EPS 131: Introduction to Physical Oceanography and Climate

Dept of Earth and Planetary Sciences, Harvard University

Homework #2

Assigned Tuesday, Feb 17 2026; due Thursday, Feb 26 at 10am via Gradescope

1. **Major ocean surface currents from sea surface height measurements:** find the sea surface height differences across the Gulf Stream (33N), Kuroshio (33N) and the circumpolar current (70E), as well as their widths, from the UMD "Simple Ocean Data Assimilation" (SODA) Reanalysis product (click "Views", then "colors with land" then zoom in as needed by dragging to select a rectangle over the map).

   - Calculate the surface currents (m/s) based on the above, using the correct f (Coriolis parameter) for each latitude.
   - Calculate the total transport of these currents (as a mass flux in units of kg/s, and as a volume flux in units of Sverdrups) assuming that the currents extend to a depth of 1 km and ignoring any dependence on depth.
   - How do these transports compare to the output of the Amazon River? All the world rivers combined?

2. **Geostrophic calculation in the presence of sea level and density variations:** Assume the temperature T and sea level h across the Gulf Stream, at a section along latitude 30N, are given by

$$T(x,z) = 0\ ^\circ\mathrm{C} + T_0 e^{z/H}\left(1+\delta\frac{x}{L}\right) \qquad 0<x<L;$$
$$h(x) = ax^2,$$

   where $T_0 = 15\ ^\circ$C, $H = 1.5$ km, $L = 100$ km, $a = 1$ m/(200^2 km^2), $\delta = 0.05$.

   (a) Write the expression for density as a function of x and z using $\rho = \rho_0(1-\alpha(T-T_0))$ and an expansion coefficient $\alpha = 2.5\cdot 10^{-4}/^\circ$C (appropriate for a temperature of 20 °C and pressure of 0 bar). Use $\rho_0 = 1029$ kg/m^3.

   (b) Derive the general expression the zonal pressure gradient $\frac{\partial P}{\partial x}$ at a depth z as a function of sea surface height and density, briefly discuss each term.

   (c) Write the full general expression derived in class for the velocity $v(x,z)$ across the section of observations. Evaluate and simplify this expression as much as possible for the above T and h observations. What is the numerical value of the velocity at $x = L/2$, $z = -500$ m? (again use the correct value of f for this latitude)

   (d) Find the depth of the level of no motion for v at $x = L/2$ and evaluate it numerically.

   (e) ****optional** extra credit:** Calculate the total transport (in units of Sv) in the north direction over the range of depths from the surface to the level of no motion, over the width L of the current.

3. ***optional challenge problem: equilibrium shape of water in a rotating bucket:

   (a) Calculate the equilibrium shape of the surface of water in a cylinder-shaped bucket rotating about its axis with angular velocity Ω, and radius r, if it contains water of uniform density ρ_0.

   (b) Describe the surface shape and interpret the results in terms of the forces acting on the water.

   (c) How big is the centrifugal force in the ocean, relative to the gravity force? Why do we not consider the centrifugal force in our discussions of rotating ocean dynamics in class?

   Hint: The two forces acting on the water are the centrifugal force and the pressure force. When the water is at rest in the rotating frame, those two forces balance each other. Use this to solve or the height of the surface of the water as a function of the radius of the bucket.

[What's the point of **optional** extra credit problems? Apart from the fun of doing them, they may bring the total HW score to up to 110% therefore counting against problems you may have missed in this or other HW assignments...]