Homework #6

Introduction to physical oceanography

1. **Buoyancy oscillations with friction:** write down an equation that expresses the following balance,

   linearized vertical acceleration = buoyancy + vertical friction

   using a non scale-selective form for the *vertical* friction. Solve for the location as function of time $\delta z(t)$ of a fluid element that is displaced from its original position within a stably stratified ocean. You may assume any convenient reasonable initial conditions.

2. **Surface Ekman Spiral:** [This is not an easy one, please just do your best...] Consider the balance of Coriolis force and vertical friction

$$\begin{aligned} -fv &= A_v u_{zz} \\ fu &= A_v v_{zz} \end{aligned}$$

   and solve it to find $u(z)$ and $v(z)$ as follows.

   (a) Derive a single equation for u by differentiating the first equation twice with respect to z and substituting the second equation into the result.

   (b) Assume an exponential solution $u = e^{az}$ and find what the exponent a is. Note that there are four solutions for a. What are they?

   (c) Let the four exponents be a_1, a_2, a_3, a_4; the general solution for $u(z)$ is then a linear combination of the four exponential functions found above,

$$u(z) = \sum_{i=1}^{4} b_i e^{a_i z}.$$

   where the coefficients b_i need to determined from our boundary conditions. Show that because we require the solution to decay to zero away from the surface (as $z \to -\infty$), two of the coefficients in the above sum (say b_3 and b_4) must be zero. Explain why physically we require the solution to decay to zero away from the surface.

   (d) Find the other two remaining coefficients b_1 and b_2 by using the boundary conditions $u(z = 0) = 1$ and $v(z = 0) = 0$. Take the real part of your solution, and write explicitly your solution for $u(z)$ and $v(z)$. Note that given $u(z)$ you can find $v(z)$ using one of the original momentum equations above.

   (e) Plot $u(z)$ and $v(z)$ from your solution. Show that it describes a spiral.

   (f) What is the expression for the exponential decay scale of the velocity from the surface down in your solution? This is called the Ekman depth. Estimate it assuming the latitude is 45°N and the vertical viscosity in the surface ocean is $A_v = 100 cm^2/sec$.

3. **Review of equations:**

   (a) Write the 3d momentum equations, including all terms derived in class in terms of the horizontal velocity vector $\vec{u}_H = (u, v)$ and the vertical velocity w, as well as in terms of $\nabla_H = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y})$ and $\frac{\partial}{\partial z}$. Do not use (u, v) at all in your answer, nor $\frac{\partial}{\partial x}; \frac{\partial}{\partial y}$, nor the 3d ∇ or the 3d velocity vector $\mathbf{u} = (u, v, w)$.

(b) Do the same to the continuity equation (mass conservation) for incompressible fluid, and for the temperature equation.

(c) Explain the physical meaning of each term in each of these equations and in which of the phenomena studied in class so far it plays a role.

(d) (i) How many equations have you written? (ii) How many scalar equations are these equations actually equivalent to? (iii) How many scalar unknowns are there? (iv) What are these unknowns? (v) Is the number of scalar/ vector equations equal to the number of scalar/ vector unknowns? if not, what's missing?