Workshop #3, outline for TF, not students
APM 115: mathematical modeling

Climate, hysteresis, multiple equilibria

Model the Thermohaline circulation using a 3 box model, similar to the Stommel two-box model discussed in class. The boxes represent the equatorial ocean (box 1, 0-30N in the North Atlantic), the mid-latitude ocean (box 2, 30-60N) and the polar ocean (box 3, 60-90N). Students may start from the Matlab program Stommel2box.m for the Stommel box model. The program Stommel3box.m contains the solution.

1. Write the equations for the salinity for all three boxes, including a transport term between boxes 1 and 2 and between boxes 2 and 3.

2. Integrate the equations using ode45, slowly varying the fresh water forcing to find a hysteresis behavior. Plot it using some appropriate variables (q12, q23, S2, etc) as function of the fresh water forcing.

3. Starting from the hysteresis plot of, say the THC between boxes 2 and 3 as function of the fresh water forcing, sketch an approximate bifurcation diagram for this model that includes a guess of where the unstable solution branch is. Do this first for the 2 box model.

4. Explain which bifurcations are seen and how they result in the observed hysteresis behavior (saddle nodes).

5. **Optional, time permitting:** Solve for the steady state (fixed point) solutions using Matlab's fsolve. Draw the bifurcation diagram for q12 and q23 based on this solution. Also, integrate the equations using ode45 for a few values of the FW forcing to find which of the calculated equilibria states are stable and which unstable.