Workshop #2
APM 115: mathematical modeling

Equilibrium Stability for Two Dimensional Dynamic Systems

1. In this workshop, we are going to analyze the stability of a system dynamical model of love affairs. After we fully characterize and solve the linear model, you are going to develop a new non-linear model of love affairs by improving on the existing model.

   Modeling Romeo's love for Juliet

   $R =$ Romeo's love for Juliet $J =$ Juliet's love for Romeo (NOTE: positive means love, negative means hate)

$$dR(t)/dt = aR + bJ \quad (1)$$
$$dJ(t)/dt = cR + dJ \quad (2)$$

   a, b, c, and d are proportionality constants that can be either positive or negative (or zero).

   (a) Find the fixed points and analyze the stability of those fixed points.

   (b) Write a matlab code that uses ode45 to solve the system with for a variety of values of a, b, c, & d. The relative signs of a, b, c, & d dictate what kind of lovers you are modeling.

      i. As an initial example, consider $b > 0$, $c < 0$, and $a = d = 0$. What kind of lovers are we modeling?

      ii. Plot the phase plane of Romio's love and Juliet's love

      iii. Change the parameter values to model a pair of "Cautious Lovers."

   (c) Now, improve the model of some particular pair of lovers by making it nonlinear.

      i. Find the fixed points and stability of the new non-linear model

      ii. Solve the new model with ode45.

      iii. FOR TF: provide the following verbal description and let the students find the equation: Romeo is excited by Juliet's love (or hate) and reciprocates. Juliet is more complicated. On the one hand she is turned off by Romeo's love. However, she is excited by her own love to Romeo, as long as his feelings (love or hate) for her are not too strong. If his feelings are getting stronger (again, whether love or hate) she cannot handle the emotional stress and feels scared of her own feelings. ah...

$$dR/dt = J \quad (3)$$
$$dJ/dt = -a(R^2 - 1)J - R \quad (4)$$

   (d) Write a script to use the matlab function quiver to plot the stability of the fixed points of the new model. Integrate the equations using ode45.