary solution for the pdf); then the expressions for the mean escape time (4.7) and the rest of the equations all the way to end of section 4, including the random telegraph process and the steady probabilities for this process;

Finally, from section 5 of Cessi (1994) with the solutions for the spectrum in the regime of small noise (linearized dynamics) and larger noise (random telegraph); For the solution in the small noise regime (equation 5.3), let $y' = y - y_a$ and then Fourier transform the equation to get $-i\omega\hat{y}' = -V_{yy}\hat{y} + \hat{p}'$ where hat stands for Fourier transform; then write the complex conjugate of this equation, multiply them together using the fact that the spectrum is $\mathcal{S}_a(w) = \hat{y}'\hat{y}'^{\dagger}$ to get equation 5.5; Show the fit to the numerical spectrum of the stochastically driven Stommel model, Figure 7;

- Zonally averaged models and closures to 2d models (Dijkstra, 2000, section 6.6.2, pages 282-286, including technical box 6.3); Atmospheric feedbacks Marotzke (1996)?

5 Dansgaard-Oeschger, Heinrich events

Downloads here.

Observed record of Heinrich and DO events: IRD, Greenland warming, possible relation between the two; synchronous collapses? or maybe not? Use Figures of obs from Heinrich_slides.pdf

Winton model: Convection, air-sea and slow diffusion: Relaxation oscillations/ Thermohaline flushes/ "deep decoupling" oscillations (Winton, 1993, section IV).

THC flushes and DO events: DO explained by large amplitude THC changes (Ganopolski and Rahmstorf, 2001); from this paper, show hysteresis diagrams for modern and glacial climates, the ease of making a transition between the two THC states in glacial climate; time series of THC during DO events; these oscillations are basically the same as Winton's deep decoupling oscillations and flushes (Figure 7 in Winton (1993), see under THC variability).

Alternatively, sea ice as an amplifier of THC variability: Preliminaries: sea ice albedo and insulating feedbacks; volume vs area in present-day climate (i.e. typical sea ice thickness in arctic and southern ocean); simple model equation for sea ice volume (Sayag et al., 2004, eqn numbers from): sea ice melting and formation (18), short wave induced melting (3rd term on rhs in 20), sea ice volume equation (20); climate feedbacks: insulating feedback (3), albedo feedback (19).

Sea ice and DO events: Figure of Camille's (Li et al., 2005) AGCM experiments (again Heinrich_slides.pdf). Sea ice as an amplifier of *small* THC variability (Kaspi et al., 2004); Possible variants of the sea ice amplification idea: Stochastic excitation of THC+sea ice=DO like variability (Fig. 5 in Timmermann et al., 2003); self-sustained DO events with sea ice amplification (Loving and Vallis, 2005).

Precise clock behind DO events? Stochastic resonance? First, (Rahmstorf, 2003): clock error, triggering error and dating error; is it significant, or can we find a periodicity for which some "clock" might fit the time series? Next, stochastic resonance: (Alley et al., 2001): consider a histogram of waiting time between DO events (Fig. 2) and find that these are multiples of 1470, suggesting stochastic resonance as a possible explanation. The bad news: no clock, (Ditlevsen et al., 2007), see their Fig. 1 and read their very short conclusions section.

DO teleconnections: Wang et al. (2001) show a strong correlation of Hulu cave in China with Greenland ice cores (show both figures). Similarly, Denton and Hendy (1994) show a correlation of Younger Dryas and glaciers in New Zealand. What is the mechanism? (1) Atmospheric teleconnections: if this wasn't covered in the El Nino section, discuss atmospheric Rossby wave teleconnections. (2) MOC/THC teleconnections: weaker NADW import to Southern Ocean due weakening of MOC (Manabe and Stouffer, 1995); see also the related THC seesaw (Broecker (1998), and this). (3) Alternatively, teleconnections may be described as the "propagation of zonally integrated meridional transport anomalies in a reduced-gravity ocean" (Johnson and Marshall (2004), Fig. 4 and discussion around eqns 1,2,10). Essentially equatorial and coastal Kelvin waves propagate anomalies due to changes in convection rate very fast from the north-west Atlantic and spread the information along eastern boundaries, and then slower Rossby waves transmit the information to the ocean interior on a decadal time scale. This mechanism also allows for inter-basin exchanges.

Heinrich events: binge-purge mechanism: following MacAyeal (1993a), and including argument for which external forcing is not likely (p 777) and heuristic argument for the time scale (p 782); then show equations for the more detailed model of MacAyeal (1993b) (from slide 22 of Heinrich_slides.pdf (or Kaspi et al., 2004)), and solution (slide 11); on the relation between DO and Heinrich events: do major DO follows Heinrich events? Do Heinrich events happen during a cold period just before DO events? When the ice sheet model is coupled to a simple ocean-atmosphere model (slide 21), can get the response of the climate system as well (slides 27,28,29) via a THC shutdown (cold event) followed by a flush (warm event). Finally, synchronous collapses? slides (33-47).

