EPS 131: Introduction to Physical Oceanography and Climate

Dept of Earth and Planetary Sciences, Harvard University

Homework #4

Assigned Tuesday, Mar 10, 2026; due Thursday, March 26, 2026, 10:00 am, via Gradescope

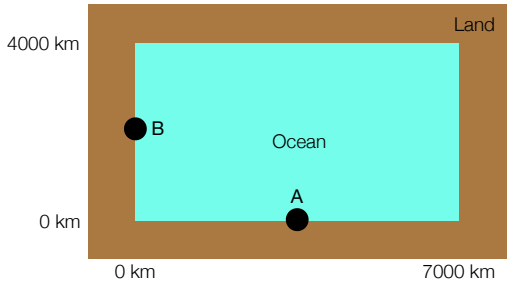
1. **Deep gravity waves:** Consider a storm that excites deep ocean waves whose amplitude as a function of wavenumber is given by $\eta_0 = \exp(-(k-k_0)^2/k_0)$ where $k_0 = 2\pi/15$ m^{-1}. You are sitting at a beach at a distance of 4000 km away; calculate and plot the frequency and amplitude of the arriving waves as a function of time, and explain your results. *Hint:* Should you use the phase or group velocities for this calculation? Note that the relevant velocity is a function of k, and therefore each k arrives at one specific time, and we can assume that it preserves its initial amplitude.

2. **Buoyancy oscillations with friction**: Study the vertical motion of a water parcel in a stably-stratified ocean that is displaced by a distance δz.

   (a) Write down an equation for the vertical displacement $\delta z(t)$ of a fluid parcel that represents the balance: vertical acceleration=buoyancy force+vertical friction. Use a non-scale selective vertical friction, $-rw$, written in terms of δz.

   (b) Solve for the position of the parcel as a function of time $\delta z(t)$ by assuming an exponential solution e^{at} and calculating a and then the full solution. Explain the solution, discussing what values of the problem parameters lead to oscillations and what the non-oscillating solutions would look like as a function of time.

3. **Kelvin waves:** consider a similar problem to that done in class, except that the continent is located at $x < 0$ (instead of $y < 0$ in class), and in the Southern rather than Northern Hemisphere. **(A)** Justify the boundary condition of $u = 0$ at the coast and assume u to vanish everywhere. Write and explain the momentum and mass conservation equations. **(B)** Find a solution for $v(x, y, t)$, $\eta(x, y, t)$. **(C)** Show that the wave propagates with the coast to its left in the Southern Hemisphere. **(D)** Consider the ocean basin shown in the figure. How long would it take a perturbation at B to propagate to A? Write your answer in terms of an ocean depth of H and then calculate a numerical value for $H = 3$ km.



4. **Ekman transport.** Follow these steps to show that if the wind blows northward along the Namibia coast, the transport will be offshore, leading to the Humboldt upwelling

system: **(i)** justify the momentum equation $0 = -fu + (\partial\tau^y/\partial z)/\rho_0$ and explain what $\tau^y(z)$ is. **(ii)** Integrate from the base of the Ekman layer to the surface, explain and use the condition that $\tau^y(z) = \tau^{y,\text{wind}}$. **(iii)** Derive the formula for the offshore volume transport. **(iv)** Assume the wind stress is 0.15 N/m^2 and that the coastline is 400 km long. Find the total offshore transport in Sv.

5. **Ekman spiral and transport:** Consider the balance of Coriolis force and vertical friction, and solve it to find $u(z)$ and $v(z)$ as follows.

   (a) Derive a single differential equation for u.

   (b) Show that the equation is satisfied by each of the following four solutions,

   $$u(z) = Ae^{z/\delta}\cos(z/\delta), Be^{z/\delta}\sin(z/\delta), Ce^{-z/\delta}\cos(z/\delta), De^{-z/\delta}\sin(z/\delta)$$

   where $\delta = \sqrt{2A_v/f}$.

   (c) Use the x momentum equation to find v in terms of the solution for u.

   (d) Find the coefficients A, B, C, D from the boundary conditions: the velocities decay with depth, as $z \to -\infty$, and the stress at the surface is equal to the wind stress, assumed to be non-zero only in the x direction,

   $$\nu\frac{\partial u}{\partial z}\bigg|_{z=0} = \frac{1}{\rho}\tau^{x,wind}, \quad \nu\frac{\partial v}{\partial z}\bigg|_{z=0} = \frac{1}{\rho}\tau^{y,wind} = 0.$$

   (i) Explain these surface boundary conditions; (ii) Show that two of the coefficients must vanish due to the condition at $z \to -\infty$. (iii) Find the remaining two coefficients by using these surface boundary conditions.

   (e) Write explicitly your solution for both $u(z)$ and $v(z)$.

   (f) ***optional extra-extra credit:** Find the net Ekman mass transport in both the x and y directions by integrating the Ekman velocities from a depth of $z = -\infty$ to $z = 0$. What is the angle of the transport relative to the applied wind stress?

6. **The Oscars!!!**

[What's the point of **optional** extra credit problems? Apart from the fun of doing them, they may bring the total HW score to up to 110% therefore counting against problems you may have missed in this or other HW assignments...]