EPS 131 Homework #1 Solutions

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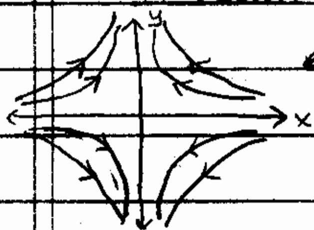
① $u(\vec{x}_o,t) = -\alpha x_o e^{-\alpha t}$, $v(\vec{x}_o,t) = y_o e^{\alpha t}$ (Lagrangian velocity)

$u(\vec{x},t) = -\alpha x$, $v(\vec{x},t) = \alpha y$ (Eulerian velocity)

Use the streamline equation $\frac{dy}{dx} = \frac{v}{u} = -\frac{y}{x} \Rightarrow y = \frac{c_1}{x}$, or use the fact that for time-independent flows (i.e., time-independent Eulerian velocity fields) streamlines are path lines so $\boxed{y = \frac{y_o x_o}{x}}$ for the streamlines



← A few streamlines

② See attached figures.

To do a rough and dirty fit of the equatorial Pacific data to $T(z) = T_o + T_1 e^{-z/h}$,

i. Find $\boxed{T_o = 1.2°C}$

ii. $T_o + T_1 e^{-500m/h} = 8.3°C \Rightarrow T_1 e^{-500m/h} = 8.3 - 1.2 = 7.1°C$

iii. $T(z = 500+h) = T_o + T_1 e^{-(500+h)/h} = T_o + (T_1 e^{-500/h}) e^{-1}$

$$= 1.2 + 7.1 e^{-1} = 3.8°C$$

$T(z = 500+h) = 3.8°C$ gives $500 + h = 1100 \Rightarrow \boxed{h = 600m}$ from data

iv. $T_1 = 7.1 e^{500m/h} = 7.1 e^{5/6} = \boxed{16.3°C}$

(I used 3 data points to approximate T_o, T_1, h)

Note that $T = 1.2°C + 16.3°C\, e^{-z/600m}$ looks similar to data in attached figures.

$\frac{dT}{dt} = \frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T = K \nabla^2 T \Rightarrow w T_z = K T_{zz}$ when $u = v = T_x = T_y = 0$

$K = hw = 600m \cdot 10^{-4}\, cm/s = \boxed{6 \frac{cm^2}{s}} = 6 \cdot 10^{-4} \frac{m^2}{s}$

② cont'd

North Atlantic: $T(z) = 3.4°C + 9.8°C\, e^{-z/500\,m}$ $\Rightarrow$ $\boxed{K = 5\ \frac{cm^2}{s}}$

Southern Ocean: $T(z) = 0.8°C + 9.3°C\, e^{-z/950\,m}$ $\Rightarrow$ $\boxed{K = 9.5\ \frac{cm^2}{s}}$

The same K, w don't fit all 3 temperature profile. Both the vertical velocity and the vertical mixing coefficient vary from place to place in the ocean.

③ a) see attached plot

b) $\frac{\partial T}{\partial y} = \frac{8 - 22°C}{54 - 31° lat} = -0.6\ \frac{°C}{° lat} \approx \boxed{-0.6 \cdot 10^{-2}\ \frac{°C}{km}}$

c) $\frac{dT}{dt} = \underbrace{\cancel{\frac{\partial T}{\partial t}}}_{\text{steady state}} + u \underbrace{\cancel{\frac{\partial T}{\partial x}}}_{\text{no}} + v \frac{\partial T}{\partial y} + \cancel{w \frac{\partial T}{\partial z}} = 0.1\ \frac{m}{s}\left(-0.6 \cdot 10^{-2}\ \frac{°C}{km}\right) = \boxed{-0.05\ \frac{°C}{day}}$

⑤ $\frac{\partial}{\partial t}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\mu & -\Omega \\ \Omega & \mu \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

define $\overleftrightarrow{M} = \begin{pmatrix} -\mu & -\Omega \\ \Omega & \mu \end{pmatrix}$

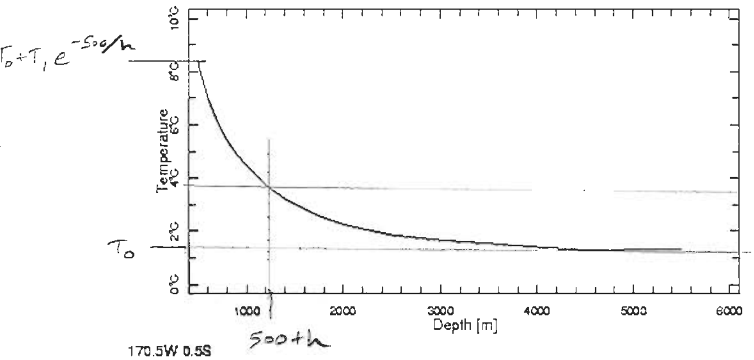
The solution is $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} e^{\overleftrightarrow{M} t}$, where $e^{\overleftrightarrow{M} t} \equiv \mathcal{U}\, e^{\Lambda t}\, \mathcal{U}^{-1}$