6 Glacial cycles

Downloads here.

6.1 Basics

Briefly: glacial cycle phenomenology: SIS intro slides

Milankovitch forcing: same, see also movies and explanations in page by Peter Huybers, here.

Basic feedbacks: lecture 8 from WH notes;

Supplement the discussion of the parabolic profile with the more accurate expression shown by the solid line in Fig 11.4 in Paterson (1994); First, rate of strain-stress relationship from Van-Der-Veen (1999): strain definition (section 2.1, p. 7-9); rate of strain is even better explained by Kundu and Cohen (2002) sections 3.6 and 3.7, pages 56-58; stress and deviatoric stress and stress-rate of strain relationship and Glenn's law from section 2.3, pages 13-15 of Van-Der-Veen (1999). Next, the ice sheet profile derivation: the following derivation is especially sloppy in dealing with the constants of integration, and very roughly follows Chapter 5, p 243 eqns 6-10 and p 251, eqns

18-22 from Paterson (1994):

$$\dot{\varepsilon}_{xz} = \frac{1}{2}\frac{du}{dz} = A\tau_{xz}^n = A(\rho g(h-z)\frac{dh}{dx})^n$$

integrate from $z = 0$ to z, and use the b.c. $u(z=0) = u_b$,

$$u(z) - u_b = 2A(\rho g\frac{dh}{dx})^n \frac{(h-z)^{n+1}}{n+1} - 2A(\rho g\frac{dh}{dx})^n \frac{h^{n+1}}{n+1}$$

Let $u_b = 0$ (no sliding) and average the velocity in z,

$$\begin{aligned}
\bar{u} &= (1/h)\int_0^h dz 2A\left(\rho g\frac{dh}{dx}\right)^n \frac{(h-z)^{n+1}}{n+1} - 2A\left(\rho g\frac{dh}{dx}\right)^n \left(\frac{h^{n+1}}{n+1}\right) \\
&= \frac{2A}{(n+1)}\left(\rho gh\frac{dh}{dx}\right)^n h\left(\frac{1}{n+2} - 1\right) \\
&= -\frac{2A}{(n+2)}\left(\rho gh\frac{dh}{dx}\right)^n h. \qquad (1)
\end{aligned}$$

Next we use continuity, assuming a constant accumulation of ice at the surface, $d(h\bar{u})/dx = c$ which implies together with the last equation

$$cx + K_1 = h\bar{u} = -\frac{2A}{(n+2)}\left(\rho gh\frac{dh}{dx}\right)^n h^2 = K_2\left(h\frac{dh}{dx}\right)^n h^2$$

where ablation is assumed to occur only at the edge of the ice sheet at $x = L$. The last eqn may be written as

$$(K_3x + K_4)^{1/n}dx = h^{2/n+1}dh$$

and solved using boundary conditions of $h(x=0) = H$ and $h(x=L) = 0$ to obtain

$$(x/L)^{1+1/n} + (h/H)^{2/n+2} = 1.$$

This last equation provides the better fit to obs in Paterson Fig 11.4 (also shown in WH notes).

6.2 Glacial cycle mechanisms

General outline: Calder model, Imbrie and Imbrie, Paillard, Le-Treut and Ghil, 100 kyr from a collapse of the ice sheets due to geothermal heat build up and induced basal melting, sea ice switch, Saltzman "earth system" models, stochastic resonance; phase locking, and a discussion of how a good fit to ice volume does not imply a correct physical mechanism; difference between locking to periodic and quasi-periodic forcing; Huybers' integrated insolation and the 41kyr problem. **Sources:** lecture 9 from WH notes, Paillard review paper, SIS slides, Pacing glacial cycle slides, Huybers slides.

Some additional details follow.

On the expected spectral characteristics of the solutions to equations 36 and 37 (Imbrie&Imbrie) from lecture 9 of the WH notes: Fourier transforming the two, we find $i\omega\hat{V}(\omega) = -k\hat{i}(\omega)$. Multiply by the complex conjugate of this equation to get the power spectrum $|\hat{V}(\omega)|^2 = k^2|\hat{i}(\omega)|^2/\omega^2$, implying that higher frequency are damped, which emphasizes the low frequencies, including 100kyr, relative to the high ones, including 20 and 41kyr. For the second model, in the simpler case where τ is constant, we similarly have $(i\omega+\tau^{-1})\hat{V}(\omega) = \hat{i}(\omega)$, leading to the spectrum $|\hat{V}(\omega)|^2 = |\hat{i}(\omega)|^2/(\omega^2+\tau^{-2})$, and now τ may be used as a tuning parameter to determine which frequencies are damped. But this still amplifies both the 100 and 400 frequencies in $\hat{i}(\omega)$, which is inconsistent with the proxies.

