Homework #9
Introduction to physical oceanography

1. **Gravity waves in a finite depth water:**

   (a) **EPS 131 Oscar competition:** Create a short video of an experiment showing that fluid particle motion in surface gravity waves is circular. One possibility is to observe a float in a wave field, but you are welcome to come up with a different approach that will show particle motion below the surface as well (side view of a transparent container?). You may do this individually or in pairs. Please email me your entry in electronic format. Entries will be posted on the course home page. A panel of experts will choose the winning entry, and yes, there is a prize!

   (b) Solve for the velocity field (u,w) for gravity waves in an ocean of finite depth H: the equation and boundary conditions were derived in class. Assume a solution of the form $\phi = \cos(kx - \omega t)(e^{az} + be^{-az})$ and find a solution for the constants a,b that satisfies all boundary conditions; also, find the dispersion relation $\omega = \omega(k)$. Write your solution for ϕ, u and w as function of x,z,t,k only (no dependence on ω). See Knauss for help.

   (c) Calculate and schematically plot the particle trajectories at different depths from the surface to the bottom; what do they look like?

2. Show that $\nabla \times \nabla\phi = 0$ (that is, that the curl of a gradient vanishes) in Cartesian coordinates, for a scalar function ϕ. Hint: the above vector relationship is equivalent to three scalar equations, and you need to show that all three are zero.

3. **Vorticity in gravity waves:** We started our derivation of gravity waves assuming that the vector vorticity is identically zero, $\vec{\zeta} = \nabla \times \vec{u} = 0$. Then we found that particle trajectories are circular for deep water waves. This question tries to find if this implies a contradiction:

   (a) Calculate the three components of vorticity vector from the gravity wave solution for u,w in deep water waves. Does this answer contradict our assumption of irrotational motion?

   (b) Plot a vector field for the instantaneous velocity vector field (u,w) using the quiver function in Matlab.

   (c) **Extra credit challenge problem:** calculate and plot the contours of the instantaneous stream function for deep gravity waves. Please describe and interpret your results.