$\mathcal{U} \equiv \begin{pmatrix} (\mu+\lambda)/\Omega & (\mu-\lambda)/\Omega \\ -1 & -1 \end{pmatrix}$, $e^{\Lambda t} \equiv \begin{pmatrix} e^{-\lambda t} & 0 \\ 0 & e^{\lambda t} \end{pmatrix}$, $\lambda \equiv \sqrt{\mu^2 - \Omega^2}$

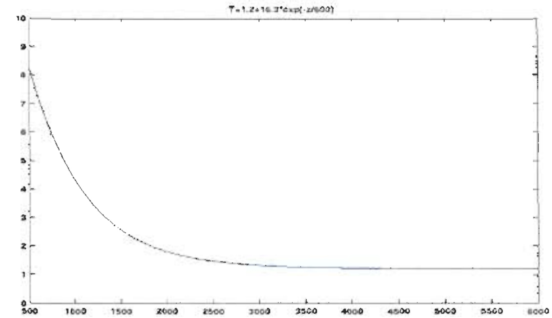
This is one of many ways to solve the linear coupled ordinary differential equation, and it illustrates the solution's dependence on $e^{\pm \lambda t}$, where $\pm\lambda$, defined above, are the eigenvalues of $\overleftrightarrow{M}$. If $\mu > \Omega$, the exponential is real, and the motion is unstable (blows up like $e^{\sqrt{\mu^2 - \Omega^2}\, t}$). If $\Omega > \mu$, the exponential is imaginary, and the motion is oscillatory (oscillates like $e^{i\sqrt{\Omega^2 - \mu^2}\, t} = \cos\sqrt{\Omega^2 - \mu^2}\, t + i \sin\sqrt{\Omega^2 - \mu^2}\, t$).

(2)

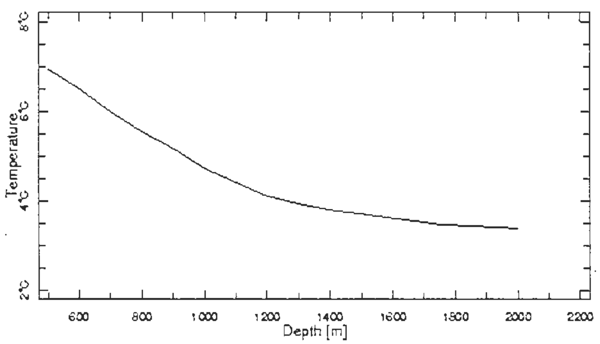
EQUATORIAL PACIFIC



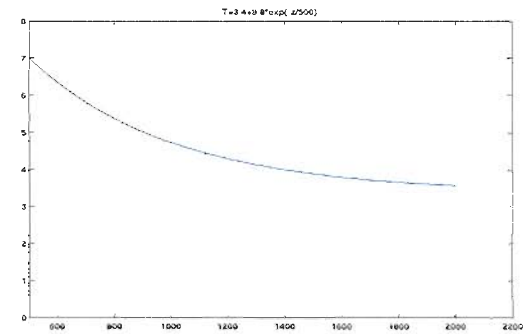
170.5W 0.5S



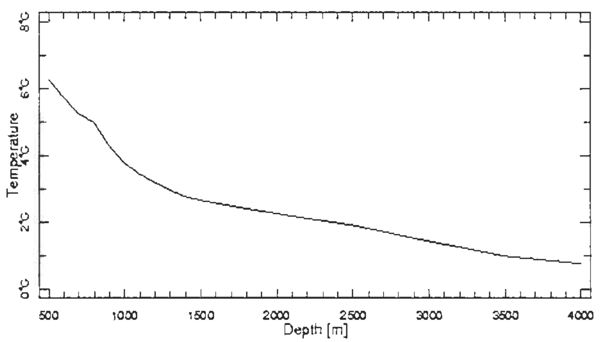
NORTH ATLANTIC



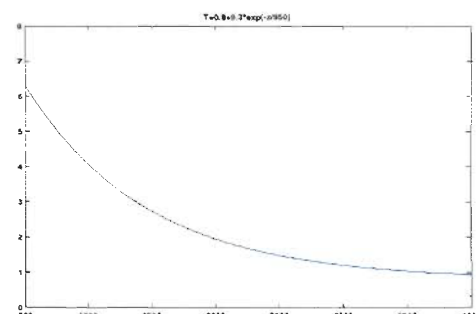
25.5W 59.5N



SOUTHERN OCEAN



149.5E 50.5S



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