6.3 CO_2

1. Basics of ocean carbonate system, including carbonate ions, prognostic equations for alkalinity, total carbon and a nutrient, from notes. [Section 4 till eqn 9; section 4.1, eqns 10-19; section 4.2 and 4.3 with an approximate analytic solution of the carbonate system and a detailed explanation of the response to calcium carbonate dissolution; then section 5, first paragraph].

2. The exact solution of this set of equation for pCO_2 as function of total CO_2 and alkalinity based on Fig 1.1.3 from Zeebe's book. Demonstrate by discussing the effect of adding CO_2 from volcanoes, of photosynthesis and remineralization ($2CO_2 + 2H_2O \rightleftharpoons 2CH_2O + 2O_2$) and of $CaCO_3$ deposition or dissolution ($CaCO_3 \rightleftharpoons Ca^{2+} + CO_3^{2-}$, or, equivalently, $CaCO_3 + CO_2 + H_2O \rightleftharpoons Ca^{2+} + 2HCO_3^-$).

3. The simple 3-box model (Toggweiler, 1999), introduce using his Fig. 1.

4. Heuristic analysis from the beginning of section 3 of (Toggweiler, 1999): (his eqns 4,5,8, or see hand written notes).

5. Results of full 3 box model for glacial CO_2 as function of ventilation by f_{dh} and high latitude biological pump P_h (section 2), Figures 2 and 3.

6. Criticism of the results: section 2, paragraphs 2, 3 on left column, page 575, or p 2 of the same hand written notes as linked above. Bottom line is that the model also predicts changes to the high latitude surface nutrients PO_{4h}, and this change hasn't been observed. Toggweiler later shows that reversing the THC in the southern ocean (to be more realistic, actually) helps with this.

7 Pliocene climate

Phenomenology (Molnar and Cane, 2007; Dowsett et al., 2010) and relevant proxies for temperature, productivity, upwelling, CO_2, etc. Pliocene as nearest analogue of future warming (although only for the equilibrium response). Observations of global temperature, permanent El Nino and warming of mid-latitude upwelling sites.

Possible mechanisms for permanent El Nino: FW flux causing a collapse of the meridional density gradient and therefore meridional ocean heat flux, which leads to a deepening of the thermocline; Hurricanes and tropical mixing; opening of central American seaway; movement of new Guinea and changes in Pacific-Indian water mass exchange; atmospheric superrotation. Mechanisms for warming of upwelling sites.

Details of all of the above in Pliocene powerpoint presentation.

8 Equable climate

Downloads here.

Earth Climate was exceptionally warm, and the equator to pole temperature difference (EPTD) exceptionally small, during the Eocene (55Myr ago), when the continent location was not dramatically different. Many explanations have been proposed, and we will briefly survey some.

First, Phenomenology from slides.

8.1 Equator to pole Hadley cell

(Farrell, 1990)

- Angular momentum conservation leading to large u in the upper branch of the Hadley cell: $M=(u+\Omega r\cos\theta)r\cos\theta$ and if a particle starts with $u(\text{equator})=0$, we find from $M(30)=M(equator)$ that
`u(30)=(6300000*2*pi/(24*3600))*(1/cosd(30)-cosd(30))=132m/sec.`

- The resulting large u_z is balanced via thermal wind by strong T_y, leading to a large EPTD (eqn 1.5 in Farrell (1990))

- To break this constraint, can dissipate some angular momentum, reduce f (as on Venus), or increase the tropopause height H.

- (Optional self-reading) The details, given in Farrell (1990), require the extension of the Held-Hou (1980) ideas to include dissipation. Vallis (2005) summarizes the frictionless theory very nicely (sections 11.2.2-11.2.3).

8.2 Polar stratospheric clouds (PSCs)

- Greenhouse effect due to PSCs (Sloan et al., 1992)

- Zonal stratospheric circulation (Vallis Fig 13.12, and p 568): SW absorption near summer pole leads to a reversed temperature gradient in summer hemisphere: e.g., $T_y<0$ in southern hemisphere during Jan. This leads to $u_z \propto T_y/f_0>0$ in southern hemisphere July, and using

$u = 0$ at top of stratosphere we get $u < 0$ (easterlies) during summer (Jan) in the southern hemisphere stratosphere. Similarly, $u < 0$ (easterlies) during summer (July) in northern hemisphere.

