EPS 131: Introduction to Physical Oceanography and Climate

Dept of Earth and Planetary Sciences, Harvard University

Homework #7

Assigned Thursday, April 16, 2026; due Tuesday, April 28, 10 am, via Gradescope

1. **Transports, transports, transports:** Consider a subtropical gyre centered at 45°N extending in the north-south direction from $y = 0$ to $y = L$, and in the east-west direction from $x = 0$ to $x = L$. The wind stress is $\vec{\tau} = (\tau^{(x)}, \tau^{(y)}) = (0.1\cos(\pi y/L), 0)$ N/m^2, $L = 3000$ km.

   (a) Find the expressions, as a function of latitude y of the Ekman transport, the Ekman pumping at the base of the Ekman layer, and the Sverdrup transport.

   (b) Plot the three vs latitude y.

   (c) Calculate the Ekman pumping, Ekman transport and Sverdrup transport at $(x, y) = (L/2, L/2)$.

   (d) Calculate the total Ekman transport across $y = L/2$ in Sv, the total vertical transport due to the Ekman pumping over the entire domain in Sv, and the total Sverdrup transport across $y = L/2$ in Sv.

   (e) Discuss your findings in the last item and explain how the wind drives the large-scale ocean circulation.

2. **Western boundary currents:**

   (a) Draw schematic subtropical gyres with eastern, then western, boundary currents.

   (b) Explain why the vorticity equation derived in class may be approximated in the western boundary current by $\beta V = -r\partial V/\partial x$

   (c) Solve this equation near the eastern boundary & then near the western boundary.

   (d) Can you obtain solutions that decay toward the interior in both cases? Interpret and explain why the Gulf Stream is in the west Atlantic.

   (e) Describe the heuristic explanation of western boundary currents based on the group velocities of short and long Rossby waves.

3. **Rossby waves:**

   (a) Start from the linearized vorticity equation with no forcing and no friction, $\zeta_t + \beta V = 0$. Assume a shallow-water ocean, and using geostrophy for the depth-integrated velocities ($-f_0 V = -gH\eta_x$, $f_0 U = -gH\eta_y$), show that this equation may be written in terms of the free surface elevation, as $\partial_t(\eta_{xx} + \eta_{yy}) + \beta\eta_x = 0$.

   (b) Find the dispersion relation for the wave solution $\eta = \eta_0 \cos(kx + ly - \omega t)$. Calculate the phase and group velocities in the x direction at latitude 30N and plot them as a function of k for $2\pi/(4000 \text{ km}) < k < 2\pi/(400 \text{ km})$, with $l = 2\pi/(1200 \text{ km})$.

(c) How long would the fastest such waves take to transport *energy* across the Atlantic Ocean at the above latitude?

(d) ***optional extra credit: derive the vorticity equation and then the Rossby wave dispersion relation including the effects of the scale-selective friction terms in the momentum equations. Start from the momentum equations $U_t - fV = -gH\eta_x + \nu(U_{xx} + U_{yy})$, $V_t + fU = -gH\eta_y + \nu(V_{xx} + V_{yy})$. Let $f = f_0 + \beta y$, $\theta_0 = 45\text{N}$, $\nu = 10\ \text{m}^2/\text{sec}$.

i. Show that $U_x + V_y \approx 0$ follows from geostrophy, $-f_0 V = -gH\eta_x$, $f_0 U = -gH\eta_y$). What does $U_x + V_y \approx 0$ mean physically?

ii. Take the curl of the momentum equations to derive a vorticity equation. Write the dissipation term as the Laplacian of relative vorticity.

iii. Use geostrophy to write the vorticity equation in terms of η.

iv. Substitute a wave solution of the form $\exp(i(kx + ly - \omega t))$, solve for ω. How does dissipation appear?

v. Calculate the amplitude damping factor by the time the waves cross the Atlantic on waves with a wavelength of 100 km vs 1000 km.

4. **El Niño:** Suppose a wind perturbation occurs in the central Pacific at longitude 180°W, corresponding to the weakening of the easterly winds there.

(a) Why is this expected to lead to an equatorial Kelvin wave that deepens the East Pacific thermocline?

(b) Why does the same wind perturbation lead to an off-equatorial Rossby wave (say at latitude 8°N) that involves a thermocline shallowing signal? (*Hint:* wind curl and Ekman pumping).

(c) Describe the main stages of the delayed oscillator mechanism, starting with weakening easterly perturbation, going through the amplification of an El Niño signal, and all the way to a La Niña signal.

(d) Assuming wave velocities $C_{\text{Kelvin}} = 2.2$ m/s, $C_{\text{Rossby}} = C_{\text{Kelvin}}/3$, calculate how long does the equatorial Kelvin wave take to propagate from the central equatorial Pacific eastward, triggering an El Niño event? How long does it take the signal starting as an off-equatorial Rossby wave to propagate from the central Pacific to the eastern boundary and start reducing the El Niño amplitude?

[What's the point of **optional** extra credit problems? Apart from the fun of doing them, they may bring the total HW score to up to 110% therefore counting against problems you may have missed in this or other HW assignments...]