Homework #9
APM 111: Introduction to numerical methods
due May 2, 2006

1. Consider the affair described by

$$dR/dt = J, \qquad dJ/dt = -R + J$$

   (a) characterize the romantic styles of Romeo and Juliet,
   (b) plot the phase plane picture using `quiver` in Matlab (you can use as an example `love_affairs.m` from the supporting material directory).
   (c) describe the evolution of the relationship starting from $R(t=0)=1$, $J(t=0)=0$. **Optional:** solve these equations using some ODE integration scheme that we covered in class.

2. **An optional challenge problem:** add nonlinear terms such as $-\varepsilon R^2$ to the R equation and $-\varepsilon J^2$ to the J equation. Explain what these mean (in terms of the romantic styles of Romeo and Juliet), plot the resulting phase space picture (quiver plot in the R,J plane) and interpret your results.

3. Eigenvectors as boundaries in phase space. Show that eigenvectors are an invariant manifold: given a system of ODEs such as

$$\frac{d}{dt}\begin{pmatrix} R \\ J \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} R \\ J \end{pmatrix}$$

   if a solution starts on an eigenvector of the matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

   it will remain on it for all times. Assume the eigenvalue/ eigenvector are both real. What is the physical (or should we say romantic) significance of the eigenvalue and of its sign?

4. Prove that a similarity transformation preserves eigenvalues. That is, if λ is an eigenvalue of a matrix A with eigenvector x, and $B = T^{-1}AT$, then λ is an eigenvalue of B. What is the eigenvector of B which corresponds to the eigenvalue λ?

5. Consider the matrix:

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \\ -1 & 3 \end{pmatrix}$$

   (a) calculate analytically (as much as you can) the singular values and right and left singular vectors of A
   (b) demonstrate the full and economic singular value decompositions explicitly using your solution for the singular vectors and singular values.
   (c) repeat the previous two sections of this question for the transpose of the matrix A.