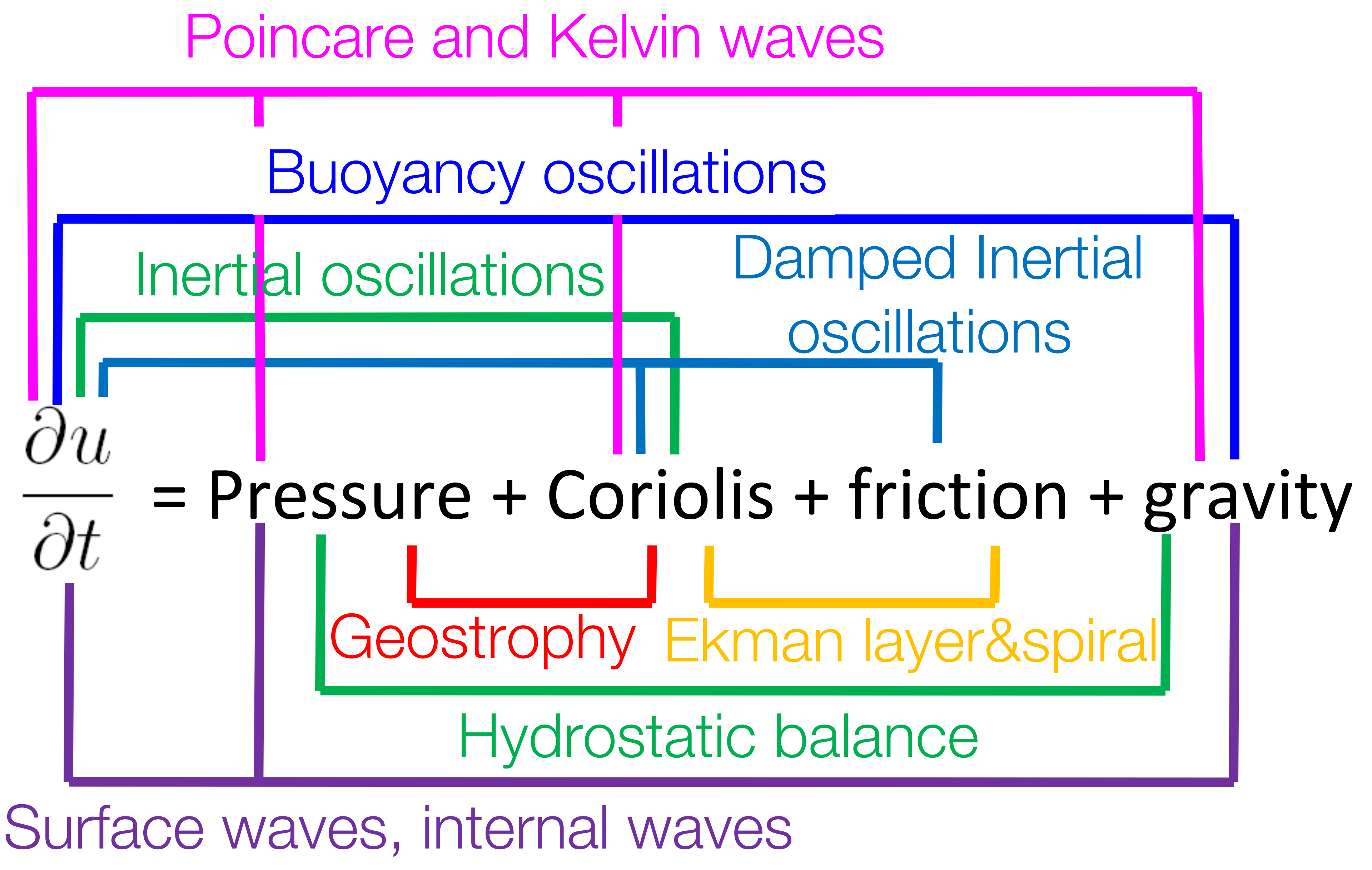
F=ma


Poincare and Kelvin waves
Buoyancy oscillations
Inertial oscillations
Damped Inertial oscillations
$\frac{\partial u}{\partial t}$ = Pressure + Coriolis + friction + gravity
Geostrophy
Ekman layer&spiral
Hydrostatic balance
Surface waves, internal waves

Momentum equations:

$$\frac{\partial u}{\partial t}+\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)-fv = -\frac{1}{\rho_0}\frac{\partial P}{\partial x}-ru+\nu_v\frac{\partial^2 u}{\partial z^2}+\nu_h\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial v}{\partial t}+\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)+fu = -\frac{1}{\rho_0}\frac{\partial P}{\partial y}-rv+\nu_v\frac{\partial^2 u}{\partial z^2}+\nu_h\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

$$\frac{\partial w}{\partial t}+\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right) = -\frac{1}{\rho_0}\frac{\partial P}{\partial z}-g\frac{\rho}{\rho_0}-rw+\nu_v\frac{\partial^2 w}{\partial z^2}+\nu_h\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)$$

In vector form: $\mathbf{u} \equiv (u,v,w), \qquad \mathbf{u}_h \equiv (u,v)$

$$\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}\cdot\nabla\mathbf{u}) = -\frac{1}{\rho_0}\nabla P-\mathbf{g}\frac{\rho}{\rho_0}-r\mathbf{u}_h+\nu_v\frac{\partial^2\mathbf{u}_h}{\partial z^2}+\nu_h\nabla_h^2\mathbf{u}_h$$

Mass conservation statements:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0; \qquad \nabla\cdot\mathbf{u}=0 \qquad \frac{\partial h}{\partial t}+\frac{\partial}{\partial x}(uh)=0$$

Temperature/ Salinity equation:

$$T_t+uT_x+vT_y+wT_z=\kappa_h(T_{xx}+T_{yy})+\kappa_v T_{zz}$$

Equation of state: $\rho(T,S)=\rho_0\left(1-\alpha(T-T_0)+\beta(S-S_0)\right)$

Vorticity!

$$\zeta = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}$$

Meridional motion and planetary vorticity

Vorticity dissipation by bottom friction

vortex stretching by Ekman pumping, due to wind curl

$$\underbrace{\frac{\partial \zeta}{\partial t}}_{} + \underbrace{\beta V}_{\text{Meridional motion and planetary vorticity}} = \underbrace{-r\zeta}_{\text{Vorticity dissipation by bottom friction}} + \underbrace{\mathrm{curl}\left(\frac{\vec{\tau}}{\rho_0}\right)}_{\text{vortex stretching by Ekman pumping, due to wind curl}}$$

Vorticity!

Reminder:

$$\frac{\partial \zeta}{\partial t} + \beta V = -r\zeta + \mathrm{curl}\left(\frac{\vec{\tau}}{\rho_0}\right)$$

$$\zeta = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}, \quad (U, V) = \int_{-H}^{0} dz\,(u, v)$$

$$\mathrm{curl}\vec{\tau} = \frac{\partial \tau^{(y)}}{\partial x} - \frac{\partial \tau^{(x)}}{\partial y}$$

Vortex spin-down

Rossby waves

$$\frac{\partial \zeta}{\partial t} + \beta V = -r\left(\frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}\right) + \mathrm{curl}\frac{\vec{\tau}}{\rho_0}$$

Western boundary current

Sverdrup balance

The End