Workshop #5, outline for TF, not students
APM 115: mathematical modeling

Traffic and delay equations:

Use the Taylor expansion method for approximately solving the traffic delay ODEs. some guidelines:

1. start from eqn 2.108 in mesterton-gibbons (scanned on course web page). set number of cars to 2, where the velocity of the first car is specified and we only need to solve for the second car.

2. linearize the log function around one,

3. Taylor expand the delay term around t to eliminate delay, keeping up to second derivative in time

$$\begin{aligned}
\frac{d\phi_2}{dt} = \phi_2'(t) &= v \ln\left[1 + x_{max}\{\phi_1(t-\tau) - \phi_2(t-\tau)\}\right] \\
&\approx v\left[x_{max}\{\phi_1(t-\tau) - \phi_2(t-\tau)\}\right] \\
&\approx v\left[x_{max}\left\{\phi_1(t-\tau) - (\phi_2(t) - \tau\phi_2'(t) + \tau^2\phi_2'')\right\}\right]
\end{aligned}$$

4. change variables $\xi(t) = \phi_2'$ to obtain a set of two 1st order ODEs

$$\xi = v x_{max}\left\{\phi_1(t-\tau) - (\phi_2 - \tau\xi + \tau^2\xi')\right\}$$

so that we have 2 equations,

$$\begin{aligned}
\phi_2' &= \xi(t) \\
\xi' &= \phi_1(t-\tau)/\tau^2 - \phi_2/\tau^2 + \xi(1/\tau - 1/(v x_{max}\tau^2))
\end{aligned}$$

5. integrate using same specified scenario for the first car and same model coefficients as used in traffic.m on the course web page and compare the solutions.

6. vary both the delay τ and the mean velocity v of the cars and solve for the perturbation displacement $phi_2(t)$ using both the delay equation and the approximate Taylor solution. Based on your calculations, plot a line in the $\tau - v$ plane that separate stable traffic from unstable traffic. Interpret your results.

7. time permitting: add another term in Taylor expansion, is accuracy relative to solution of delay equation improved?

8. time permitting: linearized stability analysis of ODEs by solving for eigenvalues of linearized matrix operator.