Topics:

1. Intermittency

   (a) Type I
   i. Saddle-node (tangent) bifurcation
   ii. \( x_{n+1} = \varepsilon + x_n + x_n^2 \)
   iii. A real eigenvalue crosses the unit circle at +1.
   iv. Laminar signal increases monotonically to chaos.
   v. \( \tilde{T}(\varepsilon) \sim \varepsilon^{\frac{1}{2}} \)

   (b) Type II
   i. Hopf bifurcation
   ii. \[
   r_{n+1} = (1 + \varepsilon)r_n + r_n^3
   \]
   \[
   \theta_{n+1} = \theta_n + 1
   \]
   iii. Two complex conjugate eigenvalues cross the unit circle simultaneously.
   iv. Laminar signal spirals toward chaos.
   v. \( \tilde{T}(\varepsilon) \sim \varepsilon^{-1} \)

   (c) Type III
   i. Inverse period doubling bifurcation
   ii. \( x_{n+1} = -(1 + \varepsilon)x_n - x_n^3 \)
   iii. A real eigenvalue crosses the unit circle at -1.
   iv. Laminar signal alternates as it approaches chaos.
   v. \( \tilde{T}(\varepsilon) \sim \varepsilon^{-1} \)
   vi. The probability distribution, \( P(l) \), for finding a certain length of laminar behavior is different from that for Type II intermittency.