- Winter hemisphere (northern Jan, southern July) has no SW at pole, temperature gradient is not reversed and winds are westerlies there.

- Stationary Rossby waves propagating vertically from the troposphere cannot propagate into easterlies, therefore can only reach stratosphere in the winter hemisphere.

- Brewer-Dobson stratospheric circulation: zonally averaged Transformed Eulerian Mean (TEM) momentum balance is $-f_0\overline{v}^* = \overline{v'q'}$ (Vallis, eqn 13.88; $q' = \zeta' + f\partial_z(b'/N^2)$, see chapter 7.2). Assuming the potential vorticity flux is down gradient (equatorward, because the gradient is dominated by β), the rhs is negative, so that the mean flow $\overline{v}^* > 0$ is poleward.

- B-D circulation warms the pole and cools the equator in the stratosphere (Vallis eqn 13.89): $N^2\overline{w}^* = \frac{\theta_E - \theta}{\tau}$, together with positive w in tropics and negative in polar areas forced by poleward B-D meridional flow. This leads to $\theta < \theta_E$ (cooling!) at equator (where $\overline{w}^* > 0$) and $\theta > \theta_E$ (warming!) at pole (where $\overline{w}^* < 0$).

- Feedback between EPTD, vertically propagating planetary waves, Brewer-Dobson stratospheric circulation and PSCs (Kirk-davidoff et al., 2002): warmer climate means weaker tropospheric EPTD, this leads to weaker mean tropospheric winds and weaker synoptic scale motions (which are, in turn, created via baroclinic instability of the mean winds and meridional temperature gradient). Both of these factors weaken the production of vertically propagating Rossby waves (forced by mean winds interacting with topography, and by synoptic motions). As a result, weaker Eliassen-Palm flux EP, weaker $\nabla \cdot EP = \overline{v'q'}$, weaker B-D circulation, and therefore colder pole and warmer equator. Colder pole allows more PSCs to develop. This, in turn, further weakens the EPTD in troposphere, providing a positive feedback.

8.3 Hurricanes and ocean mixing

- What sets maximum hurricane strength, "hyper-canes" (from Kerry's web page): rate of energy *input* per unit area into the hurricane is roughly $G = \varepsilon C_k \rho V_s L(q_0^* - q_a)$, where V_s is the max surface wind speed, ε efficiency in translating enthalpy to K.E., qs are the atmospheric surface specific humidity ocean saturation humidity, L is the latent heat of evaporation, ρ the air density and C_k the bulk coefficient for evaporation. The rate of energy dissipation per unit area is given by $D = C_D \rho V_s^3$.

- Think of the hurricane as a Carnot cycle: air acquires heat (in the form of moisture from evaporation) as it flows along the surface toward the center, it then expands adiabatically while releasing latent heat going up; it releases the heat to the environment while mixing out of the convective plumes at the top of the storm (temperature T_0, and undergoes compression at while descending back to the surface. The *efficiency* of KE generation in a Carnot cycle in terms of the temperatures of the warm and cold reservoirs involved is $\varepsilon = (T_H - T_C)/T_H$. For Hurricanes, $T_H = SST$ is temperature of the heat source (the ocean surface). $T_C = T_0$ is the average temperature at which heat is lost by the air parcels at the top of the storm. The taller a hurricane is, the lower the temperature T_0 at its top and thus, the greater the thermodynamic efficiency. For a typical hurricane, $\varepsilon \approx 1/3$.

- Setting dissipation equal to generation ($G = D$), we get $V_s^2 = \varepsilon L(q_0^* - q_a)C_k/C_D$. Assuming the atmosphere to be 85% saturated, and the ratio of the two bulk coefficients to be about one, we get
$$V_s^2 = \varepsilon L 0.15 q_0^* = \frac{SST - T_0}{SST} L 0.15 q_0^*.$$
Note that this is exponential in temperature, because of the Clausius-Clapeyron relationship.

- Finally, assuming that stronger hurricanes lead to stronger ocean mixing, and this to stronger MOC. Stronger MOC means warmer poles (Emanuel, 2002).

- Consequences on EPTD of enhanced tropical ocean mixing

8.4 Convective cloud feedback

- Moist adiabatic lapse rate (Marshall and Plumb, 2008): sections 1.3.1 and 1.3.2 on pp 4-6 for some preliminaries; section 4.3.1 on pp 39-41 for the dry lapse rate; sections 4.5.1-4.5.2 pp 48-50 for the moist lapse rate.

- Equivalent potential temperature, convective instability and moist static energy (my `moist_atmospheric_th` notes, sections 7.2, 7.3).

- Two level model, section 2 of Abbot and Tziperman (2009).

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