EPS 131: Introduction to Physical Oceanography and Climate

Dept of Earth and Planetary Sciences, Harvard University

Homework #6

Assigned Tuesday, April 7, 2026; due Thursday, April 16, 2026, 10 am, via Gradescope

1. **Vorticity:** Calculate the vertical component of the vorticity for the following two-dimensional flow fields using the appropriate expressions for the curl operator.

   (a) Polar coordinates, $(u_\theta, u_r) = (\omega\, r^2, 0)$.

   (b) Polar coordinates, $(u_\theta, u_r) = (\Gamma/r^2, 0)$.

   (c) Cartesian coordinates, $(u, v) = (-x, y)$.

   For these 2d cases, plot the vorticity using `contourf`, and superimpose the flow field using `quiver`. Discuss your results.

   (d) ****optional** extra credit:** Calculate the 3d vorticity vector for the 3d velocity field given by the spherical coordinates (longitude, latitude, radius) as $\mathbf{u} = (u^{(\phi)}, v^{(\theta)}, w^{(r)}) = (\Omega r \cos\theta, 0, 0)$ where the equator is at $\theta = 0$. What does this flow field represent? How is your result related to what we covered in class?

2. **Ekman pumping and the wind-driven circulation; the Sverdrup balance:** Given the following wind stress field,

$$\vec{\tau} = (\tau^{(x)}, \tau^{(y)}) = \left(-\tau_0 \cos\left(\frac{\pi y}{L}\right), 0\right)$$

   where $L = 3000$ km, $\tau_0 = 0.15$ N/m^2, $0 \le x \le 2L$ is the eastward distance, and $0 \le y \le L$ is the northward distance, with the center of the basin at 35°N, assume the wind-driven circulation to extend to a depth of 1 km.

   (a) Calculate the Ekman pumping velocity field,

   (b) Plot (schematically or using `contourf/ quiver`): **(1)** The wind stress as arrows in (x, y). **(2)** The Ekman transport as different color arrows in (x, y). **(3)** The Ekman pumping as a line plot as a function of y. Explain your results. Assume the Coriolis parameter is constant for this specific calculation.

   (c) Calculate the resulting northward wind-driven interior ocean velocity $v(y)$, in m/s.

   (d) Calculate and line-plot the total transport by the interior circulation (away from the western boundary) in Sverdrups as a function of y. Use the value of β and f calculated at the basin center.

   (e) Describe your solution and discuss its relation to the subpolar and/or subtropical gyres in the North Pacific, for example.

3. **Finally, ξ vs ζ!!!**

[What's the point of ****optional** extra credit** problems? Apart from the fun of doing them, they may bring the total HW score to up to 110% therefore counting against problems you may have missed in this or other HW assignments...]