

1. A firm receives an order for Q units of product to be delivered at time T . It seeks a production schedule to fill this order that minimizes the (un)discounted sum of production costs and inventory costs. Suppose the $u(t)$ denote production rate at time t , and suppose that the cost of this is $c_1(u(t))$. Let $x(t)$ be the stock produced by date t , suppose this stock incurs a storage cost of $c_2(x(t))$.
 - a) What is the Hamiltonian for this problem?
 - b) What are nice but simple functional forms for the costs, given that we want c_1 to be convex (due to decreasing returns to scale). (we want them to make $c_1(u) = \alpha u^2$ and $c_2(x) = \beta x$)
 - c) Solve the problem assuming that the non-negativity constraint on u is slack. When (for what parameter values) is this condition satisfied?
 - d) Make an intuitive/qualitative guess about what the solution looks like when the non-negativity condition on u isn't slack.
 - e) Michael tells them that the solution is delayed start-up and that after the start up date the non-negativity constraint is slack.
 - f) They are asked to solve this problem, including the determination of the start-up date- if that is too hard he could walk them through it.