EPS 131: Introduction to Physical Oceanography and Climate

Dept of Earth and Planetary Sciences, Harvard University

Homework #1

Assigned Thursday, Feb 5, 2026; due Tuesday, Feb 17, 2026, at 10:00 am, via Gradescope

1. **Effect of atmospheric pollution on the greenhouse effect!** In class, we assumed that all the incoming solar (short wave, or SW) radiation reaches the surface of the Earth. In reality, some of this SW radiation is absorbed by the atmosphere. Let the fraction of the downward SW radiation absorbed due to atmospheric pollution in the form of microscopic particles emitted by factories be $\delta = 0.15$ (15 percent). Ignore the effect of absorption on the reflected SW. The corrected equations for the energy balance in the atmosphere and at the surface are:

$$\frac{1}{4}S(1-\alpha)\delta + \sigma T^4 = 2\sigma\theta^4$$
$$\frac{1}{4}S(1-\alpha)(1-\delta) + \sigma\theta^4 = \sigma T^4$$

   where T is the surface temperature, θ is the atmospheric temperature, and the Stefan Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/(m^{2}K^4).

   (a) Explain and rationalize each term above.

   (b) Solve for T. Use $S = 1367\ W/m^2$, $\alpha = 0.25$, $\delta = 0.15$.

   (c) Suppose that reduced contamination in the atmosphere changes δ to 0.1. What effect would that have on the surface temperature?

2. **Emission height.** Suppose the CO_2 concentration quadruples instantaneously and that the resulting increase in emission height is 250 m. Satellite measurements indicate that the outgoing heat (LW) radiation has decreased by 8 W/m^2. Calculate the atmospheric lapse rate.

3. **Sea level rise:** Suppose that the ocean warms more at the surface ($z = 0$) than at depth ($z < 0$), such that the increase in temperature is given by

$$\Delta T = \begin{cases} 3e^{z/400} & z > -400 \text{ meter} \\ 0 & z < -400 \text{ meter} \end{cases}$$

   Calculate the sea level rise following a similar calculation to that was done in class. Use

$$\rho(T) = \rho_0(1 - \alpha(T - T_0))$$

   where $\alpha = 2 \times 10^{-4}$, $T_0 = 4°C$, $\rho_0 = 1025$kg/m^3.

4. **"Abyssal recipes"**, deep exponential temperature profile:

(a) Using the example Jupyter notebook code and the global annual-averaged temperature and salinity data (pickle files) given with the HW (all on Canvas under "Files"), plot vertical ocean temperature and salinity profiles for two locations:

- The North Atlantic Ocean at (30W,35N)
- The South Pacific at (240E,30S)

(b) What is different in the temperature and salinity profiles from the Atlantic and Pacific? Explain the reason for these differences in terms of the water masses that are expected in the two basins.

(c) Plot an exponential profile that is as close as possible to the North Atlantic temperature profile, between 1 and 3 km depth, by adjusting T_0, a and b in the code.

(d) Assuming that the mixing in the ocean is given by $\kappa = 10^{-4}$ m^2/s, deduce from the value of b you found what the upwelling velocity is.

5. *****Optional*** challenge problem:** N-layer greenhouse effect: suppose there are N atmospheric layers, each completely absorbing the heat (long-wave) radiation coming from below or above, and emitting it back both up and down. Our objective is to calculate the global warming factor for the surface temperature in this case, (this is a generalization of the factor $2^{1/4}$ derived in class) and to find an expression for a given atmospheric layer temperature, T_i.

   Here is some guidance on how to go about it, step by step:

   (a) Consider the schematic figure below (borrowed from a course taught by Tapio Schneider, Caltech).

   (b) Write the radiative energy balance for the entire atmosphere, by considering the incoming and outgoing fluxes at the top of the atmosphere (dash line). It should involve T_n and S_0.

   (c) Use the above radiation balance to solve for T_n

   (d) Write the radiative equilibrium temperature at the surface, T_s and using the above result, solve for T_s in terms of the atmospheric temperature at the top layer, T_n, and the temperature at the first layer, T_1.

   (e) Write the radiative energy balance in each internal atmospheric layer (denoted by $0 < i < n$ in Figure) by realizing that there is a balance between the emission from the levels above and below it and emission from the layer itself in both the upward (towards outer space) and downward (towards the surface) directions.

   (f) Use the above balance to show that the differences between the temperatures of adjacent layers raised to the 4^{th} power are constant! What does this mean?

   (g) Find an expression for T_i in terms of T_n, i, and n, only, and plot the dependence of T_s on the number of layers in our "atmosphere", n.

Discuss your results. Why can't we treat the atmosphere as a very large number of layers, which would imply a very large greenhouse effect? Which of our assumptions above breaks down as the number of layers increases?

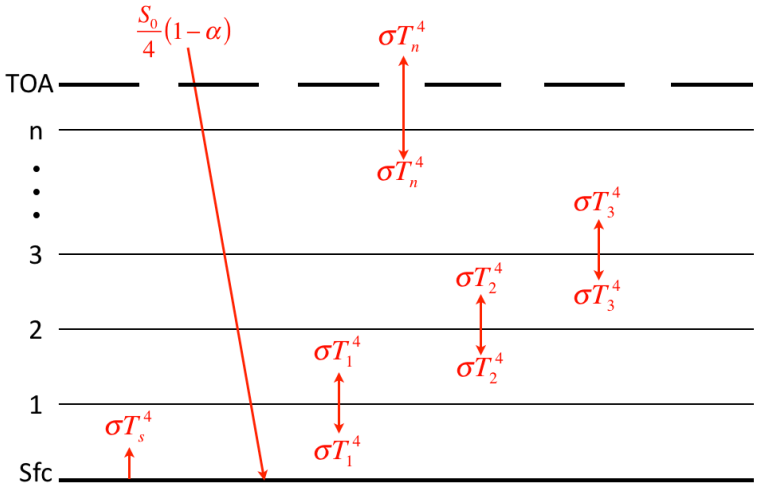


Figure 1: Example of radiative balance for a surface with an n-layer atmosphere.

Figure 1: Borrowed from a course taught by Prof. Tapio Schneider, Caltech

[What's the point of **optional** extra credit problems? Apart from the fun of doing them, they may bring the total HW score to up to 110% therefore counting against problems you may have missed in this or other HW assignments...]