EPS 131: Introduction to Physical Oceanography and Climate

Dept of Earth and Planetary Sciences, Harvard University

Homework #5

Assigned Thursday, March 26, 2026; due Tuesday, April 7, 5 pm, via Gradescope

1. **Steady states and tipping points of the MOC.** The Stommel model equations are,

$$
\begin{aligned}
V\frac{\partial S_1}{\partial t} &= |q|(S_2 - S_1) - F_s \\
V\frac{\partial S_2}{\partial t} &= |q|(S_1 - S_2) + F_s \\
q &= K(\rho_1 - \rho_2) \\
\rho &= \rho_0 - \alpha(T - T_0) + \beta(S - S_0),
\end{aligned}
$$

where $K = 2 \cdot 10^{10}$, $\alpha = 2 \times 10^{-4}$ K^{-1}, $\beta = 8 \times 10^{-4}$ ppt^{-1}, $F_s = (0.22 \text{ Sv}) \times S_0$, $\rho_0 = 1025$ kg/m^3, $T_0 = 2°$C, $T_1 = 8$ °C, $T_2 = 19$ °C, $S_0 = 35$ ppt, $A = (4000 \text{ km})^2$, $H = 4$ km, $V = AH$.

   (a) Derive these equations following the derivation in class, indicating the assumptions used at each step. In particular, consider both the case that $q < 0$, and that $q > 0$, and then show that both cases may be reduced to the above equations. Describe what each term in the above equations represents in one sentence.

   (b) Derive a single equation for the time evolution of $\Delta S = S_1 - S_2$. Find all of its steady-state solutions.

   (c) Explain why there are only three steady-state solutions and not four.

   (d) Describe the balance represented by each steady-state solution by discussing the sign and magnitude of all terms appearing in the salt budget equation: q, ΔS, and their product vs F_s. Show how the right-hand side of the salt budget equation vanishes at a steady state for each of the three solutions while each of the factors takes a different value.

   (e) Calculate and plot the value of q for the three steady-state solutions as a function of the freshwater forcing for F_s/S_0 in the range of 0 to 2 Sv.

   (f) Are there three solutions for every value of F_s? What does that imply for the existence of tipping points? What would tipping points imply for countries surrounding the northern North Atlantic?

   (g) Discuss how these steady states indicate that a hysteresis is expected, and explain what that hysteresis would look like if CO_2 is increased (assuming that F_s increases with CO_2), as it does now, and later gradually decreased.

[What's the point of ****optional** extra credit** problems? Apart from the fun of doing them, they may bring the total HW score to up to 110% therefore counting against problems you may have missed in this or other HW assignments...]